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By J. ROBERTSON,

LIBRARIAN to the Royal Society,

Late Master of the Royal Mathematical School Christ's Hospital, London;
and afterwards, for more than Eleven Years,

Head-Master of the Royal Academy, at Portsmouth.

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CHURCH HISTORY

OF THE METHODIST CHURCH

IN THE UNITED STATES

AND TERRITORIES

BY

JOHN A. ANDERSON, D.D.

NEW YORK: 1894

414

TO THE

RIGHT WORSHIPFUL

Sir ROBERT LADBROKE, Knt. Alderman,

PRESIDENT;

THE

Worshipful THOMAS BURFOOT, Esq,

TREASURER;

And the rest of the

WORSHIPFUL GOVERNORS

OF

Christ's Hospital, London:

This Book, containing the Elements of Navigation, and a Treatise on Marine Fortification, first published for the Use of the Children in the Royal Mathematical School, when they were under my Care, is, as a grateful acknowledgement for past favours, addressed by

Your WORSHIPS most humble Servant,

John Robertson.

THE

P R E F A C E.

IT having been part of my employment for many years past, to instruct youth in the theoretical and practical parts of Navigation; I was naturally led to draw up rules and examples fitted to the years and capacity of the scholar: some of the precepts, from time to time were altered, according as I had observed how they were comprehended by the majority of my pupils; until at length I had put together a set of materials, which I found sufficient for teaching this Art.

Upon my being intrusted by the governors of Christ's Hospital (in the beginning of the year 1748) with the care of the Royal Mathematical School there, founded by King Charles the second, I had a great opportunity of experiencing the method I had before used; and finding it fully answered my expectation, I determined to print it for the use of that school: but as those children are to be instructed in the mathematical sciences, on which the art of Navigation is founded, I judged it proper, on their account, to introduce the subjects of Arithmetic, Geometry, Trigonometry, &c. for which reason, this treatise is distinguished by the title of Elements of Navigation.

After my appointment (in the year 1755) to be head master of the Royal Marine Academy at Portsmouth, founded by King George the second, I also found that this book was sufficiently intelligible to beginners of middling capacities; and therefore, in the second edition, in the year 1764, the manner in which it was first composed was continued, except the removing of the book of Astronomy, from being the 8th. into the place of the 5th.; where by the books of Plane Sailing, Globular Sailing, and Days Works, which together nearly comprehend the art of Navigation, follow in succession: There were indeed some variations in the modes of expression in a few places; but the additions in every book were made rather to extend the notions of learners, than to supply any deficiency wanting in the former edition; except some of the additions in the 9th. book, which were not so well known at the time of the first impression.

The work is divided into ten parts or books, each being a distinct treatise; the preceding ones contain the necessary elements which are wanted in those that follow. The demonstration of the several propositions are given as concisely as I could contrive, to carry with them a sufficient degree of evidence: I brought about the whole of the elementary parts, brevity and perspicuity were considered; but the practical parts are more fully treated on, and intended to include every useful particular, worthy of the mariner's notice.

In the elementary parts, where it is not easy to introduce new matter, there will be found the common principles treated in a manner, which, it is apprehended, is better adapted to beginners; and such new lights thrown on several particulars, as will render them more obvious than in the view wherein they have been commonly seen.

The treatise of Fortification annexed, is the result of many years application; and is delivered in a very different mode from what other writers have taken; for among the multitude which I have seen, they generally begin with the fortifying of a town, the most difficult part of the art, and end with works the most easy to contrive and execute: Herein the works of the simplest construction are begun with, and the learner gradually advanced to the fortifying of a town: Indeed the limits chosen for this tract have caused some articles to be briefly mentioned, and others to be totally omitted; nevertheless, it is conceived that, in its present state, it may be of considerable use to Marine Officers, and even furnish some hints not altogether unworthy the notice of the Gentlemen of the Army.

The Maritime parts of these Elements, contained in the viiith and ixth books, are also delivered in a manner somewhat different from what is seen in other writers; who, for the most part copying from one another, have not much contributed towards perfecting the art of Navigation; the writers indeed have been many, but the improvers have been very few; Wright, Norwood, and Halley, having done the most of what has been discovered since a little before the beginning of the 17th century: however in the method here taken, it is apprehended that, the proper judges will find some few improvements, as well in the art itself as in the manner of communicating it to learners.

The common treatises of Navigation, which on account of their small bulk and easy price, are vended among the British Mariners, seem not to be wrote with an intention to excite in their readers a desire

desire to pursue the Sciences, farther than they are handled in those books; so that it is no wonder our Seamen in general had so little mathematical knowledge; for the person who could keep a true journal, formed on the most easy occurrences, has been reckoned a good artist; but whenever those occurrences have not happened, the journalist has been at a loss, and unable to find the ship's place with any tolerable degree of precision; and such accidents have probably contributed to the distress which many ships crews have experienced, and which a little more knowledge among them might have prevented, or at least have lessened.

About the middle of the 16th century, Navigation began to be considered as an art in a great measure dependent on the Mathematical Sciences; and on such a plan has it been cultivated by the labours of the most judicious, who have applied themselves towards its perfection; and although the art has been enriched by the observations of some learned men in different nations, yet it has so happened, that the chief of the improvements, and particularly the mathematical ones*, were first published in Britain.

In this work are collected most, if not all, of the useful and curious particulars relating to the art of Navigation; there are also interspersed historical remarks of inventions, with the names of many eminent men, and their works; these were intended as incentives to inspire learners and our Seamen with a desire not only of knowing the things herein treated of from their foundations, but of pushing their inquiries into such other parts of the sciences as may procure to themselves pleasure, profit and respect, and render them more useful to their country by the skill resulting from such acquisitions.

I have always thought that the chief motives which ought to induce a person to appear as a writer should be, either that he has something new to publish, or that he has arranged the parts of a known subject, in a method more regular and useful than had been done before; in either of these cases he cannot be a proper judge, unless he has seen the pieces extant on that subject, or at least, those of the most eminent authors already published: On these principles I was led to examine what had been done by the different writers on Navigation; and having perused most of their books, of which I

* See the following dissertation.

could get information, I had an opportunity of discovering the steps by which this art has risen to its present perfection, and consequently of knowing the most material parts of the history of its progress; among other things, I could not avoid remarking a mistake which has crept into many of the modern books of Navigation; which is, that Wright's invention of making a true sea-chart was stolen by Mercator, and published as his own. I suspect this story had its rise in a book printed in the year 1675 by Edward Sherburne, intitled "*The Sphere of Marcus Manilius made an English Poem*," with Annotations and an Astronomical Appendix."

My enquiries into these matters induced the late learned Dr. James Wilson to review and complete his observations on the same subject, and produced his Dissertation on the History of the Art of Navigation; which he was pleased to give me leave to publish with the second Edition of this work.

There are few persons, howsoever knowing and careful, but may commit, and overlook, inadvertences in their own compositions, which may be discovered by others: therefore at my request the greatest part of the manuscript for the first edition was read and examined by two of my friends*, well acquainted with the theory and practice of Navigation; who, by their judicious observations, enabled me to improve several articles: Some parts of the additions to the 2d Edition, received much elegance and perspicuity through the friendly advice and communications of the late learned Dr. Henry Pemberton, F. R. S.

The second Edition of these Elements having also been well received by the Public; Dr. Wilson took the pains to revise his Dissertation, which he improved in many particulars: And I have also endeavoured to retain their favourable opinions of my labours, by giving Compendiums for performing the operations of the new methods of finding the latitude and longitude of a ship at sea; and some other alterations and additions which I conceived would render this third Edition more generally useful.

Nov. 1, 1772.

* William Mountaine, Esq. F. R. S. and Mr. William Payne.

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THIS treatise in two volumes contains ten books; each is divided into several sections, and numbered with the Roman numerals.

The particular articles are numbered by the common figures, each book beginning with the number I.

The references made from one article of the treatise to another is of two kinds.

First. When in the same book. Then the number of the article referred to, is put in a parenthesis. Thus (27), refers to the article numbered 27 in the same book.

Secondly. When in another book. Then the number of the book, in Roman figures, and of the article in common figures, is put in a parenthesis. Thus (II. 160) refers to the 160th article of the second book.

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ERRATA.

The following oversights, mostly typographical, the reader is desired to correct with his pen, and excuse others which may have escaped the Author.

In the DISSERTATION.

Page	Line	Read	Page	Line	Read
i	last but 5	Histoire	xv	last but 5	en est
ii	18	Precession	xviii	last but 3	on est
iv	last but 9	Magnete	xxiv	24	report
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	28	matters	xxviii	last but 9	Le Neptune
					Regie.

In VOLUME I.

Page	Line	Read	Page	Line	Read
33	7	Unity	191	last but 4	Arc
44	20	two right lines	278	20	1 X 10
92	23	to the	365	last but 10	11 X 10
131	last but 2	of no other	398	12	Alexandrian
145	10	known			June 3, 1769

In VOLUME II.

Page	Line	Read	Page	Line	Read
179	In the Trav. Tab. under lat. 33° 30'	313	11	This alters the work and makes lat. 50	hour $\angle$ 54° 21'
	against dist.				6h.
304	64, 65, 66, 67	357	In the three last		14h.
	In Ex. vii. and viii. for 7h.	416	lines but two		7h.
			1		94448 to 97313

FORTIFICATION.

Page 25, line 32. Read, but is better.

DISSERTATION

ON THE

RISE AND PROGRESS

OF THE

Modern ART OF NAVIGATION.

IT has been much disputed, to whom the world was obliged for the mariner's compass. A late *Italian* writer indeed contends, after many *, that the honour of the invention is due to *Flavio Gioja* of *Amalfi* in *Compania*, who lived about the beginning of the 14th century †, though others say it came from the east, and was earlier known in *Europe* ‡. However: that may be, it is certain, this wonderful discovery gave rise to the present art of navigation; which seems to have made some progress during the voyages, that were begun in the year 1420, by *Henry Duke of Visé* §. This learned Prince, brother to *Edward King of Portugal*, was particularly knowing in cosmography, and sent for one master *James* from the island of *Majorca*, to teach navigation, and to make instruments and charts for the sea ¶.

These voyages being greatly extended, the art was improved under the succeeding monarchs of that nation. For *Roderic* and *Joseph*, physicians to King *John* the Second, together with one *Martin de Bohemia* a *Portuguese* native of the island of *Fayal*, scholar to *Regiomontanus* about the year 1485, calculated for the use of the sailors tables of the Sun's declination, and recommended the astrolabe for taking observations at sea ¶.

The famous *Christopher Columbus* is said, before he attempted the discovery of *America*, to have consulted *Martin de Bohemia*, with others, and during the course of his voyage to have instructed the *Spaniards* in

* Suitable to that verse of *Pagnormilana*,

Prima dedit nautis usum mogueri Amalphis.

† See Signor *Grigorio Grimaldi*'s Dissertation on this subject in the *Memoirs* of the *Etruscan Academy* of *Cortena*, Tom. iii. p. 193, printed at *Rome* in 1742.

‡ *Histoire des Mathematiques*, par *M. Montucla*, à *Paris*, 1758.

§ *Marianæ Hist. Hispan.* lib. xx. cap. 11. and lib. xxvi. cap. 17. *Moguntia*, 1605.

¶ *Decados d'Asia* par *J. di. Burnos*, lib. xvi. 1552.

¶ *Messin Histor. Indic.* lib. i. p. 6. printed at *Florence* in 1588.

navigation;

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navigation*; for the improvement of which art, the Emperor Charles the Fifth afterwards founded a lecture at *Seville*†.

The variation of the sea-compass could not be long a secret. *Columbus*, on the 14th of September 1492, observed it, as his son *Ferdinand* asserts, though others seem to attribute that discovery to *Sebastian Cabot*||. And as this variation differs in different places, *Gonzales d' Ovied* found there was none at the *Azores*§; where some geographers have thought fit in their maps to make their first meridian to pass through one of those islands; it not being then known, that the variation altered in time.

The use of the *Cross-Staff* now began to be introduced amongst the Sailors. This very ancient instrument being described by *John Werner* of *Nuremberg*, in his *Annotations* on the first book of *Ptolemy's Geography*, printed in 1514, he recommends it for observing the distance between the Moon and some star, in order thence to determine the longitude. *Werner* seems to have been the greatest geometer as well as astronomer of the time. In 1522, he published a tract¶, containing a specimen of the conics, with some solid problems, and also he there determined the precession of the equinox more exactly than it had been done.

But the art of navigation still remained very imperfect, from the constant use of the plane-chart, whose gross errors must have often misled the mariner, especially in voyages far distant from the equator. Its precepts were probably at first only set down on the earliest sea-charts, as that custom is continued to this day; and larger directions have been usually premised by the *Dutch*, to collections of their charts called *Wagoner's*, from the name of the publisher; also by many affected titles, such as *Fiery-Columnis*, *Sea-Beacons*, *Mirrors*, *Atlases*, &c.

At length there were published in *Spanish* two treatises, containing a system of the art, which had a great vogue; the first by *Pedro de Medina* at *Valladolid*, in 1545, called *Arte de Nauagar*; the other at *Seville*, in 1556 by *Martin Cortes*, with this title, *Breve Compendio de la Sphera, y de la Arte de Nauagar con nuevos Instrumentos y Reglas*. The author of this last tract says, he composed it at *Cadix* in 1545.

These seem to have been the oldest writers, who had fully handled this subject; for *Medina*, in his dedication to *Philip Prince of Spain*,

* La Historia general y natural de las Indias par Gonzalles de Miedo, en Sevilla, 1535. And Descriptione de las Indias Occidentale, de Antonio de Herrera, en Madrid, 1601.

† Hakluyt, in the dedication of his first volume of Voyages, printed in 1599. ‡ In Columbus's life written in *Spanish*, which is very scarce, but it was printed in *Italian* at *Venice* in 1571.

§ See *Lisvao Sanuto's Geographia*, at the same place in 1585, Dr. *William Gilbert*, de *Magneta*, London, 1600; and *Purchas's Pilgrim*, in 1625, vol. I. § *Cabot*, a *Venetian* by birth, first served our King *Henry* the seventh, then the King of *Spain*, and lastly returning to *England*, he was constituted grand pilot by King *Edward* the Sixth, with an annual salary of above 100 pounds. Of this famous navigator and his expeditions, many writers have made mention both foreigners and *English*, as *Peter Martyr*, *Ramusio*, *Herrera*, *Holinshed*, *Lord Bacon*, and particularly *Hackluyt* and *Purchas*, in their Collections of Voyages.

¶ *Opera Mathematica* at *Nuremberg*, in quarto.

laments,

laments, that multitudes of ships daily perished at sea; because there were neither teachers of the art, nor books whereby it might be learnt; and *Cortes*, in his dedication, boasts to the Emperor, that he was the first who had reduced navigation into a *compendium*, enlarging much on what he had performed*.

Medina gave ridiculous directions, how to guess at the place of the horizon, when it could not be seen; as also he defended the errors of the plane-chart, and advanced against the variation of the magnetic needle such absurd arguments, as *Aristotle* and his followers had done to prove the impossibility of the Earth's motion. But *Cortes* briefly and clearly made out the errors of the plane-chart, and seemed to reflect on what had been said against the variation of the compass, when he advised the mariner rather to be guided by experience, than to mind subtle reasonings. Besides he endeavoured to account for this variation, in imagining the needle to be influenced by a magnetic pole (which he called the point attractive) different from that of the world, and this notion has been farther prosecuted by others.

However, *Medina's* book, being perhaps the first of its kind, was soon translated into *Italian*, *French*, and *Flemish* †, serving for a long time as a guide to the navigators of foreign countries.

But *Cortes* was our favourite author, a translation of whose work by Mr. *Richard Eden* was, on the recommendation of that great navigator Mr. *Steven Burrough*, and the encouragement of the Society for making discoveries at sea, published at *London* in 1561: which underwent various impressions ‡, whilst that of *Medina*, though translated within twenty years after the other, seems to have been neglected, notwithstanding the encomiums bestowed on it by Mr. *John Frampton*, the translator.

A system of navigation at that time consisted of some such particulars as these. An account of the *Ptolemaic* hypothesis, and the circles of the sphere; of the roundness of the Earth, its longitudes, latitudes, climates, and eclipses of the luminaries; a kalendar; how to find the prime, equinox, &c. and by the last the Moon's age, and thence the tides; a description of the sea-compass, not forgetting the load-stone, with something about the variation, called its north-easting and north-westing for the discovery of which by night as well as by day *Cortes* said, might be easily contrived an instrument; tables of the Sun's declination for four years §,

* The learned Don *Nicolo Antonio*, in his *Biblioteca Hispanica*, printed at *Rome* in 1672, Tom. i. p. 323. puts down a book, intitled, *Tratado de la Sphera y del marcar con el regimiento de las alturas*, written by *Francisco Falero*, a *Portuguese*, and printed at *Seville* in 1535; but perhaps there is a mistake in the date. He also mentions an edition of *Cortes* in 1551.

† The *Italian* and *French* translations were printed in 1554, the first at *Venice*, the other at *Lyons*; the *Flemish* edition, I have seen, was at *Antwerp* in 1580; perhaps it had been printed before.

‡ In the latter editions some mistakes in the translation are corrected.

§ *Cortes* sets down the places of the Sun for a twelvemonth, with an equatorial-table to correct those places, serving for many years to come; and also another table to find the Sun's declination from his longitude being given.

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in order to find the latitude, from his meridian altitude; to do the same thing by those called the guard-stars in the north, and the crossers in the south; of the course of the Sun and Moon; the length of the days; of time and its divisions; to find the hour of the day, and by the nocturnal that of the night; and lastly, a description of the sea-chart, on which to discover where the ship is, they made use of a small table, that shewed upon an alteration of one degree in the latitude, how many leagues were run on each rumb, together with the departure from the meridian. Besides some instruments were described, especially by *Cortes*; as one to find the place and declination of the Sun, with the days and place of the Moon; certain dials, the astrolabe and cross-staff, with a complex machine to discover the hour and latitude at once.

And after this manner the art continued to be treated, though from time to time were made improvements by the following authors.

As *Werner* had proposed to find the longitude by observations on the Moon; so *Gemma Frisius*, in a tract intitled *De Principiis Astronomiæ et Cosmographiæ*, printed at *Antwerp* in 1530, advised for the same purpose the keeping the time by the means of small clocks or watches then, as he says, lately invented. He also contrived a new sort of cross-staff, which he describes in his treatise *De Radio Astronomico et Geometrico* printed at the same place 1545, and in his additions to *Peter Apian's* Cosmography, gives the figure of an instrument, he calls a *Nautical Quadrant*, as very useful in navigation, promising to write largely on the subject; accordingly, in an edition he made anno 1553, of his above mentioned book *de Principiis Astronomiæ*, &c. he delivers several *nautical axioms*, as he calls them, which with some alterations were repeated by his son *Cornelius Gemma*, in a posthumous piece of his father on the *Universal Astrolabe*, published in 1556. *Gemma Frisius* died in 1555, aged 45 years.

With us *Dr. William Cunningham*, in his *Cosmographical Glass*, printed in 1559, amongst other things briefly treats of navigating, especially shewing the use of the *Nautical Quadrant*, much praising that instrument.

But a greater genius than these undertook this subject; for the famous mathematician *Pedro Nunez*, or *Nonius*, having so early as 1537 published a book written in the *Portuguese* language, to explain a difficulty in navigation proposed to him by the commander *Don Martin Alphonso de Sosa*; which was thirty years after printed at *Basil*, in *Latin*, with the addition of a second book, the whole intitled *de Arte et Ratione Navigandi*; where he exposes both truly and learnedly the errors of the plane-chart; and besides gives the solution of several curious Astronomical Problems, amongst which is that of determining the latitude from two observations of the Sun's altitude and the intermediate azimuth being given. He also delivers many useful advices about the art of navigation, particularly how to perform its operations on the globe. He observed, that though the oblique rhumbs are spiral lines, yet the direct course of a ship will always be the arch of some great circle, whereby the angle with the meridians will continually change; all that the steersman can here do for the preserving of the original rhumb is to correct these deviations, as soon as they appear sensible. But thus the ship will in reality describe a course without the rhumb-line intended; and therefore his calculations for assigning the latitude, where any rhumb line crosses the several meridians,

ridians, will be in some measure erroneous. He also again sets down his method of division of a quadrant by concentric circles*, which he had described in his ingenious treatise *de Crepusculis*, printed in 1542, imagining it had been practised by *Ptolemy*. There were also added other tracts of his, but the completest edition of his *Latin* works was made by himself at *Coimbra*, in 1573. His treatise of *Algebra*, written in *Spanish*, was printed at *Antwerp* six years before.

In 1577 *Mr. William Bourne* published his treatise †, intitled, *A Regiment for the Sea*, which he designed as a supplement to *Cortes*, whom he frequently quotes. Besides many things common with others, *Bourne* gives a table of the places and declinations of thirty-two principal stars, in order to find the latitude and hour; as also a larger tide-table than that published by *Mr. Leonard Digges*, in 1556 ‡. He shews, by considering the irregularities in the Moon's motion, the errors of the sailors in finding her age by the epact; and also in their determining the hour from observing upon what point of the compass the Sun and Moon appeared. He advises in sailing towards high latitudes to keep the reckoning by the globe, as there the plane-chart errs most. He despairs of our ever being able to find the longitude by any instrument, unless the variation of the compass should be caused by some such attractive point, as *Cortes* had imagined. Though of this he doubts, and as he had shewn how to find the variation of the compass at all times he advises to keep an account of the observations, as useful to discover thereby the place of a ship; which advice the famous *Simon Stevin* prosecuted at large in a treatise published at *Leyden* in 1599 intitled *Portuum investigandorum Ratio Metaphraso Hugone Grotio*; the substance of which was the same year printed at *London*, in *English*, by *Mr. Edward Wright*, intitled *The Haven-finding Art*.

But the most remarkable thing in this ancient tract is, the describing the way our sailors estimated the rate a ship made in her course, by an instrument called the *log*. This was so named from the piece of wood, or log, that floats in the water, while the time is reckoned during the line fastened to it is veering out. The author of this device is not known, and I find no farther mention of it till 1607, in an *East-India* voyage, published by *Purchas*; but from that time its name occurs in other voyages, that are amongst his collections. And henceforward it became famous, being taken notice of, both by our own authors, and by foreigners; as by *Gunter* in 1623, *Snellius* in 1624, *Metus* in 1631, *Oughtrid* in 1633, *Herigone* in 1634, *Saltonstall* in 1636, *Norwood* in 1637, *Furnier* in 1643; and indeed by almost all the succeeding writers on navigation, of every country. And it continues to be still in use as at first, though attempts have been often made to improve it, and other contrivances pro-

* The admirable division now so much in use is a very great improvement of this; so that when the famous *Dr. Edmund Halley*, the Royal Astronomer, revived that by adapting it to his *Mural Arch*, some body here named it *A Nonius*, in which he has been followed by many.

† He had ten years before published what he calls *Rules of Navigation*, as appears from his *Almanac*, printed in 1571.

‡ In his treatise intitled, *A Prognostication everlasting*, fol. 23.

posed to supply its place. Many of these have succeeded in quiet water, but proved useless in a troubled Sea.

A following edition of this book was revised by the author, where, in the preface, he sets forth the gross ignorance of the old ship masters, repeating some of the insipid jests they made use of to justify their want of knowledge in their art. Amongst the additions, he enlarges on the account of the log-line. And at the end subjoins an *Hydrographical Discourse touching the five several Passages into Cathay*.

Bourne published other tracts, as one called *Inventions or Devices*, where he describes a method by *wheel-work* of measuring the velocity of a ship at Sea, which artifice he attributes to one Mr. *Humfrey Cole*.

At *Antwerp*, in 1581, *Michael Coignet*, a native of the place, published a small treatise intitled, *Instruction nouvelle des Points plus excellents & necessaires touchant l'Art de Naviger* *. This served as a supplement to *Médina*, whose mistakes *Coignet* well exposed. He there shewed, that as the rhumbs are spirals, making endless revolutions about the poles, numerous errors must arise from their being represented by straight lines on the sea-charts; and expressed his hopes of discovering a rule to remedy those errors; saying, that most of the speculations, delivered by the great mathematician, *Peter Nonius*, for that purpose, were scarce practicable; and therefore in a manner useless to sailors. In treating of the Sun's declination, he took notice of the gradual decrease in the obliquity of the ecliptic, a point long disputed, but now settled from the theory of attraction. He also described the cross-staff with three transverse pieces, as it is at present made, which he acknowledged to be then in common use amongst the mariners; but he preferred that of *Gemma Frisius*. He likewise gave some instruments of his own invention, which are now quite laid aside, except perhaps his nocturnal. As the old sea-table, mentioned above, erred more and more in advancing towards the poles; he set down another to be used by such as sailed beyond the 60th degree of latitude. At the end of the book is delivered a method of sailing on a parallel of latitude by means of a ring dial, and a 24 hour glass; on which the author very much values himself.

The same year Mr. *Robert Norman* † published a discovery, he had long before made, of the dipping of the magnetic needle, in a small pamphlet, called *The Neue Attractiue*, where he shews how to determine its quantity; and in speaking of the loadstone, he disputes against *Carter's* notion, that the variation of the compass was caused by a point fixed in the heavens, contending that it should be sought for in the earth, and proposes how to discover its place. He also treats of the various sorts of compasses, setting forth at large the dangers that must arise from the then prevailing practice of not fixing, on account of the variation, the wire directly under the *flower-de-luce*; as compasses made in different

* It had been published in *Flemish*, but the *French* edition is the fullest. *Coignet* died in 1623, leaving many mathematical manuscripts, see *Valerii Andree Bibliotheca Belgica*, printed at *Louvain* in 1643.

† He is commended for an excellent Artift by our authors, as *Bourne*, *Burroughs*, *Sir Humfrey Gilbert*, *Huet*, *Potter*, *Blundeville*, *Wright*, and *Dr. Gilbert*. countries

countries have it placed differently. *Bourne* indeed had warned against this abuse, and there are many things common to both authors.

To *Norman's* piece is always subjoined, *Mr. William Burrough's Discourse of the Variation of the Compass or Magnetick Needle*. The author had been a famous navigator, having used the sea from fifteen years of age, and for his merit promoted to be Controller of the navy by *Queen Elizabeth* *. He shews how to determine the variation several ways, setting down many observations of it made by an azimuth-compass of *Norman's* invention, but improved by himself. He demonstrates the falshood of the rules commonly used, to find the latitude by the guard-stars. He particularizes many errors in the then sea-charts, occasioned by the neglect of the variation; adding, *But of these coastes* (towards the north), *and of the inward partes of the countries of Russia, Muscovia, &c. I have made a perfect plot and description, by myne owne experience in sundrie voyages and travailes, bothe by sea and lande to and fro in those partes, which I gave to her Majestie in anno 1578*. And lastly, he justly finds fault with *Coignet's* instrument, called a nautical hemisphere; but speaks too severely against the writers of navigation, concluding thus,

But as I have already sufficientlie declared, the compass sheweth not alwaies the pole of the worlde, but varieth from the same diversly, and in saying describeth circles accordingly. Whiche thing, if Petrus Nonius, and the rest that have written of Navigation, had jointlie considered in the tractation of their rules and Instruments, then might they have been more availeable to the use of Navigation; but they perceiving the difficultie of the thing, and that if they had dealt therewith, it would have utterly overwhelmed their former plausible conceits, with Pedro de Medina (who, as it appeareth, having some small suspicion of the matter, reasoneth very clerkly, that it is not necessary that such an absurdity as the Variation, should be admitted in such an excellent art as Navigation is) they have all thought best to passe it over with silence.

The Spaniards too continued to publish treatises of the art, particularly at *Seville* was printed in 1585 an excellent Compendium by *Roderico Zamorano*; which is written clearly and with brevity, not being incumbered with such idle speculations as abound in *Medina* and *Cortes*. The author was Royal Lecturer at *Seville*, and contributed much to the reforming the sea-charts; as we are told by his successor, *Andres Garcia de Céspedes*, who also published a treatise of navigation at *Madrid*, in 1606.

As globes may be very serviceable for the mariner, *Mr. Edward Mulinæux* set forth, in 1592, at the charges of *Mr. William Sanderson* merchant, a pair much larger than those the famous geographer *Gerard Mercator* published in 1541. On the terrestrial one were described many new discovered countries, and traced out the respective voyages round the world by *Sir Francis Drake* in 1577, and *Mr. Thomas Candlish* in 1586, with the progress *Sir Martin Frobisher* had made towards the north in 1576 to a place called his *Straits*.

* *Hakluyt's Voyages*, Vol. i. p. 417. printed in 1599.

† *Mr. Sanderson* was commended for his knowledge as well as generosity to ingenious men.

These globes were accompanied with a tract containing their uses written in *English*; but in 1594 Mr. *Robert Hues* published a more elaborate one in *Latin*; wherein, amongst others, he solves by the globe the problem of determining the latitude from two heights of the Sun observed with the intermediate time being given*; and in the last part of his book, he performs the usual questions in navigation, premising a very sensible discourse on the rhumb-lines, where he refutes the error of *Gemma Frisius*, who had asserted, that they met in the poles. At the conclusion he highly praises a treatise of Mr. *Thomas Hariot*, hoping it would be soon published, in which that author had treated of this subject upon geometrical principles, with great sagacity and judgment. But all the manuscripts of that great mathematician were lost, except his *Artis Analyticæ Praxis*, which was published long after his death in 1631; wherein is first advanced that idea of algebraic equations, which has been ever since followed.

Hues† was a person of letters, and besides had been far at sea. Amongst other curious particulars, he gives a good account of the attempts that had been made at various times to measure the Earth. In the *Épître* to Sir *Walter Rawley* he takes an occasion to enumerate the many discoveries of our mariners in very different parts of the world. His book was received with great applause, and has been indeed a pattern for such as afterwards handled the same subject. It has been often printed abroad, particularly in 1617 with the notes of *John Isaac Pontanus*, who omitted the *Épître* and the mentioning of *Hariot*. However from this mutilated edition it was translated into *English* by one Mr. *John Chilmead* and published in 1639.

Amongst our sailors, none were more famous than Captain *John Davis*‡, who gave name to the straits he discovered; and great matters were expected from his long experience and skill. In 1594 he published a small treatise, intitled, *The Seaman's Secrets*. This is writ with brevity, though somewhat pedantically, and was esteemed in its time, an eighth edition being printed in 1657; so it seems to have supplanted *Cortés*. *Davis* treats of plane sailing, calling it *horizontal*, and sets down the form of keeping a reckoning at Sea. He likewise shews how to sail by the globe, and boasts of what he intended to do; much commending great circle sailing, without describing it, as also what, he calls *paradoxal*;

* This problem has been discussed by Dr. *Henry Pemberton*, in the *Philosophical Transactions*, Vol. ii. part 2d. 1760, p. 910, where he has also given some improvements in trigonometry.

† There is an account of *Hues* and *Hariot* in *Anthonj Wood's Athen. Oxon.* Vol. i. printed in 1721, as being both members of that University.

‡ Several of his voyages are in *Hackluyt's* and *Purchas's* collections. He and Captain *Abraham Kendall* are greatly praised by Sir *Robert Dudley*, in his *Arcano del Mare*, as keeping a perfect reckoning by the way of longitude and latitude, where are given two of their *Journals*. This *Dudley* was a natural son of the great Earl of *Leicester*, and had commanded, in 1594, a fleet against the *Spaniards*; but retiring to *Florence*, he assumed the titles of Duke of *Northumberland* and Earl of *Warwick*. His *Arcano* was printed at that place in two volumes, in 1646 and 1647.

that

that is, by a projection on the plane of the equator with spiral thumbs, saying, he will publish a chart for that purpose. But above all, he extols the use of calculations in the cases of navigation, and promises to handle that subject.

At the end of the book is given the figure of a staff of his contrivance, to make a back observation. Of this the author is so vain as to say, *Then which instrument (in my opinion) the seaman shall not find any so good, and in all climates of so great certainty, the invention and demonstration whereof, I may boldly challenge to appertain unto my selfe (as a portion of the talent which God hath bestowed upon me) I hope without abuse or offence to any.*

This instrument seems to have for some time been in use; for *Adrian Metius* in his treatise intitled *Astronomia Institutio*, printed in 1605, gives a figure of it from an original, in the possession of *M. Frederic Hautman*, governor of *Amboyna*. But it soon yielded to one of a more commodious form, which is now commonly called *Davis's Quadrant**; as if it was also of his invention, and that perhaps only because a back observation is made by both instruments, so the quadrant itself was at first styled a *Staff* and *Back-Staff*.

The famous traveller *Signor Pietro della Valle* passing, in 1623, from *Ormus* to *Surat* aboard an *English* vessel, where observing this quadrant much practised by the sea-men, as it was quite new to him, takes an occasion to shew its use very distinctly, and says, they told him, that it had been lately invented and called *David's Staff* † from its author. Also Captain *Charles Saltonstall*, in his *Navigation*, describes it under the name of a *Back-Staff*; and in Captain *Thomas James's* famous voyage for discovering a north-west passage, begun in 1631, amongst the many instruments, he carried along with him, are mentioned two of *Mr. Davis's Back-Staffs*, which were doubtless these quadrants.

Contemporary with *Davis* was *Mr. Richard Pelter*, who, it is said, had been a principal master aboard the *Royal Navy*. He wrote a very small book intitled *The Pathway to Perfect Sailing*, where, from an observation he made in 1586, he would infer †, that different loadstones communicated different degrees of variation to the magnetic needle, and therefore despises the publishing observations of that kind, as needless. His book was not printed till 1644, nor did it deserve to be published at all, as it abounds with mistakes, is written fantastically, obscurely, and arrogantly.

But all this while the plane chart, notwithstanding its errors were frequently complained of, continued to be followed; as its use is easy, and serves tolerably well in short voyages, especially near the equator.

* It is called by the *French* *Quartier Anglois*.

† *David Staff*, *cbe in lingua Inglesse wale à dir legnodi David Viaggi*; Part 3. Letter 1. à *Roma*. This author not only praises the Captain *Nicholas Woodcock*, and other officers, but also the common sailors, for their care and skill; and says, the *Portuguese* lose great number of ships for not being so exact in their observations as the *English*.

‡ Perhaps he should have thence concluded the variation altered, as was discovered afterwards.

However,

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However, a way to remedy these errors had, for some time, been inquired after. And *Gerrard Mercator* seems to be the first, who conceived the means of effecting this, in a manner convenient for seamen, by continuing to represent both the meridians and parallels of latitude by parallel straight lines, as in the plane-chart; but gradually augmenting the distances between the degrees of latitude in advancing from the equator towards either pole, that the rhumbs also might be extended into straight lines, so that a straight line drawn between any two places, laid down in this chart by their longitudes and latitudes, should make an angle with the meridians, expressing the rhumb leading from one to the other. But though *Mercator*, in 1569, set forth an universal map thus constructed *, it does not appear upon what principles he proceeded; probably, by observing in a globe furnished with rhumbs, what meridians the rhumbs passed at each degree of latitude. That he knew not the genuine principles, I shall make evident; our countryman, Mr. *Edward Wright*, was certainly the first who discovered them.

Wright infinuates, but without sufficient grounds, that this enlargement of the intervals between the parallels had been suggested before by *Cortes* †, and even by *Ptolemy* himself.

As to *Cortes*, he speaks of the number of the degrees of latitude, and not the extent of them: for his expression amounts to no more than this, that the degrees of latitude are to be numbered from the equator, and consequently both northwards and southwards from that line the numbers affix to them must continually increase; and from any place having latitude (suppose Cape *St. Vincent* in *Spain*, which is his instance) the degrees of latitude will be denoted by numbers increasing towards the pole, and decreasing towards the equator. He had before expressly directed, that they should be all equal by measurement on a scale of leagues adapted to the map ‡.

The passage in *Ptolemy* §, referred to by *Wright* §, does indeed relate to the proportion between the distances of the parallels and meridians, but contains no shadow of *Mercator*'s scheme: for instead of proposing any gradual enlargement of the distances of the parallels in a general chart; that passage relates only to particular maps, and is more distinctly explained in the first chapter of his last book; where he advises explicitly not to confine a system of such maps to one and the same scale, but to plan them out by a different measure, as occasion shall require, with this only caution, that the degrees of longitude should in each bear in some measure that proportion to the degrees of latitude, which the magnitude of the respective parallels bear to a great circle of the sphere; and subjoins, that in particular maps, if this proportion be observed in regard to the

* See his life, written by his intimate friend, *Gualterus Ghymini*, which was prefixed to an enlarged edition of his *Atlas*, published at *Duisburg*, in 1593, by *Rumoldus* his son, a year after his father's death. *Gerrard Mercator* was born in 1512.

† See the 2d chapter of *Wright*'s book.

‡ *Part* 3d. cap. 2d. fol. 58.

§ *Geograph.* lib. ii. cap. 1.

§ In an advertisement set down on his universal map, at the end of his second edition of his book; and in this mistake he has been followed by others middle

middle parallel, the inconvenience will be not great, though the meridians should be straight parallels to each other; wherein his design is plainly no other, than that the maps should in some sort represent the figures of the countries they are drawn for. *Mercator*, who drew maps for *Ptolemy's* tables*; understood him in no other sense, thinking it an improvement not to regulate the meridians by one parallel, but by two; one distant from the northern, and the other from the southern extremity of the map by a fourth part of the whole depth; whereby in his maps, though the meridians are straight lines, they are generally drawn inclining to each other towards the pole.

But *Mercator's* universal map, mentioned above, though the author designed it for the benefit of sailors, was so far from being readily adopted, that some of the most skilful amongst them objected to its usefulness. Thus *Mr. Burrough* says of it—*By augmenting his degrees of latitude towards the poles, the same is more fitte for suche to beholde, as studie in cosmographie, by readinge outbours upon the lands, then to bee used in Navigation at the sea.*

And *Mr. Thomas Blundeville*, in his *Briefe Description of Universal Mappes and Cardes*, first printed in 1589, gives an account of this map, observing that *Barnardus Puteanus* of *Bruges* had published, in 1579, one altogether like it. And though *Blundeville* is so particular, as to set down numbers expressing the distances between each parallel of latitude in those maps, yet he seems to slight them, by saying, that no better rules than those given by *Ptolemy* can be devised. But what is delivered by the *geographer* about the construction of a general map, is a very indifferent performance, altogether unworthy the author of the *Almagest*, and not in the least corresponding with the sagacity shewn in two treatises on the *Planisphere* and *Analemma*, which the *Arabians* have handed down to us as *Ptolemy's*†.

Maginus also, at the end of his *Geographia Universalis* (the former part of which is a translation of *Ptolemy's*) first printed at *Venice*, in 1596, mentions this map of *Mercator*, and gives even a sketch of it; but seems to have no distinct conception of the author's design.

That *Mercator's* map was not rightly described, is manifest from the numbers given by *Blundeville*; and that he was ignorant of a genuine method of dividing the meridian, appears from a passage in his life, where the writer says, *Mercator* often assured him, that this extending a sphere into a plane answered to the quadrature of the circle, as that nothing seemed to be wanting but the demonstration.

However, our authors now began to entertain favourable thoughts of it, perhaps from the report that *Mr. Wright* was about to treat on that subject, *Dr. Thomas Hood*, to his first edition he made, in 1592, of *Bourne's Regiment*, added a *Dialogue* of his own, called *The Mariner's Guide*, written only to shew the use of the plane chart, where he acknowledges and sets forth its errors, and highly praises *Mercator's*, saying, he had composed a treatise

* In an edition he made of *Ptolemy's Geography*, in 1584.

† These were published by *Fed. Commandinus*, one at *Venice*, in 1518, the other at *Rome*, in 1562.

concerning it; but the indistinct account he gives thereof, shews it would not be this author's lot to render it fit for the use of navigation. And Mr. *Blundeville*, in the following editions of his above-mentioned tract, omitted the commendation he had given of *Ptolemy's* method of delineating an *universal map*.

Mercator's scheme was not indeed contrived for representing the parts of a country in a just proportion to each other; but is appropriated to the use of mariners, who sail upon rhumbs by the guidance of the compass; which our countryman, Mr. *Edward Wright*, perfected*, by discovering a true way of dividing the meridian. An account of this he sent from *Caius college*, in *Cambridge*, where he was then a Fellow, to his friend the above-mentioned Mr. *Blundeville*, containing a short table for that purpose, with a specimen of a chart so divided, together with the manner of dividing it. All which *Blundeville* published, in 1594, amongst his *Exercises*, in that part which treats of navigation†; where he has well delivered what had been before written on that art; inasmuch that his book was long in great repute, a seventh edition having been printed in 1636. To the second edition, Anno 1606, and following ones, was added his former discourse of universal maps.

In 1597, the Reverend Mr. *William Barlowe*, in his *Navigator's Supply*, gave a demonstration of this division as communicated by a friend; saying, *This manner of carte has been publiquely extant in print these thirtie yeeres at least ‡, but a cloude (as it were) and thicke misse of ignorance doth keepe it hitherto concealed: And so much the more, because some who were reckoned for men of good knowledge, have by glauncing speeches (but never by any one reason of moment) gone about what they could to disgrace it.*

This book of *Barlowe's* contains descriptions of several instruments for the use of navigation, the principal of which is an *azimuth-compass*, with two upright sights §; and as the author was very curious in making experiments on the loadstone, he discourses well and largely on the sea-compass; and still farther handles that subject in a tract he published some years after, intitled, *Magnetical Advertisements*.

At length, in 1599, Mr. *Wright* himself printed his famous treatise, intitled, *The Correction of certain Errors in Navigation*, which had been written many years before; where he shews the reason of this division ||,

* Some of our modern writers have said, *Mercator* took the hint from *Wright*, but that is a mistake; for *Mercator's* map was published thirty years before *Wright's* book, who frequently refers to it. See *Edward Sherburne's* translation of the first book of *Manilius*, in 1675, p. 86.

† Chap. 29.

‡ He should have said 28 only.

§ Many of these instruments are in the *Arcano del Mars*, together with the demonstration above-mentioned.

|| Maps with their meridians thus divided had been published at *Amsterdam* by *Jodocus Hondius*, who when in *London*, working as an engraver, learnt the manner of doing it from Mr. *Wright's Manuscript*; the fourth chapter of which he had transcribed into one of his maps. *Hondius* afterwards in his letters, both to Mr. *Briggs*, and also to Mr. *Wright*, begged pardon for not having acknowledged.

the manner of constructing his table, and its uses in navigation, with other improvements: A book, as Dr. Halley says, *well deserving the perusal of all such as design to use the sea*.*

In the preface, *Wright* complains of the obstinacy of our mariners, for not liking an improvement in their art, saying, that they were like those whose ignorance Master *Bourne* had exposed, repeating *Bourne's* very words.

"Though this great improvement in navigation by *Wright* has been embraced and followed by all proper judges; yet some undiscerning persons have of late, even amongst us, found fault with it, as one *Henry Wilson*, by a proposal for a *curvilinear sea-chart*, in 1720; and one *Mr. West*, of *Exeter*, in a posthumous piece, printed in 1762. But their cavils were sufficiently obviated; those of the first by *Mr. Harsden*, in his *Mercator's Chart*, and in his *Reply*, both printed in 1722; and of the second, by *Mr. William Mountaine*, in the *Philosophical Transactions*, vol. LIII. p. 69. *Anno* 1763."

In 1610 was made a second edition, dedicated to Prince *Henry*, his royal pupil †, where the author inserted farther improvements; particularly, he proposed an excellent way of determining the magnitude of the Earth; at the same time recommending very judiciously the making our common measures in some settled proportion to that of a degree on its surface, that they might not depend on the uncertain length of a barley-corn.

Some of his other improvements were; The Table of Latitudes for dividing the meridian, computed to minutes; whereas before it was but to every tenth minute, and the short table sent by him to *Blundeville* to degrees only: An Instrument, he calls the Sea-rings, by which the variation of the compass, altitude of the Sun, and time of the day, may be determined readily at once in any place, provided the latitude be known: The correcting the errors arising from the excentricity of the eye in observing by the cross staff: A total amendment in the Tables of the declinations and places of the Sun and stars from his own observations, made with a six-foot quadrant, in the years 1594, 95, 96 and 97: A sea-quadrant, to take altitudes by a forward or backward observation, and likewise with a contrivance for the ready finding the latitude by the height of the pole-star, when not upon the meridian. And that his book might be the better understood by beginners, in this edition is subjoined a translation of the above-mentioned *Zamorano's Compendium*; he correcting some mistakes in the original, and adding

acknowledged the obligation. See *Wright's* preface, where he complains of *Hondius's* proceeding; and farther relates, how his book, a copy of which having been presented to the Earl of *Cumberland*, had liked to have come out under the name of a famous navigator, whom, from some circumstances there mentioned, I imagined to have been *Abram Kendall*.

* *Philosophical Transactions*, for 1696. N^o 219.

† In 1657 a 3d edition was published by *Mr. Joseph Moxon*, where the dedication is unadvisedly left out, and at the end is added by the editor the above-mentioned *Haven-finding Art*, as also *Wright's* universal map, improved by the discoveries made since his time.

a large

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a large table of the variation of the compass observed in very different parts of the world, to shew it is not occasioned by any magnetic pole.

This excellent person was allowed fifty pounds a year (no inconsiderable sum at that time of day) by the *East India Company*, for reading a lecture of navigation; he also projected the conveying water to *London*, but was prevented from executing his scheme by designing men, which is frequently the case. Whilst he led a studious and retired life, his reputation was so far known, that Queen *Elizabeth* granted in 1589, a *dispensation* for his absence from the university, in order to accompany the Earl of *Cumberland* in the expedition to the *Azores*; as I am informed by Sir *James Burrough*, Master of *Caius College*, whose fine taste in architecture, part of the new buildings in *Cambridge* shew, they rendering the rest of those buildings a disgrace to that famous seat of learning, which has produced many great men, as, (to mention here only mathematicians) *Wright*, *Briggs*, *Oughtred*, Dr. *Pell*, *Foster*, *Horrox*, *Bainbridge*, Bishop *Ward*, Dr. *Wallis*, Dr. *Barrow*, *Roche*, Sir *Isaac Newton*, *Cotes*, and Dr. *Brook Taylor*.

Wright's improvements on *Mercator's* chart became soon known abroad.

In 1608 were published the *Hypomnemata Mathematica* of the above-mentioned *Simon Stevin*, composed for the use of Prince *Maurice*. In the part concerning navigation, the author, having treated of sailing on a great circle, and shewn how to draw mechanically the rhumbs on a globe, sets down *Wright's* two tables of latitudes and of rhumbs, in order to describe those lines more accurately; and in an appendix he commends *Hues*, shews a mistake committed by *Nonius* in relation to the rhumbs, and pretends to have discovered an error in *Wright's* latter table; but *Wright* himself, in the second edition of his book, has fully answered all *Stevin's* objections, demonstrating that they arose from his gross way of calculating.

And in 1624 the learned *Walebrardus Snellius*, Professor of the Mathematics at *Leyden*, published his *Typhis Batavi**, a treatise of navigation on *Wright's* plan, written somewhat obscurely. In the introduction are praised *Nonius*, *Mercator*, *Stevin*, *Hues*, and *Wright*. But since what had been performed by our artists on this subject, is not there particularly declared, as are the improvements made by the others; it has happened that some have attributed *Wright's* principal discovery to this author. Thus *Albert Girard*, who in 1634 published a *French* translation of *Stevin's* Works with notes, in one of them observes, that *Snellius* had calculated, what he calls, *Tabulæ Canonice Parallelorum*, to minutes as far as 70 degrees; whereas *Wright* had set forth in 1610 such a table so calculated to 89 degrees 59 minutes; notwithstanding which *M. de Lagny*, in the Memoirs of the Royal Academy of Sciences at *Paris* for 1703, treating of the *Corrected Chart*, says, *c'est Willebrord Snellius qui*

* In 1617 had been published his *Eratosthenes Batavi*, where is given an account of his measuring the Earth.

n'est l'inventeur. But the French writers now acknowledge our countrymen to have been its author*.

Snellius was followed in *Holland* by *Adrian Metius*, in a treatise, intitled, *Primum Mobile*, printed at *Amsterdam*, in 1631; and in *France* by the learned *Peter Hérigone*, in his *Cursus Mathematicas*, where, in the dedication of the fourth tome to the Marshal *Bassompierre*, the author says, *Artem navigandi in censu Mathematicas non reposuere plerique nostrum, neque sanè in hunc ordinem ascribi meruit, quando cetera tantam nautarum prædilebrata est; nunc vero cum iuventis tabulis loxodromicis (quas nos primum Gallis exhibemus) formam certam firmisque Reges acceperit sine injuriâ omitti non potest.* But to return to our countrymen.

Mr. Wright, in the 12th chapter, having shewn how to find the place of a ship on his chart, observed, the same might be performed more accurately by calculation; but considering, as he says, that the latitudes, and especially the courses at sea, could not be determined so precisely, he forbore setting down particular examples; as the mariner may be allowed to save himself this trouble, and only mark out upon his chart, when truly constructed, the ship's way after the manner then usually practised.

However, in 1614 †, *Mr. Raphe Handson*, among his nautical questions, subjoined to a translation of *Pitiscus's Trigonometry*, solved very distinctly every case of navigation, by applying arithmetical calculations to *Wright's* table of latitudes, or of meridional parts, as it has since been called.

And besides, though the method *Wright* discovered for determining the change of longitude by a ship sailing on a rhumb, is the adequate means of performing it; *Handson* proposed two ways of approximation for that purpose, without the assistance of *Wright's* division of the meridian line. The first was computed by the arithmetical mean between the co-sines of both latitudes; the other by the same mean between their secants, as an alternative, when *Wright's* book was not at hand, though this latter is wider from the truth than the first; and farther he shewed by the foresaid calculations, how much each of these *compendiums* deviates from the truth, and also how erroneously the computations, on the principles of the plain-chart differ from them all.

There is another method of approximation, by what is called, The Middle Latitude ‡, which, though it errs more than that by the arithmetical mean between the co-sines; yet being less operose, is the only one now used by our sailors; notwithstanding the arithmetical mean between the logarithmic co-sines, equivalent to the geometrical mean between the co-sines themselves, had been since proposed by *Mr. John Bossut* ||, which in high latitudes is somewhat preferable.

* — c'est qu'on appelle les cartes réduites, invention admirable, de la quelle on est redevable à Edouard Wright, quoiqu'on l'ait souvent attribuée à Mercator. Hist. de l'Acad. Royale des Sciences, An. 1753. P. 275.

† It was reprinted in 1630.

‡ Gunter's works, first printed in 1623.

|| About 1630, in a dialogue which was published after the author's death, in an appendix to the *Pathway to perfect Sailing*. *Bossut* had been a teacher

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The computation by the middle latitude, will always fall short of the true change of longitude; that, by the geometrical mean, will always exceed; but that, by the arithmetical mean, fall short in latitudes above 45 degrees, and exceed in lesser latitudes. However, none of these methods, when the change in latitude is sufficiently small, will deviate greatly from the real change in longitude.

About this time logarithms * began to be introduced into the practice of the mathematics; and as they are of excellent use in the art of navigation, we shall here say something about their original.

These were invented by *John Napier*, Baron of *Marchisoun* in *Scotland*, as appears from his treatise, intitled, *Mirifici Logarithmorum Canonis Descriptio*, first printed in 1614 †. Soon after, the author communicated to Mr. *Henry Briggs*, Professor of Geometry at *Gresham College* in *London* ‡, another form of logarithms; with which Mr. *Briggs* was so well pleased, that he immediately set about computing a very large table of them, which he published in 1624, with his *Arithmetica Logarithmica* §. But in the mean time, as a specimen, he printed in 1617 a few copies for his own use and that of his friends, of a very small one, not exceeding a thousand natural numbers.

From this table Mr. *Edmund Gunter*, Mr. *Briggs*'s colleague in *Astronomy*, computed one of artificial sines and tangents to every minute of the quadrant; which he published in 1620, being the first of its kind §. And when he made an edition of his works three years after, both these tables were subjoined to his book.

of navigation at *Cbatam*, and well made out what he undertook, that a ship would return to the place it departed from, by sailing on the same rhumb, contrary to what *Fuller* and others had maintained. At the end of this discourse, he applies his compendium to the three principal problems in sailing.

* The foundation of logarithms is a property of two serieses of numbers, one in arithmetical, the other in geometrical proportion; which property is declared by *Archimedes* in his *Arenarius*.

† In 1619 was made, after the author's death, a second edition, with his farther improvements in Spherical Trigonometry.

‡ He was in 1619 appointed by Sir *Henry Saville*, his professor of geometry at *Oxford*.

§ *Adrian Vlacq* made an edition of this book at *Tergou*, in 1618, where the table of logarithms was continued by him to one hundred thousand numbers, though the logarithms themselves are but to ten places, whereas in *Briggs*'s book they were to fourteen. Some copies of *Vlacq*'s tables were purchased by our booksellers, and published at *London*, with an *English* explanation premised, dated 1631.

§ *Vlacq* also published, at the same place, in 1633, his *Trigonometria Arithmetica*, with tables of logarithmic sines and tangents to every tenth second of the quadrant. *Vlacq*'s tables have a great reputation for their exactness, as have amongst us *Sherwin*'s first edition in 1706, and *Gardiner*'s in 1742. *M. de Fontenelle*, in the History of the Academy of Sciences for 1717, commends an edition of *Vlacq*'s smaller tables, made at *Lyons*, in 1670, as does *M. de la Lande*, in his *Astronomy*, printed at *Paris*, in 1764, tables published there in 1760.

There

There he applied to navigation, according to *Wright's* table of meridional parts, as well as to other branches of the mathematics, his admirable Ruler *, on which were inscribed the logarithmic lines for numbers and for sines and tangents of arches. He also greatly improved the Sector † for the same purposes. And he shewed how to take a back observation by the cross-staff, whereby the error, arising from the exactness of the eye, is avoided; describing likewise an instrument of his invention, named by him. A Cross-Bow, for taking altitudes of the Sun or stars, with some contrivances for the more ready collecting the latitude from the observation ‡.

The discoveries relating to the logarithms were carried to *France* by Mr. *Edmund Wingate*, who, going to *Paris* in 1624, published in that city two small tracts in *French* ||, and dedicated them both to *Gaston*, the King's only brother. In the first he teaches the use of *Gunter's* ruler, and in the other, of the tables of logarithms and artificial sines and tangents, as modelled according to *Napier's* last form, attributed by *Wingate* to *Briggs*, which is a mistake; as appears from the dedication of *Napier's Rabbologia*, printed in 1616, and from what Mr. *Briggs* himself said in the preface of his *Aritbmetica Logarithmica*.

The Reverend Mr. *William Oughtred* projected this ruler into a circular arch, shewing fully its uses in a treatise first printed in 1633, intitled, *The Circles of Proportion*; where in an appendix are well handled several important points in navigation. It has been made in the form of a Sliding Ruler. See *Seth Patridge's* use of the double scale in 1662.

As by the logarithmic tables all trigonometrical calculations are greatly facilitated; so the first author, who, I find, has applied them to the cases of sailing, was Mr. *Thomas Addison*, in his treatise, intitled, *Aritbmetical Navigation*, printed in 1625. He also gives two traverse tables with their uses, the one to quarter points of the compass, the other to degrees.

Mr. *Henry Gellibrand*, Mr. *Gunter's* successor at *Gresham College*, published his discovery of the changes in the variation of the compass in a small quarto pamphlet, intitled, *A Discourse Mathematical on the Variation of the Magnetical Needle*, printed in 1635. This extraordinary phenomenon he found out by comparing the observations made at different times near the same place by Mr. *Burrough*, Mr. *Gunter*, and himself,

* This Ruler is so constantly in the practice of our artists, that it has got the name of *The Gunter*.

† The uses of a sector had been shewn by Dr. *Robert Hood*, in a tract he published in 1598.

‡ This ingenious person died in 1626, aged 45 years. His works have been several times reprinted with successive additions; the second edition was made in 1636 from his own manuscript; then from those of Mr. *Samuel Foster* Professor of Astronomy at *Gresham College*, again by Mr. *Henry Bond*, and Mr. *William Leybourn*. The fullest and last, being the fifth, was in 1673.

|| These were afterwards printed at *London* in *English* with improvements.

all persons of great skill and experience in these matters. And this discovery was soon known abroad *, for father *Athanasius Kircher*, in his treatise intitled *Magnes*, first printed at *Rome* in 1641, says, our countryman Mr. *John Graves* had informed him of it, and then gives a letter of the famous *Marinus Mercator* containing a very distinct account thereof. *Gellibrand* had been famous for the part he bore in the *Trigonometria Britannica* of his deceased friend Mr. *Briggs*, which was printed in 1633, at *Wagou*, under the care of *Adrian Vlacq*. *Gellibrand* also, in 1635, published in English an *Institution Trigonometrical*.

In 1631 Mr. *Richard Norwood* had published an excellent treatise of *Trigonometry*, adapted to the invention of logarithms, particularly in applying *Napier's* general canons †. The author having, as he says, acquired his knowledge in the mathematics at sea ‡, especially shewed the use of trigonometry in the three principal kinds of navigation. And towards the farther improvement of that art, he undertook a laborious work for examining the divisions of the log-line.

As altitudes of the Sun are taken on ship-board, by observing his elevation above the visible horizon; to collect from thence the Sun's true altitude with correctness, *Wright* observes it to be necessary, that the dip of the horizon below the observer's eye should be brought into the account, which cannot be calculated without knowing the magnitude of the earth. Hence he was led to propose different methods for finding this; but complains, that the most effectual was out of his power to execute; and therefore contented himself with a rude attempt, in some measure sufficient for his purpose; and the dimensions of the Earth deduced by him corresponded so well with the usual divisions of the log-line, that as he writ not an express treatise on navigation, but only for the correcting such errors, as prevailed in general practice, the log-line did not fall under his notice. But Mr. *Norwood*, for regulating this instrument upon genuine principles, put in execution the method Mr. *Wright* recommends, as the most perfect for measuring the dimension of the Earth, with the true length of the degrees of a great circle upon it; and, in 1633, actually measured the distance between *London* and *York*; from whence, and the summer-solstitial altitudes of the Sun observed on the meridian at both places, he found a degree on a great circle of the Earth to contain 367196 *English* feet, equal to 57200 *French* fathoms or toises, which is very exact; as appears from many measures, that have been made since that time.

* In the History of the Royal Academy of Sciences at *Paris* for 1712, p. 19. it is said by M. de *Fonitelle*, that the learned *Pier Gassendi* was the principal discoverer of this property; but *Gassendi* himself acknowledged that he had before received information of *Gellibrand's* discoveries. *Gassendi* Oper. vol. ii. p. 142. *Ludg.* 1658.

† A very advantageous reprint of it was made by M. *Mariotte* at a meeting, in 1668, of the Academy. *Du Hamel*, Hist. Acad. Scient. p. 51. 1701.

‡ From a sailor he became a teacher, styling himself before his books, a Reader of the Mathematics in *London*.

Of this affair Mr. Norwood gives a full and clear account in his treatise, called *The Seaman's Practice*, first published in 1637. There, with unaffected modesty, he apologizes for the hardness of a private person's undertaking so difficult a task; and very cautiously points out the true reason, how so great a mathematician as *Snellius* had failed in his attempt. He also shews various uses of his discovery, particularly for correcting the gross errors hitherto committed in the divisions of the log-line. But such necessary amendments have been little attended to by the sailors, whose obstinacy in adhering to inveterate mistakes has been always complained of by the best writers on navigation.

Farther, Mr. Norwood likewise there describes his own excellent method of *setting down and perfecting a Sea-Reckoning*, using a traverse-table, which method he had followed and taught for many years; and besides, shews how to rectify the course, by the variation of the compass being considered; as also how to discover currents, and to make proper allowance on their account.

This treatise and that of *Trigonometry* were continually reprinted, as the principal books for learning scientifically the art of navigation. What he had delivered, especially in the latter of them concerning this subject, was contracted as a manual for sailors, in a very small piece, called his *Epitome*, which useful performance has gone through numberless editions.

No alterations were ever made in *The Seaman's Practice*, till in the 12th edition, printed in 1676, after the author's decease, there began to be inserted, at page 59, the following paragraph in a smaller character, [*About the year 1672 Monsieur Picart has published an account in French, concerning the measure of the Earth, a breviate whereof may be seen in the Philosophical Transactions, N° 112, wherein he concludes one degree to contain 365184 English feet, nearly agreeing to Mr. Norwood's experiment.*] And this advertisement is continued in the subsequent editions, as I find it in one printed so lately as 1732.

Norwood's measure therefore, though it was not known to the great Sir *Isaac Newton* in his youth, was not buried in oblivion, on account of the confusions occasioned by our civil wars, as M. de *Voltaire* has been pleased to say *; on the contrary, it has been constantly commended by our writers on navigation: as by Mr. *Henry Bond*, soon after its publication, in a note at page 107 of the *Seaman's Kalender*, which ancient book he reprinted and improved, whose use, through numberless editions, is continued amongst our sailors to this day; by Mr. *Henry Phillips* in his *Geometrical Seaman* in 1652, and in his *Advancement of Navigation* in 1657; by Mr. *John Collins* in his *Navigatio* by the *Plane Scale*, in 1659; by the reverend Dr. *John Newton* in his *Mathematical Elements*, in 1660; Mr. *John Seller* in his *Practical Navigation*, in 1669; Mr. *John Brown* in his *Triangular Quadrant* in 1671.

* *Element de la Philosophie de Newton*, chap. xviii. printed at Paris in 1738.

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And in the *Philosophical Transactions* for 1676, N^o 126, there is given a very particular account of it. Nor had it escaped the royal notices; for when King *James*, in 1690, honoured the observatory at *Paris* with a visit, he informed the gentlemen, then present, of this measure of the Earth; and upon their acquainting his Majesty how that had been determined by M. *Picard*, the King wished the two measures might be compared together †.

But that it was not commonly known in *France* is no wonder; seeing our books were not then so much inquired after as at present by that polite and learned people.

In the *Journal des Sçavans* for December 1666, it was observed of Dr. *Hooke's Micrographia*, qu'il est écrit en une langue que peu de personnes entendent; but long after in the same *Journal* for February 1750, it is said of the *English* tongue, that it was une langue que tous les vrais sçavans devoient savoir. And now, as *Norwood* is taken notice of in the latter editions of Sir *Isaac Newton's Principia*, his name and merit indeed are become universally known. Inasmuch that a particular account of his measure is given by M. *de Moutpertuis*, in the preface to his Treatise of the figure of the Earth, printed at *Paris* in 1738; wherein he describes his method of determining the length of a degree on the Earth in *Lopland*; and *Norwood* is mentioned by two learned *Spanish* sea officers, D. *Forge Juan*, and D. *Antonio d'Ullua*, in their voyage printed at *Madrid* in 1748, which was undertaken, as they were appointed to accompany the *French* mathematicians, sent to measure a degree near the equator.

About the year 1645 Mr. *Bond* published in *Norwood's Epitome* a very great improvement in *Wright's* method, by a property in his meridian line, whereby its divisions are more scientifically assigned, than the author himself was able to effect; which was from this theorem, That these divisions are analogous to the excesses of the logarithmic tangents of half the respective latitudes augmented by 45 degrees above the logarithm of the radius

This he afterwards explained somewhat more fully in the third edition of *Gunter's* works, printed, in 1653, where, after observing that the logarithmic tangents from 45° upwards increase in the same manner (as he expresses it) that the secants added together do, if every half degree be accounted as one whole degree of *Mercator's* meridional line; his rule for computing the meridional parts appertaining to any two latitudes (supposed on the same side of the equator) is laid down to this effect; To take the logarithmic tangent, rejecting the radius, of half each latitude augmented by 45 degrees, and dividing the difference of those numbers by the logarithmic tangent of 45° 30', the radius being likewise rejected, and the quotient will be the meridional parts required, expressed in degrees. And this rule is the immediate consequence from the general theorem, That the degrees of latitude bear to 1 degree (or 60 minutes, which in *Wright's* table stands as the meridional parts for 1 degree) the same proportion as the logarithmic tangent of half any latitude augmented by 45 degrees, and the radius neglected, to the like tangent

* *Du Hamel*, Hist. Academ. Regal. Scient. p. 285.

of half a degree augmented by 45 degrees, with the radius likewise rejected.

But here was farther wanting the demonstration of this general theorem, which was at length supplied by that great mathematician, Mr. *James Gregory of Aberdeen*, in his *Exercitationes Geometricæ*, printed at *London* in 1668; and since more concisely demonstrated, together with a scientific determination of the divisor, by Dr. *Halley*, in the *Philosophical Transactions* for 1695, N^o 219, from the consideration of the spirals into which the rhumbs are transformed in the stereographic projection of the sphere upon the plane of the equinoctial; which the excellent Mr. *Roger Cotes* has rendered still more simple, in his *Logometria*, first published in the *Philosophical Transactions* for 1714, N^o 388.

It is moreover added in *Gunter's* book, that if $\frac{1}{2}^\circ$ of this divisor (which does not sensibly differ from the logarithmic tangent of $45^\circ 1' 30''$ curtailed of the radius) be used, the quotient will exhibit the meridional parts expressed in leagues: and this is the divisor set down in *Norwood's Epitome*.

After the same manner the meridional parts will be found in minutes, if the like logarithmic tangent of $45^\circ 0' 30''$ diminished by the radius be taken, that is, the number used by others * being 12633, when the logarithmic tables consist of eight places besides the index.

This Mr. *Bond*, who introduced so useful a discovery into the art, was a teacher of the mathematicks in *London*, and employed to take care of and improve the impressions of the current treatises of navigation. In an edition of the *Seaman's Kalender*, p. 103, he declared, he had discovered the longitude, by having found out the true theory of the magnetic variation; and to gain credit to his assertion, he foretold, that at *London* in 1657 there would be no variation of the compass, and from that time it would gradually increase the other way, which happened accordingly. Again, in the *Philosophical Transactions* for 1668, N^o 40, he published a table of the variations for 49 years to come.

This joyful news to all sailors acquired Mr. *Bond* a great reputation; inasmuch that the treatise he had composed, called *The Longitude found*, was in 1676 published by the special command of King *Charles the Second*, and ushered into the world with the approbation of several of the most eminent mathematicians of that time †.

But it was soon opposed, there being published at *London* a book in 1678, called *The Longitude not found*, written by one Mr. *Beckborow*. And indeed as *Bond's* hypothesis did not in any wise answer its author's sanguine expectations, the famous Dr. *Halley* again undertook this affair; and from a multitude of observations he would conclude, that the mag-

* See Mr. *Perkins's Treatise of Navigation* in Vol. I. of Sir *Jonas Moore's New System of the Mathematicks*, p. 208, printed at *London* in 1681. *Perkins's* book was published by itself the year following under the title of the *Seaman's Tutor*.

† In the *Philosophical Transactions* for the same year, N^o 130, it is said, the Lord *Brouncker's* name was inserted by mistake.

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netic needle was influenced by four poles. His speculations on this subject are delivered in the *Philosophical Transactions* for 1683, N^o 148, and for 1692, N^o 195. But this wonderful phenomenon seems to have hitherto eluded all our researches.

However, that excellent person in 1700 published a general map, on which were delineated curve lines expressing the paths, where the magnetic needle had the same variation. This was received with universal applause*, as it may lead to some discovery in so abstruse an affair, and at present be useful on many occasions in determining the longitude. The positions of these curves will indeed continually suffer alterations; but then they should be corrected from time to time; as they have been for the year 1744, and 1756, by two ingenious persons, Mr. *William Mountaine* and Mr. *James Dodson*, Fellows of the Royal Society. The latter died not long after he had been chosen, for his merit, mathematical master, at *Christ's Hospital*, in *London*.

Dr. *Halley* also gave in the *Philosophical Transactions* for 1690, N^o 183, a dissertation on the monsoons, containing many observations very useful for all such as sail to the places that are subject to those winds.

The true principles of navigation having been settled by *Wright*, *Norwood*, and *Bond*, many authors amongst us trod in their steps, making some little improvements. It would be impossible to enumerate each particular. Of the writers already mentioned, *Phillips* and *Collins*, in the title-pages of their books, declare what they aimed at; *Phillips* also in his tract, called the *Advancement of Navigation*, recommends a pendulum instead of a half-minute glass, to estimate the time the log-line is running out. He also proposes to do the same thing by wheel-work. Besides, in the *Philosophical Transactions* for 1668, N^o 34, he delivers a better method to determine the tides than what was commonly practised; for which purpose Mr. *John Flamsteed*, the royal astronomer, still gave more perfect directions in the same *Transactions* for 1683, N^o 143; as likewise he first ordered a glass lens to be fixed on the shade vane, in what is called *Davis's* quadrant†, which contrivance Dr. *Robert Hook*, Professor of Geometry at *Gresham* college, had before thought of‡.

Seller's Practical Navigation, though without demonstrations, has the rules of sailing in the different kinds, as performed by calculation, by the plane scale, by the *Gunter*, and by the sinical quadrant, with various other matters relative to the art; as also the use of the azimuth-compass as now modelled, the ring-dial, the sea-ring, cross-staff, *Davis's* quadrant,

* It is particularly commended in the History and Memoirs of the Royal Academy of Sciences at *Paris*, for the years 1701, 1705, 1706, 1708, and 1710. See also Mr. *Robins's* Reflections in his Introduction to Lord *Apsley's* Voyage round the World, made in 1743, &c. as also in the ninth chapter of the first book, and eighth of the third.

† See the above-mentioned *Parkinson's Navigation*, page 250.

‡ See Bishop *Thomas Sprat's* excellent History of the Royal Society in 1667. pag. 246; and *Hooke's* Posthumous Works, published by *Richard Waller*, Esq. in 1705, p. 557.

plough

plough, nocturnal, inclinatory needle and globe, together with all the necessary tables; the whole being delivered in a manner so well adapted to the general humour of mariners, that it has undergone numberless editions: the last, I have seen, was in 1739: but some late writers seem to have abated the run of this book.

As in sailing especial regard ought to be had to the lee-way a ship makes, so many authors have touched upon this point; but the allowances usually made on that account are very particularly set down by Mr. *John Buckler*, and published in a small tract first printed in 1702, entitled, *A New Compendium of the whole Art of Navigation*, written by Mr. *William Jones*.

We ought not here to pass over in silence the very useful invention of Dr. *Gowin Knight*, which is the making *artificial magnets*, that are of greater efficacy than the natural ones. Though the Doctor has not thought fit to reveal his secret; yet others have found it out, who have made it public, particularly Mr. *John Michel*, and Mr. *John Canton*; the first in a treatise of *Artificial Magnets*, printed in 1750; the other in the Philosophical Transactions, vol. XLVII. *Ann.* 1751.

The Earth being now universally agreed to be not a perfect globe, but a spheroid, whose diameter at the poles is shorter than any other; the Rev. Dr. *Patrick Murdoch* published a tract in 1741, where he accommodated *Wright's* sailing to such a figure; and Mr. *Colin Maclaurin*, the same year, in the Philosophical Transactions, N^o 461, gave a rule to determine the meridional parts of a spheroid, which speculation he farther treats of in his book of *Fluxions*, printed at *Edinburgh*, in 1742.

Though Sir *Isaac Newton* in his *Principia*, first printed in 1686, had demonstrated from the theory of gravity, that this must be the real form of the Earth, as it revolved about an axis; yet in the year 1718 M. *Cassini* again * undertook from observations to shew the contrary, and that the Earth was a spheroid, having its longest diameter passing through its poles †; and in 1720 M. *de Mairan* advanced arguments, supposed to be strengthened by geometrical demonstrations, to farther confirm M. *Cassini's* assertion. But in the *Philosophical Transactions* for 1725, N^o 386, 387, 388. Dr. *Desaguliers* published a dissertation, wherein he made appear the weakness of the reasoning, and the insufficiency of the observations, as they were managed, to settle so nice an affair. He there also proposed a proper method for adjusting this point, when he says, *If any consequence of this kind could be drawn from actual measuring, a degree of latitude should be measured at the equator, and a degree of longitude likewise measured there; and a degree very northerly, as for example, a whole degree might be actually measured upon the Baltic sea, when frozen, in*

* In the Memoirs of the Royal Academy of Sciences at *Paris*, his father in 1701, and he in 1713, attempted to prove the Earth was an oblong spheroid.

† M. *John Bernoulli* in his *Essai d'une Nouvelle Physique Celeste*, printed at *Paris* in 1735, triumphs over Sir *Isaac Newton*; vainly imagining these precious observations could invalidate what Sir *Isaac* had demonstrated.

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the latitude of sixty degrees. There, according to M. Cassini's last supposition, a degree would be 56653 toises; whereas at the equator it would be of 58019 toises, the difference being 1366 toises, about the two and fortieth part of a degree, which must be sensible; and likewise the degree of longitude would according to him be of 56817 toises, less by 1202, or the forty-eighth part, than a degree of latitude at the same place.

On this admonition, in 1735 were sent from France two sets of mathematicians, members of the Royal Academy of Sciences; one towards the pole, the other to the equator, in order to measure at each place the length of a degree on the meridian. The report they brought home quite overset, what had been urged in favour of the oblong figure; a degree towards the north, in the latitude of $66^{\circ} 20'$ being found to contain about 57438 toises, and near the equator but 56750.

This unwelcome news caused again a degree to be measured in France, which at length came out to be consonant with those which had been brought from very distant parts of the world. Thus these mathematicians confirmed by painful observations, what Sir Isaac Newton had, as M. de Maupertuis used to say, determined in his elbow-chair; Sir Isaac making the length of a degree under the pole to be 57382, and at the equator 56637 toises. And perhaps no observations can be exact enough to determine this matter more precisely.

But let us mention some of the foreign writers on navigation.

At Rome, in 1607, came forth a treatise, intitled, *Nautica Mediteranea*, written in Italian by Bartolomeu Crescenti, the Pope's engineer. The author misses no opportunity of exposing the errors of Medina; but scarce gives any thing of his own, except a machine to serve to measure the way a ship made.

As the Jesuits have treated of most branches of learning, so this art has not been beneath their consideration; the three following authors having been of their society.

At Paris, in 1633, Father George Fournier, published an *Hydrography*, principally relating to navigation. The author would persuade us, that one of Dieppe had corrected the plane chart; and that the *Hollanders* learnt of the French the making charts so corrected; whereas this had been engraved long before at Amsterdam, by Isodorus Hondius, and others.

John Baptist Riccioli, in his *Geographia & Hydrographia Reformata*, printed at Bologna in 1661, inserts a treatise of navigation, collecting his materials from almost every writer, as he does in his *Almagest* and *Chronology*, which is indeed the chief merit of his works.

Father Millet Dechalles wrote on this subject after a more masterly manner, both in his *Curfus Mathematicus*, first printed at Lyons in 1674, and in a French treatise, published in 1677, intitled, *L'Art de Naviger démontré par Principes*.

These three authors, besides treating of the different kinds of sailing, abound in methods for taking of altitudes, finding the variation, and estimating the way a ship makes, &c. They also describe a machine resembling that of Crescenti. Riccioli gives a very faulty measure of the Earth, made by himself; and Dechalles advises the use of a pendulum in reckoning by the log-line, as also of wheel-work for the same purpose as Phillips and Cole had done.

But

But there were writers in France between Pournier and Debolles. As in 1666, and the following years, had been printed at Dieppe several tracts handling different parts of navigation, composed by M. G. Dany, which have been often reprinted.

And in 1671 the Sieur Blandel S. Aubin published a book, called, *L'Art de Naviger par le Quartier de Reduction*, describing an instrument much in use amongst the French sailors, by which may be performed, as by the sinical quadrant, the operations of navigation, though not much more speedily than by the traverse table, and not at all so accurately. He also published in 1673 his *Treſor de la Navigation*, where the art is well treated of, particularly by calculations.

M. Saverien in his *Marine Dictionary*, printed at Paris in 1758, says, that M. Daffier seems to have been the first of the French writers that shewed the use of Gunter's scales [*échelles Angloises*] in his *Pilote expert*, printed in 1683.

At Paris, ten years after, was published the first part of a pompous work, intitled, *La Neptune François*, by order of the French king, consisting of sea-charts, according to Wright's scheme, made from the latest observations, and reviewed by Mess. Pene, Cassini, and others. As this contained the charts of Europe only, there were added others of different parts of the world, printed the same year at Amsterdam. The whole was preceded by a discourse of M. Sauvour, who had formed some of the charts, where he shews how to perform the problems of astronomy and navigation by scales; which discourse had been published by itself at Paris, in 1692.

M. John Bouguer composed by authority his *Traité Complet de la Navigation*, first printed in 1698, which was well received, as containing most of the practices then known; and Father Pézenas, Jesuit and Royal Professor of Hydrography at Marseilles, published there, in 1733, a tract, called, *Elémens de Pilotage*; and at Avignon, in 1741, a larger work, intitled, *Pratique du Pilotage*. This author shews, how to find the meridional parts by the *Artificial Tangents*, an old discovery amongst us, declared so long ago as 1645, in *Norwood's Epitome*; he also has been industrious in translating several of our mathematical books into French.

But in 1753 M. Peter Bouguer, son of the former, published a very elaborate treatise on this subject, intitled, *Nouveau Traité de Navigation*, which is written sensibly, the author being an excellent mathematician, and famous for other productions. He there gives a variation-compass † of his own invention, and attempts to reform the log, as he had done in the Memoirs of the Academy of Sciences for 1747. He is also very

* It is only a kind of skeleton of Wright's universal map.

† Many of these sorts of compasses have been proposed at different times, as by M. Buache, in the Memoirs of the French Academy of Sciences for 1752, page 377; Captain Christopher Middleton, in the Philosophical Transactions, No 450, *Ann.* 1738; and Dr. Knight, as improved by the ingenious Mr. John Swanton, *ibid.* No 495, *Ann.* 1750.

particular

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particular in determining the lunations more accurately than by the common methods, and in describing the corrections of the dead reckoning.

The excellent astronomer, M. de la Caille, in 1760, made an edition of M. Bouguer's book, which he somewhat abridged and improved.

In 1766, at Paris, came out a treatise, with this title, *Abregé du Pilote divisé en deux parties, ou un traité principalement des Amplitudes, des Latitudes, dans l'hypothèse de la Sphère et de Sphéroïde, des mortes, des variations de l'aiguille*.

The former part of this book was first published in 1693. Here the whole is improved by M. le Moine.

Though the Spaniards were the earliest writers on navigation, yet they were very backward to adopt its improvements. Indeed *Antonio de Navegacion y practica*; in 1628, a treatise, intitled, *Navegacion especulativa y practica*; where, though the author rectifies the tables of the Sun and fixed stars, from Tycho Brahe's observations, he proceeds no farther in the theory of navigation than what had been advanced by *Navius* as followed by *Céspedes*. But of late, in 1712, was printed at the same place, *Arte de Navegar por Manuel Bimantel*; where is shewn the use of *Wright's* chart, which, in imitation of the French, the author calls *Charta Reducida*. He likewise describes *Davis's* quadrant, and mentions *Norwood* and *Picard's* measures of the Earth. In 1757 was printed at Cadix a treatise, intitled, *Compendio de Navegacion para el uso de los Cavalleros Guardias Marinas*, written by the ingenious gentleman mentioned above, Don *Jorge Juan*. This is a good performance, delivering very distinctly the several parts of the art, as now improved. Some things are here omitted, that usually occur in books on this subject; but for the knowledge of such particulars, references are made to tracts composed expressly for the use of the society of gentlemen, destined for the sea-service.

Bouguer and *Jorge Juan*, describe and commend the method of dividing instruments for taking of angles, published by *Peter Vernier*, in a treatise, intitled, *La Construction, &c. du quadrant nouveau*, printed at Brussels, in 1631. This division, an improvement of *Curtius's*, as that of *Ererenius's* is of the division by diagonals* readily shews from the first Lemma of *Clavius's* treatise on the *Astrolabe* †, as has been observed by *Piscenus*, in a book he published at Avignon in 1765, intitled, *Astronomie des Marins*.

As to their treating of *Wright's* chart, I mentioned above *Sædlius* and *Motius*. To an edition, in 1665, of *Vlaaq's* small tables of logarithms, &c. is added, by *Abraham de Genul*, one of meridional parts, whose use he shews, with other parts of navigation, in his *Course of Mathematics*, written in Dutch, and printed at Amsterdam in 1676, as had been done by *John Viriet*, in his *Flembeau reüssissant ou Tresor de la Navigation*, at the same place, in 1677.

* See Mr. Robin's Mathematical Tracts, where these divisions are largely treated of.

† First printed at Rome, in 1693.

The Dutch are great navigators, and have been famous for their *Atlasses*, before which are premised treatises of navigation, as has been already observed. The oldest I have seen of these, was published at Leyden in 1584, intitled, *Spiegel der Zee-Vaert* (or *Mirror of Navigation*), by Lucas Jansz. Wagenaer. In their later *Atlasses* there is described an instrument to be used after the manner of *Davis's* quadrant, but where instead of circular arches are substituted straight lines.

Notwithstanding all the improvements hitherto mentioned, the *searunkings*, though kept by such as were deemed very skilful mariners, are often found widely different from the truth. But this often happens through negligence, as I have heard Dr. Halley, who had used the sea, say.

These errors would be avoided, if from time to time the latitude and longitude could be determined. The first is generally obtained by the meridian altitude and declination of the Sun being given. The declination is got by the help of tables of the Sun, with an easy trigonometrical work.

The first could not be very exact, before the famous *Kepler* had determined the true form of the earth's orbit*. Hence were fabricated his *Tabule Rudolphinæ*. Next, those of Mr. *Thomas Street* were in great request†. But they now yield to Dr. *Halley's*‡; which, however, will want to be corrected hereafter: For, as Sir *Isaac Newton* has shewn, that all bodies mutually attract one another, the Earth will be disturbed in its motion by the actions of some of the other planets.

To find the longitude is a much more difficult affair. For this end, at present, the societies of learned men in *Europe* offer, from time to time, rewards to such as shall best treat of particular subjects in mathematics or physics. Some of these have been relating to navigation, when *Poleni*, *Bernoulli*, *Bouguer*, and others have obtained the prizes. And it is hoped, this institution may contribute to the advancement of the art.

Eclipses of the moon were used of old; and *Kepler* recommended those of the sun as preferable§.

The satellites of *Jupiter* were no sooner discovered by the great *Galileo S.*, than the frequency of their eclipses recommended them for this purpose; and amongst those who attempted this subject, none were more successful than Signor *Dominic Cassini*.

This great astronomer in 1688 published at *Bologna* tables for calculating the appearances of their eclipses, with directions for finding thence the longitudes of places; and being invited to *France* by *Lewis* the Fourteenth, he there published corrected tables in 1693. But the mutual attractions of the satellites on one another rendering their motions excessively irregular, the tables soon ran out; inasmuch that they require to be renewed from time to time, which is performed by ingenious persons,

* In his treatise de *Motu Maris*, in 1609.

† In his *Astronomia Carolina*, in 1661.

‡ In 1719, many years after their construction.

§ *Tabule Rudolph.* printed at *Ulm* in 1627, cap. xvi. & xxvii.

§ In his *Sydeus Nuncius*, first printed at *Venice* in 1610.

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as Dr. James Pound, Dr. James Bradley, M. Cassini the son, and M. Peter Wargentin †; so that now many of the common *Almanacks* set down, when these eclipses happen throughout the year.

Mr. Nevil Maskelyne, the Royal Astronomer, has published annually, ever since 1767, by order of the Commissioners of the Longitude, a *Nautical Almanack*, and *Astronomical Ephemeris*, containing not only the eclipses of the satellites, but also many other tables, to enable the mariner to determine the longitude. The same is attempted in the *Connoissance des Temps*, especially for these latter years, by M. de la Lande.

The large reward granted by the *Parliament* for a practical way of discovering the longitude at sea, has put many upon the search: in so much that several idle and absurd schemes have been offered by ignorant and wrong-headed men. But the perfecting the methods proposed long ago by John Werner and Gemma Frisius, seems at present to engage the attention of the public.

The theory of the moon, though much amended by the noble Tycho Brahe and Mr. Jeremy Horrox*, was found to be insufficient to answer this end. But the causes of her various irregularities having been discovered by Sir Isaac Newton, and her theory thence improved beyond expectation, gave great hopes of success; which are now increased by the farther improvements made on his principles by M. Euler and others; as they have enabled M. Tobias Mayer of *Göttingen* ‡, to compose at length tables agreeing to the moon's motion in every part of her orbit, with very surprising exactness. And this ingenious person has left behind him tables still more exact †, for which the *Parliament* have rewarded his widow, as also Mr. Euler. These tables were published in 1770, by Mr. Maskelyne.

As to the method of Gemma Frisius, M. Huygens was persuaded it might be accomplished by his inventions of pendulum clocks and watches; a description of the first he published in a small tract, printed at the Hague, in 1658, and of the second as improved in the *Journal des Savans* for the month of February, 1675. And great expectations of success had been raised from some trials made in a voyage with

¶ He succeeded Dr. Halley at *Greenwich*, where he made a complete set of Astronomical Observations, which, as they are most accurate, it is hoped they will not be lost. He became famous on publishing an account of an apparent motion in the fixt stars, which was immediately exhibited by the great mathematician Dr. Brook Taylor, according to the exact theory of the Earth's motion. See Mr. Robins's *Mathematical Tracts*, vol. II. page 276.

† Wargentin's tables are much esteemed; they were first published at Stockholm in the *Acta Societatis Regia Scientiarum Upsalensis* for the year 1741, but since more correct from a new copy of the author's at Paris in 1759, by M. de la Lande.

* This great genius died in 1641, scarce 23 years old. See his *Opera Posthuma*, published by the famous Dr. John Wallis at London, in 1673. Horrox first observed the Transit of Venus and the Sun in 1639. He wrote an account of this *Phenomenon*, which was published by the great astronomer Hevelius, at Danzig, in 1661.

‡ *Cam. Societ. Reg. Göttingens.* tom. II. page 283.

† See his *Elogium* in the *Nova Acta Eruditorum*, for March 1762.

these

these watches of the first construction, by Major *Holmes*; an account whereof is given in the Philosophical Transactions, *Ann.* 1669. But the various accidents those movements are liable to, soon caused that way to be laid aside.

Notwithstanding which, the ingenious Mr. *John Harrison* has for many years past employed himself in contriving a machine, that shall be free from all imaginable inconveniencies; and his endeavours have been so well approved of by gentlemen of the greatest knowledge in these subjects, that the commissioners for the longitude have thought fit to allow him some gratifications for his pains. He has been since farther considered, upon disclosing the internal structure of his instrument*, and he still perseveres in pushing for the whole reward.

The difficulty of making observations at sea with sufficient exactness, was feared to be insurmountable; but to overcome which has been attempted. In the History of the Royal Society, at page 246, is mentioned an invention in these words: *A new instrument for taking angles by reflection, by which means the eye at the same time sees the two objects both as touching the same point, though distant almost to a semicircle; which is of great use for making exact observations at sea.* A figure of this instrument, drawn by Dr. *Hook*, the inventor, is given in the Doctor's posthumous works, with a description, at page 503. But here as one reflection only is made, it would not answer the purpose. However, this was at last effected by Sir *Isaac Newton*, who communicated to Dr. *Halley* a paper of his own writing, containing a description of an instrument with two reflections, which soon after the Doctor's death was found among his papers by Mr. *Jones*, who communicated it to the Royal Society, and it was published in the Philosophical Transactions N^o 465, *Ann.* 1742.

How it happened that Dr. *Halley* never mentioned this in his lifetime, is very extraordinary; seeing *John Hadley*, Esq. † had described, in N^o 430, *Ann.* 1731, an instrument grounded on the same principles, which is so well esteemed, that our shops abound with them, accommodated with *Vernier's* division, as they are made by our most skilful workmen; and are now in general use amongst the skilful seamen of most of the Maritime nations.

Though *Medina's* method for finding the place of the horizon was absurd; yet, for this end, several plausible ones have been proposed by ingenious persons, as Mess. *Elton*, *Hadley*, *Godfrey*, and *Leigh*; and that chiefly by applying a level to *Davis's* quadrant. Their devises are described in the Philosophical Transactions for 1732, 33, 34, and 37.

* May not a machine be produced that shall answer to the words, but not to the intention of the act of parliament? That act was procured by *Whiston* and *Ditton*, two very weak men; and their project was treated with the utmost scorn.

† Mr. *Hadley* being well acquainted with Sir *Isaac Newton*, might have heard him say, *Hook's* proposal could be perfected by means of a double reflection. However, Mr. *Hadley*, being a very ingenious person, might have hit on the same thought; as well as Mr. *Godfrey* of Pennsylvania; to whom the invention of this admirable instrument has been ascribed by some gentlemen of that colony: This is not the only case, wherein different persons have produced similar inventions. And

xxx DISSERTATION ON THE RISE, &c.

And, lastly, an *Horizontal Top*, invented by the late Mr. Serfen, who was unfortunately lost at sea aboard the *Victory* man of war, has been approved of, and published by Mr. Smeaton in the Philosophical Transactions, vol. XLVII. for 1752. part ii. page 352.

Some methods used for obtaining the place of the horizon, and of observing with Mr. Hadley's, *Reflecting Sector*, are described by Mr. Robertson, in his *Elements of Navigation*; which treatise has deservedly met with the approbation of the public.

Thus have I endeavoured to trace out the principal steps by which the art of navigation has advanced to its present height; nor without hopes that the attempt may not prove altogether unacceptable to those, whose business or curiosity lead them to be acquainted with this very useful branch of the mathematicks: on the successful practising of which depends, in an especial manner, the flourishing state of our country.

This Dissertation, written at first by desire, is now reprinted with alterations. Though I may be thought to have dwelt too long on some particulars not directly relating to the subject; yet I hope that what is so delivered, will not be altogether unentertaining to the candid reader. As to any apology for having handled a matter quite foreign to my way of life, I shall only plead, that very young, living in a sea-port town, I was eager to be acquainted with an art that could enable the Mariner to arrive across the wide and pathless ocean at his desired harbour.

London,

JAMES WILSON.

A D V E R T I S E M E N T.

AS it may be expected that four kinds of readers will look into this book, it was thought convenient to point out to some of them, the places where they may meet with what they more particularly want.

FIRST. Those who having made a proficiency in the mathematics, will, it is likely, examine in what manner the subjects are here treated, and whether any thing new is contained therein: it is conceived that such readers will find some things which may recompence them for their trouble, in almost every one of the books.

SECONDLY. Those learners, who are desirous of being instructed in the art of Navigation in a scientific manner, and would chuse to see the reason of the several steps they must take to acquire it: To such persons, it is recommended that they read the whole book in the order they find it; or, if the learner is very young, he may omit the IVth and Vth books till after he is master of the VIth and VIIth.

THIRDLY. That class of readers, which, with too much truth may be said, comprehends most of our mariners, who want to learn both the elements and the art itself by rote, and never trouble themselves about the reason of the rules they work by: As it is probable there ever will be many readers of this kind, they may be well accommodated in this work; thus, if they are not already acquainted with Arithmetic and Geometry, let them read the five first rules of Arithmetic, to page 20th; thence proceed to the definitions and problems in Geometry, from page 43 to 58. In the book of Trigonometry, read pages 89, 90, 91, 92, 98, 99, and from 104th to 114th: the whole of book VI. In book the VIIth they may read to page 35, and as much more as they please. In book VIII, let them read the sections III, IV, V, VI, from page 194 to page 230. In book V, they may read section III, and as many problems in the Vth and VIth sections as they can; and let them read the whole of the ninth book.

FOURTHLY.

FOURTHLY. That set of readers who will not be at the pains of learning any thing more than how to perform a day's work; such may herein meet with the practice almost independent of other knowledge. Let such persons make themselves acquainted with section IV of book VI, and the use of the table at page 374; then learn the use of the Traverse Table at the end of book VII, which they will find exemplified between pages 8 and 35 Vol. II; also they must learn the use of the Table of Meridional parts at the end of Book. VIII After which, they may proceed to book IX, where they will find ample instructions in all the particulars which enters into a day's work. But with this scanty knowledge of things, they will be obliged to omit some parts, which is is well worth their pains to be acquainted with.



THE ELEMENTS OF NAVIGATION.

BOOK I. OF ARITHMETICK.

SECTION I.

Definitions and Principles.

1. **ARITHMETICK** is a science which teaches the properties of numbers; and how to compute, or estimate the value of things.

2. An **UNIT** or **UNITY**, is any thing considered as one.

3. **NUMBER**, in general, is many units.

4. **DIGITS** or **FIGURES** are the marks by which numbers are denoted or expressed, and are the nine following.

Digits, 1. 2. 3. 4. 5. 6. 7. 8. 9.

Names, One. Two. Three. Four. Five. Six. Seven. Eight. Nine.

And with these is used the mark 0, called cypher, which of itself stands for nothing; but being annexed to a digit, alters its value.

Thus 40 signifies forty; and 400 stands for four hundred; &c.

5. **INTEGER**, or **WHOLE NUMBERS**, are such as express a number of things, whereof each is considered as an unit.

Thus four pounds, twelve miles, thirty-four gallons, one hundred days, &c. is, in each case, called an integer number, or whole number.

6. **FRACTIONAL NUMBERS**, are such which express the value of some part or parts of an unit.

Thus one half, one quarter, three quarters, &c. are each the fractional parts of an unit.

B

7. **NOTA.**

7. **NOTATION** is the expressing by digits or figures any number proposed in words; and the reading of any number that is expressed by figures, is called **NUMERATION**.

8. **DECIMAL NOTATION** is that kind of numbering wherein ten units of any inferior name are equal in value to an unit of the next superior.

9. Every number is said to consist of as many places as it contains figures.

10. The value of every digit in any number is changed according to the place it stands in; and the reading of any number, consists in giving to each figure therein its right name and value.

11. The right hand place of an integer number is called the place of units; and from this place all numbers begin, whether *whole* or *fractional*; the integers increasing in order from the unit place towards the left; and the fractions decreasing in order from the unit place towards the right; And to distinguish decimal fractions from integers, there is always a point or comma (,) set on the left hand side of the fractional number; so that the integers stand on the left hand side of the mark, and the fractions on the right hand.

12. For the more convenient reading of numbers, they are divided into periods of six places each, beginning at the unit place; and each period into two degrees of three places each, whose names and order are as follow: Where X stands for the word tens, C for hundreds, and Th. for thousands.

13.

Integers						Decimal fractions					
Second period			First period			First period			Second period		
Degree	Degree	Degree	Degree	Degree	Degree	Degree	Degree	Degree	Degree	Degree	Degree
Billions.	Th. Millions	Th. Millions	Th. Millions	Th. Millions	Th. Millions	Th. Millions	Th. Millions	Th. Millions	Th. Millions	Th. Millions	Th. Millions
X	C	X	C	X	C	X	C	X	C	X	C
5	4	3	2	1	2	3	4	5	6	7	8
Decimal Fractions are also thus named,						Primes	Seconds	Thirds	Fourths	Fifths	Sixths
						Sevenths	Eighths	Ninths	Tenths	Elevenths	Twelfths
						&c.					

The name of the first period, is units; of the second, millions; of the third, Billions; of the fourth, Trillions; &c.

In the above order it may be observed, that each degree contains the names of Units, Tens, Hundreds; the first degree of a period contains the units of that period, and the second contains the thousandths thereof: So that from hence it will be easy to read a number consisting of ever so many places by the following directions.

14. RULE,

14. RULE. 1st. Suppose the number parted into as many sets or degrees of three places each, beginning at the units place, as it will admit of; and if one or two places remain, they will be the units and tens of the next degree.

2^d. Beginning at the left hand, read in each degree, as many hundreds, tens, and units, as the figures in those places of the degree express, adding the name thousands, if in the second degree of a period; and adding the name of the period, after reading the hundreds, tens, and units in its first degree.

Thus the integer number in the preceding table will be read,

Five billions, four hundred thirty two thousand, one hundred twenty three millions, four hundred fifty six thousand, seven hundred eighty nine.

15. All fractional numbers consist of two parts, which are usually wrote one above the other with a line drawn between them: The number below the line, called the *denominator*, shews into how many equal parts the unit is divided: The number above the line, called the *numerator*, shews by how many of these equal parts the value of that fraction is expressed.

Thus 9 pence, is 9 parts in twelve of a shilling; and may be wrote thus, $\frac{9}{12}$, when a shilling is the unit.

16. Such fractions whose denominators are 10, or 100, or 1000, or 10000, or 100000, &c. are called *decimal fractions*: But fractions with any other denominators, are called *vulgar fractions*.

The vulgar fractions that most frequently occur, are these:

$\frac{1}{4}$, which is read one fourth, or one quarter.

$\frac{1}{3}$ - - - - one third.

$\frac{1}{2}$ - - - - one half.

$\frac{2}{3}$ - - - - two thirds.

$\frac{3}{4}$ - - - - three fourths, or three quarters.

17. As decimal fractions are parts of an unit divided into either 10, 100, 1000, 10000, &c. parts, according to the places in the fractional number; therefore they are read like whole numbers, only calling them so many parts of 10, or of 100, or of 1000, &c.

Thus a decimal	fraction of	$\left\{ \begin{array}{l} \text{one} \\ \text{two} \\ \text{three} \\ \text{four} \\ \text{\&c.} \end{array} \right\}$	$\left\{ \begin{array}{l} \text{places, will be so} \\ \text{many parts of} \\ \text{10, Hundred.} \\ \text{100, Thousand.} \\ \text{1000, Ten Thousand,} \\ \text{\&c.} \end{array} \right\}$

1^o. Cyphers on the right hand of integers increase their value: on the left hand of a decimal fraction diminish its value: But on the left hand of integers, or on the right hand of fractions, do not alter their value.

Thus $\left\{ \begin{array}{l} 8 \text{ is } 8 \text{ units.} \\ 80 \text{ } 8 \text{ tens.} \\ 800 \text{ } 8 \text{ hundreds.} \end{array} \right\}$ And $\left\{ \begin{array}{l} 0,8 \text{ is } 8 \text{ parts in } 10 \\ 0,08 \text{ } 8 \text{ parts in } 100 \\ 0,008 \text{ } 8 \text{ parts in } 1000 \end{array} \right\}$ of an unit so divided.

When a fraction has no integer prefixed, it is convenient to put 0 in the place of units.

19. A MIXED NUMBER, is when a fraction is annexed to a whole number.

Thus five and a half is called a mixed number, and is wrote $5\frac{1}{2}$; or thus, 5,5; which is thus read, five and five tenths.

20. Like names in different numbers, are such figures as stand equally distant from the place of units; or have the same denomination annexed to them.

Thus all numbers of pounds sterling are like names, and so are all numbers of shillings; the like of any numbers of miles, &c.

21. Besides the decimal notation explained in article 8, there are other kinds in common use; such as the duodecimal, wherein every superior name contains 12 units of its next inferior name: The Sexagenary, or Sexagesimal, wherein sixty of an inferior name make one of its next superior. The former is used by workmen in the measuring of artificers works in building; and the latter is used in the division of a Circle, and of Time.

22. The following characters or marks are frequently used in Arithmetical computations, briefly to express the manner of operation.

The mark + (more) belongs to addition; and shews that the numbers it stands between are to be added together.

Thus $12 + 3$, expresses the sum of 12 and 3; or that 3 is to be added to 12, and is thus read, 12 more 3.

The mark - (less) is for subtraction; and shews that the number following it, or on the right hand, is to be taken from the number preceding it, or on the left hand.

Thus $12 - 3$, expresses the difference between 12 and 3; or that 3 is to be subtracted from 12, and is thus read, 12 less 3, or 12 lessened by 3.

This mark $\times$ (into) for multiplication, shews that the numbers on each side of it, are to be multiplied the one by the other.

Thus 12×3 , denotes the product of 12 into 3; or that 12 is to be multiplied by 3.

Division is expressed by setting the divisor under the dividend with a line drawn between them, like a fraction.

Thus $12 \over 3$, expresses the quotient of 12 by 3; or that 12 is to be divided by 3.

This sign = (equal) shews that the result of the operation by the numbers or quantities on one side of it, is equal either to the numbers or quantities on the other side, or to the result of the operation by these numbers or quantities.

Thus $12 + 3 = 15$; and $12 - 3 = 9$; and $12 \times 3 = 36$; and $12 \over 3 = 4$; severally shews the value of the preceding expressions.

23. TABLES OF ENGLISH MONEY, WEIGHTS AND MEASURES.

MONEY.		TROY WEIGHT.	
Farthings	Pence	Shill.	Pound
960 =	240 =	20 =	1 £.
48 =	12 =	1s.	
4 =	1d.		
<i>Note, 1, 2, 3 farthings, are thus wrote</i> $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}.$			
AVERDUPPOIZE WEIGHT.		WINE MEASURE.	
Drams	Ounces	Pounds	Hund. Ton.
573440 =	35840 =	2240 =	20 = 1
28672 =	1792 =	112 =	1 C.
256 =	16 =	1 lb.	
16 =	1 oz.		
<i>Note, Provisions, Stores, &c. are weighed by the Averduppoize, or great weight.</i>			
DRY MEASURE.		CLOTH MEASURE.	
Pints	Gall.	Pecks	Bush. Quarter
512 =	64 =	32 =	8 = 1
64 =	8 =	4 =	1
16 =	2 =	1	
8 =	1		
<i>Note, 4 bush. = 1 Comb: 10 qrs. = 1 Wey, 12 Weys = 1 Last of Corn.</i>			
36 Bushels =	1 Chaldron of Coals.		

LONG MEASURE.

Barley-corns	Inches	Feet	Yards	Poles	Furl. Mile.
190080 =	63360 =	5280 =	1760 =	320 =	8 = 1
23760 =	7920 =	660 =	220 =	40 =	1
594 =	198 =	16½ =	5½ =	1	
108 =	36 =	3 =	1		
36 =	12 =	1			
3 =	1				

Also 3 miles make 1 league.

And 20 leagues or 60 Sea miles make a degree. *Ditto* XVIII.

But a degree contains about 69½ miles of statute measure.

A Fathom = 6 feet = 2 yards.

TIME.

Seconds	Minutes	Hours	Days	Year
31556937 =	525948 =	8766 =	365½ =	1
86400 =	1440 =	24 =	1 day	
3600 =	60 =	1 hour.		
60 =	1			

Pence Table.

Pence Sh.	Pence Pence	Sb.	Pence	s	d	Even parts of a Pound Sterling.			
20 =	1	8	70 =	5	10	10	0	is	1
30 =	2	6	80 =	6	8	6	8	is	1
40 =	3	4	90 =	7	6	5	0	is	1
50 =	4	2	100 =	8	4	4	0	is	1
60 =	5	0	110 =	9	2	3	4	is	1
						2	6	is	1
						2	0	is	1

or of a Pound Sterling.
or of a shilling.
or of a Pound.

24. SECTION II. ADDITION.

ADDITION is the method of collecting several numbers into one sum.

RULE. 1st. Write the given numbers under each other, so that like names stand under like names; that is, units under units, tens under tens, &c. and under these draw a line.

2^d. Add up the first or right hand upright row, under which write the overplus of the units of the second row, contained in that sum.

3. Add these units to the sum of the second row, under which write the overplus of the units of the third row, contained in that Sum.

And thus proceed until all the rows are added together.

EXAMPLES.

Ex. I. *Add 28—76—47—18 and 12 together.*

These numbers being wrote under each other will stand thus.

	28
Say 2 and 8 is 10, and 7 is 17, and 6 is 23, and 8 is 31;	76
then, because 10 units in the right hand row make an unit in the	47
next row; therefore in 31 there are 3 units of the second row,	18
and an overplus of 1; write down the 1, and add the 3 to the	12
second row, saying, 3 that is carried and 1 is 4, and 1 is 5, and	—
4 is 9, and 7 is 16, and 2 is 18, in which is one unit of the third	181
row (had there been a 3d.) and an overplus of 8; write down the	
8, and add the 1 to the third row: But as there is no third row, the 1	
carried, must be wrote on the left hand of the 8; and 181 will be the	
sum of the five given numbers.	

Ex. II. *Add 476 — 3784 —*
 18329 — 290 — 75 — 7638
— and 46 together.

The given numbers }
 set in order will stand }
 thus

476
3784
18329
290
75
7638
46

The Sum

30638

Ex. III. *Add the numbers, 10768*
 — 3489 — 28764 — 289 — 6438
— 19 and 438 together.

The given numbers }
 placed as the rule di- }
 rects, stand thus

10768
3489
28764
289
6438
19
438

The Sum

50205

Ex. IV. *Add these numbers together.*

3720,45
25,036
4179,802
3,6284

Sum

7928,8840

Ex. V. *Add the following numbers together.*

15836,071
20,09
34,7
583,27008

Sum

16474,13108

In

In the two last examples, where are integer numbers and fractional numbers, it may be observed; that like integer places, and like fractional places, stand under each other; and the manner of adding them together, is the same as explained in the first Example.

25. It frequently happens, that numbers are to be added together, whose names do not increase in a tenfold manner as in the last Examples; such as in adding different sums of money, weights or measures; in which, regard is to be had to the number of those of a lower name, contained in one of its next greater name, as shewn in the preceding tables: Examples of which follow.

Ex. VI. Add the following sums of money together.

£.	s.	d.
353	14	8½
276	10	4
89	17	5½
34	12	10½
754	15	4½

Ex. VII. Add the following sums of money together.

£.	s.	d.
7683	08	2½
954	19	9½
682	10	7½
63	15	6½
9384	14	2½

In these two examples, the carriage is by 4 in the farthings; by 12 in the pence; by 20 in the shillings; and by 10 in the pounds.

Ex. VIII. Add the following Troy Weights together.

lb.	oz.	dwt.	gr.
218	10	13	18
176	9	19	23
85	11	17	11
24	8	15	21
506	5	07	01

Carry for 24, 20, 12, 10.

Ex. X. Add the following parts of Time together.

Weeks.	Da.	Ho.	Min.	Sec.
21	4	18	37	59
11	6	13	25	47
19	3	23	59	28
38	4	08	22	39
91	5	16	25	53

Carry for 60, 60, 24, 7, 10.

Ex. IX. Add the following Averdupoise Weights together.

Tons.	Cwt.	qrs.	lb.	oz.
535	17	3	22	11
94	19	1	27	13
158	12	0	18	15
7	15	2	13	08
797	05	0	26	15

Carry for 16, 28, 4, 20, 10.

Ex. XI. Add the following parts of a Circle together.

Deg.	"	"	"	"
176	32	59	43	25
85	59	27	31	59
114	28	45	59	14
67	12	38	24	47
444	13	51	39	25

Carry for 60, 60, 60, 60, 10.

Explanation of Example VI.

Three farthings and 1 farthing is 4 farthings, and 2 farthings is 6 farthings; which is a penny halfpenny, set down ½ and carry 1.

Then 1 and 10 is 11, and 5 is 16, and 4 is 20, and 8 is 28 pence; which is 2 shillings and 4 pence; set down 4, and carry 2.

Again, 2 and 12 is 14, and 17 is 31, and 10 is 41, and 14 is 55 shillings; which is 2 pounds 15 shillings; set down 15 shillings, and carry 2 pounds. The rest is easy.

B 4

26. S.F.C.

26. SECTION III. SUBTRACTION.

SUBTRACTION is the method of taking one number from another, and seeing the remainder, or difference, or excess.

The *subducent* is the number to be subtracted, or taken away.

The *minuend* is the number from which the subducent is to be taken.

RULE. 1st. Under the minuend write the subducent, so that like names stand under like names; and under them draw a line.

2d. Beginning at the right hand side, take each figure in the lower line from the figure standing over it, and write the remainder, or what is left, beneath the line, under that figure.

3d. But if the figure below is greater than that above it, increase the upper figure by as many as are in an unit of the next greater name; from this sum take the figure in the lower line, and write the remainder under it.

4th. To the next name, in the lower line, carry the unit borrowed And thus proceed to the highest denomination or name.

EXAMPLES.

Ex. I. *From 43656874 the minuend,
Take 249853642 the subducent,
Remains 186712232 the difference.*

Here the five figures on the right of the subducent may be taken from those over them: But the 6th figure, viz. 8, cannot be taken from the 5 above it. Now as an unit in the 7th place makes 10 in the 6th place, therefore borrowing this unit makes the 5, 15; then say, 8 from 15 leaves 7, which set down; and say 1 carried and 9 is 10, 10 from 6 cannot, but 10 from 16 leaves 6, set it down; then 1 carried and 4 is 5, 5 from 13 leaves 8, set it down; then 1 carried and 2 is 3, 3 from 4 leaves 1.

Ex. II. *From 7620908
Take 3875092
Remains 3745816*

Ex. III. *From 327,9563
Take 49,8697
Remains 278,0866*

Ex. IV. *From 30007,295
Take 2536,876
Leaves 27470,419*

Ex. V. *From 5000,
Take 479,6378
Leaves 4520,3622*

Ex. VI. *Borrowed 24 14 6½
Paid 18 12 4½
Remains 6 02 2½*

Ex. VII. *Lent 294 15 9¼
Received 89 18 10¼
Remains 204 16 10¼*

Ex. VIII.

Ex. VIII. In Sexagesimal.

Ex. IX. In Sexagesimals.

From	76	28	37	49	32
Take	65	29	16	53	45
Leaves	10	59	20	55	47

From	218	46	32	59	18
Take	149	52	47	53	29
Leaves	68	53	44	56	49

27.

QUESTIONS to exercise Addition and Subtraction.

QUEST. I. The share of Jack's prize money was 148*l.* 17*s.* 6½*d.*; And Tom received as much, beside 7*l.* 18*s.* Smart money: How much money did Tom receive?

	£.	s.	d.
Tom's prize money	148	17	6½
Smart money	7	18	0
Tom received	156	15	6½

QUEST. III. What year was King George born in, he being 67 years old in the year 1749?

Current year	1749
Age	67
Year born in	1682

QUEST. II. The Spanish invasion was in the year 1588, and the French attempted an invasion in the year 1744: How many years were between these fruitless attempts?

French	1744
Spanish	1588
Years between	156

QUEST. IV. Two ships depart from the same port, one having sailed 835 miles, is got 48 miles a-head of the other: Required the aftermost ship's distance?

The first ship's distance	835
Their difference	48
Second ship's distance	787

QUEST. V. A seaman who had received 46*l.* 17*s.* 6*d.* for wages, prize money, &c. meeting with bad company was tricked out of 18 guineas: Now John had reckoned to pay his wife's debts of 13*l.* 16*s.* 6*d.* and his landlady's bill of 16*l.* 12*s.* Required whether he can fulfil his intentions, and what will the difference be?

	£.	s.	d.
Money lost	18	18	0
Wife's debt	13	16	6
Landlady's bill	16	12	0
Total	49	6	6
Money received	46	17	6
He will want	2	9	0

QUEST. VI. Will and Frank talking of their ages in the year 1749: Will said he was born in the year of the Rebellion, in 1715, and Frank said he remembered he was ten years old the year King George the second was crowned, in 1727: Required the age of each, and the difference of their ages?

Current year	1749
Will was born	1715
Will's age	34
Current year	1749
King George crowned	1727
Years since	22
Frank's age then	10
Frank's age	32
So Will was oldest by two years.	

28. SEC.

28. SECTION IV. MULTIPLICATION.

MULTIPLICATION is the method of finding what a given number will amount to, when repeated as many times as is represented by another number.

The number to be multiplied, is called the *Multiplicand*.

The number multiplying by, is called the *Multiplier*.

And the number which the multiplication amounts to, is called the *Product*.

Both *Multiplicand* and *Multiplier* are called *Factors*.

Before any operation can be performed in Multiplication, it is necessary that the learner should commit to memory the following table.

29.

The MULTIPLICATION TABLE.

times	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Observe, that in multiplying any figure in the upper line by any figure in the left hand column, the product will stand right against the figure used in the left hand column, and under that used in the upper line. Thus was 6 to be multiplied by 9, seek the greater figure 9 in the upper line, and right under it, against 6 in the left hand, stands 54 for the Product. And so of others.

The foregoing table being well known, the work of Multiplication will be performed as follows.

To multiply any number, as

By any single figure, as by

$$\begin{array}{r} 37256 \\ 7 \end{array}$$

Set them as in the margin, and proceed

$$\begin{array}{r} 37256 \\ 7 \end{array} \quad \begin{array}{r} 260792 \end{array}$$

thus, 7 times 6 is 42, set down 2 and carry 4; 7 times 5 is 35 and 4 carried is 39, set down 9 and carry 3; 7 times 2 is 14 and 3 carried is 17, set down 7 and carry 1; 7 times 7 is 49 and 1 carried is 50, set down 0 and carry 5; 7 times 3 is 21 and 5 carried is 26, which set down, and the work is done. But for compound Multiplication take the following.

30. RULE. 1st. Write the Factors so, that the right hand place of the Multiplier stands under the right hand place of the Multiplicand.

2d. Multiply the Multiplicand severally by every figure of the Multiplier, setting the first figure of each line under the figure then multiplying by.

3d. Add the several lines together; and their sum is the Product.

4th. From the right hand of the Product, point off for fractions, as many places as there are fractional places in both Factors; and those to the left of the mark of distinction are integers.

5th. If

5th. If the number of places in the Product are not as many as the number of fractional places in both Factors, maké up that number by cyphers wrote on the left hand, and to these prefix the mark of distinction.

EXAM. I. Multiply 742 by 53.

The lesser Factor being wrote under the greater Factor, as here shewn, and a line drawn under them; say 3 times 2 is 6, write 6 under the 3; then 3 times 4 is 12, write down 2 and carry 1; and 3 times 7 is 21, and 1 carried is 22, write the 22: Again, 5 times 2 is 10, write 0 under the 5, and carry 1; then 5 times 7 is 35, and 1 carried is 37, write down the 37: Now add the two lines together found by multiplying by 3 and by 5, and their sum 39326 is the Product required.

Multiplicand 742 } Factors.
Multiplier 53 }

2226
3710

39326

Product

EXAMPLE II.

Multiply 28704
by 8631

28704

86112

172224

229632

247744224 Product.

EXAMPLE III.

Multiply 3684,2795
by 7,594

147371180

331585155

184213975

257899565

27978,4183230

Here, because there are 4 fractional places in the Multiplicand, and 3 in the Multiplier, which together make 7, therefore 7 places are pointed off on the right of the product for fractions.

EXAMPLE IV.

Multiply 936,287
by 607,02

1872574

65540090

56177220

568344,93474

The cyphers in the Multiplier of this example are thus managed. Having multiplied by the 2 as before, say 0 times 7 is 0, write 0 under the 0, and proceed to the next figure 7, by which multiply as before; then coming to the second 0, say, 0 times 7 is 0, write 0 under the place of the second 0, and proceed to the next figure 6, by which multiply as before.

EXAMPLE V.

Multiply 0,34796
by 0,0258

278368

173980

69592

0,008977368

Here, because there are 5 fractional places in one Factor, and 4 in the other, there should be 9 fractional places in the Product; and there arising but 7, therefore 2 cyphers are set on the left hand to make 9 places.

31. SECTION V. DIVISION.

DIVISION is the method of finding how often one number is contained in another; or may be taken from another.

The number to be divided, is called the *Dividend*.

The number dividing by, is called the *Divisor*.

The *Quotient* is the number arising from the division, and shews how many times the Divisor is contained in the Dividend.

The operations in Division are performed as follow.

32. RULE. 1st. On the right and left of the Dividend draw a crooked line; write the Divisor on the left side, and the Quotient, as it arises, on the right side of the Dividend.

2d. Seek how often the Divisor may be taken in as many figures on the left hand of the Dividend, as are just necessary; write the number of times it may be taken, in the Quotient; and there will be as many figures more in the Quotient, as there are figures remaining in the Dividend then not used.

3d. Multiply the Divisor by this Quotient-figure, set the Product under that part of the Dividend used; subtract, and to the right hand of the remainder bring down the next figure of the Dividend: Divide as before; and thus proceed until all the figures of the Dividend are used.

4th. If there is a Remainder, to its right hand side annex a cypher or cyphers, as if brought down from the Dividend, and divide as before; and thus it is that fractions arise, viz. from the Remainders in Division.

5th. When any figure of the Dividend is taken down, or annexed, as before shewn, and the Divisor cannot be taken in the number thus increased; put 0 in the Quotient, and take down, or annex, another figure; and proceed in like manner till the Divisor can be taken from the number.

6th. When fractions are concerned: From the number of fractional places used in the Dividend, take those in the Divisor; count the number of remaining places from the right of the Quotient, put the mark there; and those to the left are integers, those to the right fractions.

7th. If there arise not so many places in the Quotient as the 6th article requires, supply the places wanting with cyphers on the left, and to these prefix the fractional mark.

Ex. I. Divide 3656*l.* among 8 persons.

Set the given numbers as in Art. 1st. Now the two left hand figures contain 8; then say 8 is contained in 36, 4 times; set 4 in the Quotient, and say 4 times 8 is 32, set 32 under 36, subtract, there remains 4, to which bring down the next figure of the Dividend 5, makes 45; then say 8 is contained in 45, 5 times; set 5 in the Quotient, and say 5 times 8 is 40; write 40 under 45, subtract, and to the Remainder 5 take down 6, the next figure of the Dividend, makes 56; then say 8 is contained in 56, 7 times; write 7 in the Quotient, multiply 8 by 7 makes 56, which write under the other 56, and subtracting there remains 0: So it may be concluded, that 3656 contains 8, 457 times: Or, if 3656*l.* be divided among 8 persons, the share of each would be 457*l.*

Ex. II.

Ex. II. Divide 3125 by 25.

$$\begin{array}{r} 25 \overline{) 3125} \\ 25 \\ \hline 62 \\ 50 \\ \hline 125 \\ 125 \\ \hline 0 \end{array}$$

Ex. III. Divide 95296 by 47.

$$\begin{array}{r} 47 \overline{) 95296} \\ 94 \\ \hline \end{array}$$

See precept 5th. 126

$$\begin{array}{r} 94 \\ \hline \end{array}$$

$$\begin{array}{r} 329 \\ \hline \end{array}$$

$$\begin{array}{r} 329 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \hline \end{array}$$

Ex. IV. Divide 5859 by 124.

$$\begin{array}{r} 124 \overline{) 5859} \\ 496 \\ \hline \end{array}$$

$$\begin{array}{r} 899 \\ \hline \end{array}$$

$$\begin{array}{r} 868 \\ \hline \end{array}$$

310 for the Remaind.

248 See precept 4th.

$$\begin{array}{r} 620 \\ \hline \end{array}$$

$$\begin{array}{r} 620 \\ \hline \end{array}$$

Ex. V. Divide 337,27368 by 6,28.

$$\begin{array}{r} 6,28 \overline{) 337,27368} \\ 3140 \\ \hline \end{array}$$

the Quotient.

See precept 6th.

$$\begin{array}{r} 2327 \\ \hline \end{array}$$

$$\begin{array}{r} 1884 \\ \hline \end{array}$$

$$\begin{array}{r} 4433 \\ \hline \end{array}$$

$$\begin{array}{r} 4396 \\ \hline \end{array}$$

See precept 5th. 3768

$$\begin{array}{r} 3768 \\ \hline \end{array}$$

Ex. VI. Divide 2,3596 by 673,4.

$$\begin{array}{r} 673,4 \overline{) 2,3596} \\ 20202 \\ \hline \end{array}$$

Quot. 0,0035

33670 See precepts

33670 4th, 6th, 7th.

In Ex. VI. the 4 fractional places given in the Dividend, and the 0 used with the Remainder, make 5 fractional places; from which 1 place in the Divisor being taken, leaves 4 fractional places for the Quotient; but in the Quotient are only the two places 35, therefore 2 cyphers are prefixed, and makes ,0035, before which, for form sake, an 0 is set for the place of units.

33. When the Divisor does not exceed the number 12, the Division may be performed in one line; by making the Multiplication and Subtraction mentally, or in the mind, and carrying the Remainder, as so many tens, to the next figure.

34. In all operations of Division, it must be observed, that the Product of the Divisor by the Quotient-figure must not exceed that part of the Dividend then using; and the Remainder, by subtracting the Product, must ever be less than the Divisor.

As the Quotient multiplied by the Divisor makes the Dividend; So the Product of two numbers being divided by one of them, will give the other; that is, Division is proved by Multiplication, and Multiplication is proved by Division.

35. SECTION VI. REDUCTION.

REDUCTION is the method of reducing numbers from one name, or denomination, to another; retaining the same value.

CASE I. To reduce a number consisting of several names, to their least name.
RULE. 1st. Multiply the first, or greater name, by the parts which an unit thereof contains of the next less name; adding to the Product the parts of the second name in the given number.

2^d. Multiply this sum by the parts that an unit thereof contains of the next less name; adding to the Product the parts of the third name contained in the given number: And thus proceed, until the least name in the given number is arrived at.

Ex. I. In 23*l*. 14*s*. 6½*d*. how many farthings?

<i>l</i> .	<i>s</i> .	<i>d</i> .
23	14	6½
20		
	474	Shillings.
	12	
	5694	Pence.
	4	

Answer 22778 Farthings.

Ex. II. In 8*lb*. 10*oz*. of gold, how many grains?

<i>lb</i> .	<i>oz</i> .
8	10
12	
	106 Ounces.
	20
	2120 Pennyweights.
	24
	8480
	4240
	5080 Grains.

Ex. III. In a cannon weighing 2 Tons, 14*C*. 3*qr*. 19*lb*. how many pounds?

T.	C.	Qrs.	lb.
2	14	3	19
20			
	54	C. weight.	
	4		
	219	Qrs.	
	28		
	1771		
	438		
	6151	Pounds.	

An explanation of the first Ex. will make all the rest plain. Since pounds is the greatest name in the given number, and an unit thereof contains 20 of the next less name, or shillings; therefore multiply the pounds by 20, saying 0 times 3 is 0, to which adding the 4 in the 14*s*. makes 4; then 2 times 3 is 6, and the 1, in the place of tens in the shillings, make 7; then 2 times 2 is 4: Now multiply 474*s*. by 12, saying 12 times 4 is 48, and the 6 in the pence make 54, write 4 and carry 5; then 12 times 7 is 84 and 5 is 89, 8*9*. Lastly, multiply the 5694 pence by 4, saying 4 times 4 is 16, and the 2 farthings in the given number is 18, write 8 and carry 1, 8*9*.

Ex. IV. In 36 deg. 48'. 27". 59". how many thirds?

°	'	"
36	48	27
60		56
	2208	Minutes.
	60	
	132507	Seconds.
	60	
	7950476	Thirds.

36. CASE

36. CASE II. A number of an inferior name being given; to find how many of each superior denomination are contained therein.

RULE. 1st. Divide the given number, by such a number thereof, as makes one of the next superior name.

2d. Divide this Quotient by the parts making one of the next name.

3d. Divide this Quotient by the parts making one of the next name: And proceed in this manner, until the highest name is obtained.

4th. Then the last Quotient, and the several Remainders, will be the parts of the different names contained in the given number.

Ex. I. In 22778 farthings, how many pounds, shillings, and pence?

$$\begin{array}{r} 4 \overline{) 22778} \quad 2 \text{ Farthings.} \\ \underline{8} \\ 12 \end{array}$$

$$\begin{array}{r} 12 \overline{) 5694} \quad 6 \text{ Pence.} \\ \underline{72} \\ 20 \end{array}$$

$$\begin{array}{r} 20 \overline{) 47,4} \quad 14 \text{ Shillings.} \\ \underline{80} \\ 7 \end{array}$$

Answer 23*l.* 14*s.* 6½*d.*

Ex. III. In 6151 pounds, how many Tons, Hundreds, Quarters, Pounds?

$$\begin{array}{r} 28 \overline{) 6151} (219 \\ \underline{56} \\ 55 \\ \underline{56} \\ 1 \\ 28 \overline{) 5,4} \quad 14 \text{ C.} \\ \underline{112} \\ 42 \\ \underline{42} \\ 0 \end{array}$$

Answer 2*T.* 14*C.* 3*Qrs.* 19*lb.*

Ex. II. In 7950476 thirds of a degree, how many ° ' " "

$$\begin{array}{r} 6,0 \overline{) 795047,6} \quad 56 \text{ Thirds.} \\ \underline{36} \\ 13 \end{array}$$

$$\begin{array}{r} 6,0 \overline{) 13250,7} \quad 27 \text{ Seconds.} \\ \underline{18} \\ 12 \end{array}$$

$$\begin{array}{r} 6,0 \overline{) 220,8} \quad 48 \text{ Minutes} \\ \underline{12} \\ 0 \end{array}$$

Answer 36° 48' 27" 56'''

Ex. IV. In 50880 grains, how many Pounds, Ounces, Pennyweights, Grs.

$$\begin{array}{r} 24 \overline{) 50880} (2120 \\ \underline{48} \\ 2,0 \overline{) 212,0} \quad 0 \text{ dwt.} \\ \underline{28} \\ 24 \\ \underline{24} \\ 0 \overline{) 106} \quad 10 \text{ Oz.} \\ \underline{100} \\ 6 \end{array}$$

Answer 8*lb.* 10*oz.*

Explanation of Ex. I. Since 4 of the given number make one of the next name, pence; then 22778 divided by 4, give 5694 pence, and a Remainder of 2 farthings; then 5694 pence divided by 12, the number of pence in one of the next name, shillings, the Quotient is 474 shillings, and a Remainder of 6 pence; then 474 shillings divided by 20, the number of shillings in one of the next name, pounds, the Quotient is 23 pounds, and a Remainder of 14 shillings. And by the 4th precept, the answer is collected.

A like operation will solve the other examples, having regard to the increase of the different names.

37. In any Division, if the Divisor has one or more cyphers on the right hand, those cyphers may be pointed off; but then as many places must be pointed off from the Dividend, which places are not to be divided, but reckoned with the Remainder. See the above examples.

38. CASE

38. CASE III. To reduce a vulgar fraction to its equivalent decimal fraction.

RULE. To the Numerator annex one or more cyphers, divide this by the Denominator, and the Quotient will be the fraction sought. If the Division does not end when six figures are found in the Quotient, the work need not be carried any further.

EXAM. I. To reduce $\frac{423}{1269}$ to its equivalent decimal fraction.

Here 423 the Denominator is made the Divisor, and 15 the Numerator is set for the Dividend, to which annexing a cypher or two for fractional places, seek how often the Divisor can be had in 15 the integral part of the Dividend; and as it cannot be taken, put 0 in the Quotient for the place of units: Then taking in one fractional place, seek how oft the Divisor can be had in 150, say 0 times, and put another 0 in the Quotient for the place of primes: Now taking in two fractional places to the 15, the Divisor will be contained therein thrice, and thus proceed until the Division ends, or till 6 places arise in the Quotient: But in this example, as the 6th place would be 6, it is omitted, because cyphers on the right hand of decimal fractions are of no signification, as will evidently appear, Notation of Fractions being well understood.

$$\begin{array}{r}
 423 \overline{)15,00(0,03546} \\
 \underline{1269} \\
 2310 \\
 \underline{2115} \\
 1950 \\
 \underline{1692} \\
 2580 \\
 \underline{2538} \\
 420
 \end{array}$$

Ex. II. Reduce $\frac{2}{3}$ to a decimal fraction.
 $2 \overline{)1,0(0,5}$ Answer.

Ex. III. Reduce $\frac{4}{11}$ to a decimal fraction.
 $4 \overline{)1,00(0,25}$ Answer

Ex. IV. Reduce $\frac{3}{4}$ to a decimal fraction.
 $4 \overline{)3,00(0,75}$ Answer.

Ex. V. Reduce $\frac{5}{8}$ to a decimal fraction.
 $8 \overline{)5,000(0,625}$ Answer.

Ex. VI. Reduce $\frac{1}{3}$ to a decimal fraction.
 $3 \overline{)1,00(0,33 \text{ \&c.}}$ Answer.

Ex. VII. Reduce $\frac{7}{12}$ to a dec. fraction.
 $12 \overline{)7,0000(0,5833 \text{ \&c.}}$ Ans.

39. In the two last Quotients, it may be observed, that 3 would continually arise; such decimal fractions are called circulating, or recurring fractions: These have a peculiar kind of operation belonging to them, which the inquisitive reader will find in a book entitled *A General Treatise of Mensuration*,* the third edition, published in the year 1767; and also in other books.

* By the Author of these Elements.

40. CASE IV. *To reduce a number consisting of different names, to a decimal fraction of its greatest name.*

RULE. 1st. Write the given names orderly under one another, the least name being uppermost; and on their left side draw a line: Let these be reckoned as Dividends.

2^d. Against each name, on the left hand, write the number making one of its next superior name: And let these be the Divisors to the former Dividends.

3^d. Begin with the upper one, and write the Quotient of each division as fractions, on the right of the Dividend next below it; then let this mixed number be divided by its Divisor, &c.

And the last Quotient will be the decimal fraction sought.

Ex. I. *Reduce 15s. 9½d. to the fractional part of a pound sterling.*

First set the three farthings, the 9 pence, the 15 shillings 43 things set 4; against the pence set 12, and against the far- 12 9,75 shillings, 20; then the 3 with 0s. supposed to be annexed, being divided by 4, the Quotient, 75 is wrote on the right hand of the 9 pence; and the mixed number 9,75 with 0s. annexed as they are wanted, being divided by 12, the Quotient, 8125 is wrote on the right hand of the 15s. then this mixed number 15,8125 being divided by 20, the Quotient 0,790625 $\frac{1}{2}$ is the answer.

Ex. II. *Reduce 1s. 2½d. to the fractional part of a pound sterling.*

$$\begin{array}{r} 4 \overline{) 12} \quad 2,25 \\ 20 \overline{) 1,1875} \\ \hline 0,059375 \end{array}$$

Answer 1s. 2½d. = 0,059375 $\frac{1}{2}$.

Ex. IV. *Reduce 8oz. 15dwt. 18gr. to the fractional part of a pound troy.*

$$\begin{array}{r} 4 \overline{) 24} \quad 4,18 \\ 20 \overline{) 15,75} \\ 12 \overline{) 8,7875} \\ \hline 0,732291 \end{array}$$

Ans. 8oz. 15dwt. 18gr. = 0,732291lb.

41. Here because 24 is a number too great to divide by in one line, therefore it is broke into the parts 4 and 6, which multiplied together make 24.

Ex. III. *Reduce 48'. 17". 53''' to the fractional part of a degree.*

$$\begin{array}{r} 60 \overline{) 53} \\ 60 \overline{) 17,88333} \\ 60 \overline{) 48,298055} \\ \hline 0,804967 \end{array}$$

Ans. 48'. 17". 53''' = 0,804967 Deg.

Ex. V. *Reduce 3qrs. 19lb. 14oz. to the fractional part of a C. weight.*

$$\begin{array}{r} 16 \overline{) 4} \quad 4,14 \\ 28 \overline{) 19,875} \\ 4 \overline{) 3,709821} \\ \hline 0,927455 \end{array}$$

Ans. 3qr. 19lb. 14oz. = 0,927455C.

Here the 16 is broke into the numbers 4 and 4; and 28 into 4 and 7; and 14 is divided by 4; and the Quotient 3,5 by 4; &c.

42. CASE V. To reduce a decimal fraction of a superior name, to its value in inferior denominations:

RULE. 1st. Multiply the given fraction by the number that an unit of its name contains units of the next lesser name; from the right hand of the Product point off as many places as are in the given fraction.

2d. Multiply the places, so pointed off, by as many as an unit of this name contains of the next less name; point off as before.

And thus proceed until the multiplication is made by the least name. 3d. Then the integers, or the numbers on the left of the distinguishing marks in each Product, will be the parts in each name, which together are equal to the given fraction.

EXAMPLE I. What number of shillings, pence, and farthings, are equal in value to 0,790625 £ . Sterling?

Here an unit of the given name £ . contains 20 of the next less name shillings; then multiplying by 20, and pointing off 6 places on the right, because the given number 0,790625 contains 6 fractional places, the Product is 15,812500 shillings; then the fractions of this number, viz. 812500 multiplied by 12, the number that an unit of this name contains of the next less name, and the Product pointed as before, there arises 9,750000 pence; the fractions of this number multiplied by 4, gives 3,000000 farthings; then the parts pointed off on the left, viz. 15s. 9 $\frac{1}{4}$ d. are the value of the given fraction.

$$\begin{array}{r} \text{£. } 0,790625 \\ \hline \text{s. } 15,812500 \\ \hline \text{d. } 9,750000 \\ \hline \text{far. } 3,000000 \end{array}$$

EXAMPLE II. What is the value of 0,956285 £ . Sterling?

$$\begin{array}{r} \text{£. } 0,956285 \\ \hline \text{s. } 1,125700 \\ \hline \text{d. } 1,5084 \\ \hline \text{far. } 2,0336 \end{array}$$

EXAMPLE IV. What is the value of 0,732291lb. troy?

This example worked as above, by multiplying by 12, 20, 24, the value will be found to be

8oz. 15dwts. 18gr. nearly.

EXAMPLE III. What is the value of 0,58695 degrees?

$$\begin{array}{r} \text{Deg. } 0,58695 \\ \hline \text{Min. } 35,21700 \\ \hline \text{Sec. } 13,0200 \\ \hline \text{Thirds } 0,120 \end{array}$$

EXAMPLE V. What is the value of 0,927455 part of a C. weight?

By operating as above, multiplying by 4, 28, 16, the answer will be

3qrs. 19lb. 14oz. nearly.

43. QUESTIONS

43. QUESTIONS to exercise the preceding rules.

QUEST. I. *A sloop with the captain and 26 hands take a prize which sold for 1578*£*. whereof each seaman had 45*£*. and the captain the rest: How much was his share?*

26 Men.
45 *£*. to each

130
104

subtract 1170*£*. the crew's share.
from 1578*£*. the whole prize.

remains 408*£*. the captain's share.

QUEST. III. *A seaman, whose wages is 35*s*. 6*d*. a month, returns home at the end of 29 months; he having taken up 12*£*. 18*s*.: How much has he to receive?*

s. d.
35 6
mult. by 12

426 pence a month,
mult. by 29 months,

3834
852

12) 12354 pence

2,0) 102,9 6*d*.

from 51*£*. 9*s*. 6*d*. = wages
take 12*£*. 18*s*. 0*d*. received,

remains 38*£*. 11*s*. 6*d*. to receive.

QUEST. V. *In 306 crowns, how many half crowns and pence?*

Answer { 612 half crowns.
18360 pence.

QUEST. VII. *A seaman's share of a prize was 14 guineas, 32 moidores, 12 thirty-six shilling pieces, and 52 pistoles at 17*s*. each: How much sterling did the whole come to?*

Answer 123*£*. 14*s*.

QUEST. II. *A boat's crew of 15 men got by plunder 321*£*. How much was the share of each?*

15) 321(21 *£*.

30

21

15

remains 6*£*. which
mult. by 20*s*. in 1*£*.

15) 120*s*. (8*s*.
120.

Answer 21*£*. 8*s*. to each.

QUEST. IV. *Six mess-mates, who propose to live well during an East-India voyage of 22 months, agree to expend among them 5*s*. a day, besides the ship's allowance: Now one of them having but 25*s*. a month, how will matters stand with him at the end of the voyage?*

Now 28 days, at 5*s*. a day, makes 140*s*. or 7*£*. a month; which for 22 months, is 154*£*.

Then a sixth part of 154*£*. is 25*£*. 13*s*. 4*d*. for each man.

Also 25*s*. a month for 22 months makes 27*£*. 10*s* for wages; which will overpay his expences, by 1*£*. 16*s*. 8*d*.

QUEST. VI. *In 30 chaldiers of coals, each of 36 bushels, how many pecks?*

Answer 4320 pecks.

QUEST. VIII. *Suppose a ship sails 5½ miles an hour for 14 days: How many degrees and minutes has she sailed in the whole; 60 sea miles making one degree?*

Answer 30 deg. 48 min.

C 2

S E C.

SECTION VII. OF PROPORTION: OR, THE RULE OF THREE.

44. Four numbers are said to be proportional, when by comparing them together by two and two, they either give equal Products or equal Quotients.

Suppose these four numbers 3 8 12 32.

In comparing them together by multiplication,

The Product of 3 and 8 is 24; of 12 and 32 is 384, unequal.
 of 3 and 12 is 36; of 8 and 32 is 256, unequal.
 of 3 and 32 is 96; of 8 and 12 is 96, equal.

Therefore 3 8 12 32, are called proportional numbers.

Now let them be compared together by division.

The Quotient of 8 by 3 is 2,6 $\frac{2}{3}$. of 32 by 12 is 2,6 $\frac{2}{3}$. equal.
 of 12 by 3 is 4 of 32 by 8 is 4, equal.
 of 32 by 3 is 10,6 $\frac{2}{3}$. of 12 by 8 is 1,5, unequal.

Therefore by this comparison, the numbers are said to be proportional.

In this kind of comparing four numbers together, there is no need to try for more equal Products, or Quotients, than one set of either sort; for either case will determine the proportionality independent of the other.

But it must be observed, that among four proportional numbers, there will be but one set of equal Products, and two sets of equal Quotients, the smaller numbers being Divisors.

45. When four numbers are to be wrote as proportionals, they must be placed in such order, that the Product of the first and fourth be equal to the Product of the second and third.

A question is said to belong to the *Rule of Three*, when three numbers or terms are given to find a fourth proportional, which is the answer to the question.

And in order to resolve such questions, the three given terms must be first placed in a proper order, which is called stating the terms of the question.

46. Questions in the *Rule of Three* are stated, and resolved by the following precepts.

1st. Consider of what kind the fourth term, or number sought, will be, whether money, weight, measure, time, $\&c.$ and among the three numbers given in the question, let that which is of the same kind with what is required, be placed for the third term.

2d. From the nature of the question, determine whether the number sought will be greater or less than the number which is placed for the third term.

3d. If

QUEST. III. *What will 1836 lb. of raisins come to, at the rate of 6 s. 8 d. for 24 lb.?*

Here as money is the thing sought, money must be the 3d term: And as 6 s. 8 d. consists of two names, they must be reduced to one name, viz. pence.

$$\begin{array}{r} \text{lb.} \quad \text{lb.} \quad \text{s.} \quad \text{d.} \\ 24 \text{ — } 1836 \text{ — } 6 \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 1836 \quad 12 \\ 8 \text{ q.} \quad \hline \end{array}$$

24) 146880 (6120 d. = 4th term.

$$\begin{array}{r} 144 \quad \hline 28 \quad \hline 24 \quad \hline 48 \quad \hline 0 \quad \hline \end{array}$$

$$\begin{array}{r} 12 \quad 6120 \\ 24 \quad 2,0 \quad 51,0 \quad 10 \text{ s.} \\ \hline \end{array}$$

$$\begin{array}{r} 48 \quad 25 \text{ £.} \\ 48 \quad \hline 0 \quad \hline \end{array}$$

Here the 2d term being multiplied by the 3d, and the Product divided by the 1st, the Quotient is 6120 pence; which being valued, gives 25 £. 10 s.

QUEST. IV. *If 20 yards of cloth 5 quarters wide will serve to hang a room: How many yards of 4 quarters wide will serve to hang the same room?*

Here yards of length are required; then 20 yards must be the 3d term.

$$\begin{array}{r} \text{qrs.} \quad \text{yds.} \\ 4 \text{ — } 5 \text{ — } 20 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \text{ — } 5 \text{ — } 20 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \text{ — } 100 \text{ (} \\ \hline \end{array}$$

Answer 25 yards.

QUEST. V. *What will 420 yards of cloth come to, at 14 s. 10½ d. for 1 ell English?*

The term sought being money, the 14 s. 10½ d. must be the 3d term, and be reduced to farthings; also the 1st and 2d terms are to be reduced to quarters of yard.

$$\begin{array}{r} \text{Ell Eng.} \quad \text{yds.} \quad \text{s.} \quad \text{d.} \\ 1 \text{ — } 420 \text{ — } 14 \quad 10\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 1 \quad 420 \quad 14 \quad 10\frac{1}{2} \\ 5 \quad \hline 5 \quad \hline \end{array}$$

$$\begin{array}{r} 1680 \quad 178 \\ 715 \quad \hline 8400 \quad \hline 1680 \quad \hline 11760 \quad \hline \end{array}$$

$$\begin{array}{r} 5 \text{) } 1201200 \text{ (} \\ \hline 4 \text{) } 240240 \text{ farthings = 4th term.} \\ \hline 12 \text{) } 60060 \text{ pence.} \\ \hline 2,0 \text{) } 500,5 \quad 5 \text{ shillings,} \\ \hline 250 \text{ pounds.} \\ \hline \end{array}$$

$$\begin{array}{r} 715 \text{ far. = 3d term.} \\ \hline \end{array}$$

4) 240240 farthings = 4th term.

$$\begin{array}{r} 12 \text{) } 60060 \text{ pence.} \\ \hline \end{array}$$

$$\begin{array}{r} 2,0 \text{) } 500,5 \quad 5 \text{ shillings,} \\ \hline 250 \text{ pounds.} \\ \hline \end{array}$$

$$\begin{array}{r} 250 \text{ £. } 5 \text{ s.} \\ \hline \end{array}$$

The Divisor 5 being a single digit, the Quot. is wrote under the Divid.

QUEST. VI. *A owes to B 463 £. but compounds for 7 s. 6 d. in the pound: How much must B receive for his debt?*

Here composition money is the thing sought; then the 3d term must be the composition money, viz. 7 s. 6 d.

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 1 \text{ — } 463 \text{ — } 7 \quad 6 \\ \hline 12 \text{) } 41670 \text{ 6d. } 90 \text{ pence.} \\ \hline 2,0 \text{) } 347,2 \text{ 12s.} \\ \hline 173 \quad \hline \end{array}$$

$$\begin{array}{r} 173 \quad \hline \end{array}$$

Answer 173 £. 12 s. 6 d.

48. As it will be more convenient in most cases to reduce such numbers, or terms, which consist of several names, to the fractional parts of their greatest name, than to reduce them to their lowest name; therefore in the solution of some of the following questions, the inferior parts of the given terms are reduced by Case IV. of Reduction; and the answers are valued by Case V.

QUEST.

QUEST. VII. If 8 lb. of pepper cost 4 s. 8 d.: What will 7 C. 3 qrs. 14 lb. come to at that rate?

lb.	C.	qrs.	lb.	s.	d.
8	—	7	3	14	—
4	—	—	—	—	8
—	—	—	—	—	12
31	—	—	—	—	—
28	—	—	—	—	56
—	—	—	882 lb.	56	—
262	—	—	—	—	—
62	—	—	—	5292	—
—	—	—	—	4410	—
882	—	—	—	—	—
—	8	49392	—	—	—
—	—	—	12	6174	6d.

2,0151,4 14s.

Answer 25£. 14s. 6d.

QUEST. IX. What is the interest of 584£. for a year, at 5 per cent. per annum: Or at the rate of 5£. for the use of 100£. for a year?

Here interest is the term required; therefore 5£. the interest of 100£. is to be the 3d term: And as the 4th term, or the interest of 584£. is greater than the 3d term; then the 2d term is to be greater than the 1st.

£.	£.	£.
100	—	584 — 5

1,00) 29,20 (See Case V. of Reduction.

20
4,00

Answer 29£. 4s.

QUEST. XI. What is the interest of 542£. 10s. for 219 days, at 5£. per cent. per annum?

To solve this question, find the interest for 1 year; multiply this interest by 219, and divide the Product by 365. the Quotient will be the answer; and is 16£. 5s. 6d.

QUEST. VIII. One bought 4 Hbds. of sugar, each containing 6 C. 2 qrs. 14 lb. at 2£. 8s. 6d. for each C. weight: What did the whole come to?

C.	qrs.	lb.	£.	s.	d.
1	—	6	2	14	—
—	—	—	—	—	8 6

Now 1 C. weight is 112 lb.
And 4 Hbds. at 6C. 2q. 14b. = 2968 lb.
Also 2£. 8s. 6d. is 582d.
Then the Product of the 2d and 3d terms is 1727376.

Which divided by the 1st term 112, the Quotient is 15423 pence, whose value is 64£. 5s. 3d.

QUEST. X. What is the interest of 387£. 12s. for 3 years and 4 months, at 3½ per cent. per annum?

Find the interest for 1 year; then thrice that, together with ¼ of one year, will be the interest sought.

£.	£.	£.
100	—	387,6 — 3,5

19380
11628

100) 1356,60
13,566 for 1 year.

3

40,698 for 2 years.

⅓ of 1 year = 4,522 for 4 months.

The sum 45,220 is the interest.

Answer 45£. 4s. 5d.

QUEST. XII. For how long must 487£. 10s. be at simple interest, at 4½£. per cent. per annum, to gain 95£. 1s. 3d.?

Find what will be the interest of 487£. 10s. for 1 year; divide 95£. 1s. 3d. by this interest, and the Quotient will be 4½ years.

QUEST. XIII. One bought 14 pipes of wine, and is allowed 6 months credit: But for ready money gets it 6 d. in a gallon cheaper: How much did he save by paying of ready money?

Answer 44*l.* 2*s.*

QUEST. XV. One bought 3 tons of oil for 153*l.* 9*s.* which having leaked 74 gallons, he would make the prime-cost of the remainder: How must it be sold per gallon?

Now 1*T.* = 252 Gal. And 3*T.* = 756
Subtract the gallons leaked = 74

Remains 682

G. G. *l.*

Then 682 — 1 — 153,45
Answer 4*s.* 6*d.* a gallon.

QUEST. XVII. At 13*l.* for 100 lb. of goods: What will 895 lb. come to, allowing 4 lb. upon every 100 lb. for tret, or waste?

Since 4 lb. is to be allowed on the 100 lb. therefore 104 lb. is given for 100.

lb. lb. *l.*
Then 104 — 895 — 13

Answer 111*l.* 17*s.* 6*d.*

QUEST. XIX. If 100 pounds of sugar be worth 36*s.* 8*d.* What will be the worth of 875 lb. rebating 4 lb. upon every 100 lb. for tare?

Here the buyer has 100 lb. on paying for 96 lb.

lb. lb. lb.
Then 100 — 96 — 875

And the 4th term will be 840 lb.

Also 100 — 840 — 1,833333
Then the 4th term will be 15*l.* 8*s.* and so much will the sugar come to.

QUEST. XIV. A clothier sold 50 pieces of kersey, each piece containing 34 ells Flemish, at the rate of 8*s.* 4*d.* per ell English: What did the whole come to?

Answer 425*l.*

QUEST. XVI. A broker sold $\frac{2}{3}$ of $\frac{1}{4}$ of a ship for 147 *l.* 11*s.* 3*d.*: How much was the whole ship valued at?

Now $\frac{2}{3}$ of $\frac{1}{4}$ = $\frac{1}{6}$ $\times$ $\frac{1}{4}$ = $\frac{1}{24}$ = $\frac{6}{144}$ by art. 38.
For 2 $\times$ 3 = 6, a new numerator.

And 5 $\times$ 4 = 20, a new denominator.
Also 147*l.* 11*s.* 3*d.* = 147,5625 art. 40.
share share *l.*

Then 0,3 — 1 — 147,5625 art. 46.

Answer 491*l.* 17*s.* 6*d.*

QUEST. XVIII. One has cloth which cost 2*s.* 8*d.* a yard: For how much must it be sold a yard on 3 months credit, to gain 25*l.* per cent. per annum?

mon. mon. *l.* *l.*
First 12 — 3 — 25 — 6,25

l. *l.* *l.*

Secondly 100 — 6,25 — 0,133333

By multiplying and dividing, the 4th term will be found 2*d.*

Then 2*s.* 8*d.* + 2*d.* = 2*s.* 10*d.* a yard, the selling price.

QUEST. XX. A chapman bought 81 kerseys for 135*l.* How must he sell them per piece to gain 15*l.* per cent.?

Find how much 135 *l.* will be advanced to, at 15*l.* per cent.

Then this sum divided by 81 will be the selling price of each piece.

l. *l.* *l.* *l.*

Now 100 — 115 — 135 — 155,25

Then 81 155,25 (1,916666 *l.*

Answer 1*l.* 18*s.* 4*d.* a-piece.

QUEST.

QUEST. XXI. *A merchant who is to receive a sum of money, is offered ducats at 6 s. 4 d. which are worth but 6 s. 2½ d. or chequins at 8 s. 2 d. each, that are worth but 8 s. By which specie will he sustain the least loss?*

6 s. 2½ d. = 74, 5d. } the real value.
 8 s. 0 d. = 96
 6 s. 4 d. = 76 } the advan. val.
 8 s. 2 d. = 98

r. val. r. val. ad. va. ad. va.
 Then 74, 5 — 96 — 76 — 97, 93
 But the chequins are valued at 98 d.
 Therefore the ducats are most advantageous.

QUEST. XXIII. *A person wants 750 pieces of foreign coin, each worth 11 s. 4 d. How much will they come to, allowing the broker the worth of 2 pieces upon every 100?*

Now 100 — 102 — 750 — 765.
 He must pay for 765 pieces, which will come to 433 £. 10s.

QUEST. XXV. *A grocer bought 4½ C. of pepper for 15 £. 17 s. 4 d. which proving to be damaged, he is willing to lose 12½ £. per cent. For how much must he sell it a lb.?*

Since he is to lose 12½ per cent. he must take 87 £. 10 s. for 100 £.
 Now diminish the 15 £. 17 s. 4 d. in this proportion, and this sum divided by the pounds in 4½ C. will give 7 d. for what each pound is to be sold at.

The Rule of Proportion is of almost universal use in all business where computation is required; as in buying and selling, values of stocks and their dividends; the interest and discount of money; the customs and duties on goods, &c. But the designed brevity of this book will not permit farther illustrations.

QUEST. XXII. *One who had sold a parcel of cloths at 2 s. 10 d. a yard on 3 months credit, found he had gained 25 £. per cent. per annum: What did the cloth cost per yard?*

mo. £. mo. £.
 Now 12 — 25 — 3 — 6, 25
 And 100 + 6, 25 = 106, 25 £.

£. £.
 Then 106, 25 — 100 — 0, 14166

The fourth term to which will be a fraction, whose value will be 2s. 8d. which is the prime-cost per yard of the cloth.

QUEST. XXIV. *A gentleman would exchange 729 pieces of 4 s. 2 d. each into sterling money: How much will he receive for them, allowing the broker 1½ £. per cent.?*

P. P. £. £.
 Now 1 — 729 — 0, 208333 — 151, 875
 the worth of the pieces.
 Then 101, 25 — 100 — 151, 875 — 150 £.
 He will receive 150 £. for them.

QUEST. XXVI. *Suppose 42 gallons of honey be valued at 2 £. and the duty is 15 £. per cent. on this value, and a draw back of 5 £. per cent. on the duty for prompt payment: What will the ready money duty of 672 gallons come to?*

Now 42 G. : 672 G. :: 2 £. : 32 £.
 And 100 £. : 15 £. :: 32 £. : 4, 8 £.
 Also 100 £. : 95 £. :: 4, 8 £. : 4, 56 £.

Answer 4 £. 11 s. 2½ d.

SECTION VIII. OF THE POWERS OF NUMBERS,
AND OF THEIR ROOTS.

49. *The Power of a number, is a product arising by multiplying that number by itself, the product by the same number, this product by the same number, &c. to any number of multiplications.*

50. The given number is called the first power or root.

The Product of the 1st power by itself, is the second power, or square.

The Product of the 2d power by the 1st, is the 3d power, or cube.

The Product of the 3d power by the 1st, is the 4th power, &c.

51. Here follows the 1st, 2d, and 3d powers of the nine digits.

Roots, or 1st power	1	2	3	4	5	6	7	8	9
Squares, or 2d power	1	4	9	16	25	36	49	64	81
Cubes, or 3d power	1	8	27	64	125	216	343	512	729

Ex. I. *What is the 2d power, or square of the number 24?*

Ex. II. *What is the 3d power, or cube of 38?*

Now $38 \times 38 = 1444$ the 2d power.

Then $1444 \times 38 = 54872$ the 3d power.

$24 \times 24 = 576$ is the 2d power.

The figure, or number, shewing the name of any power, is called the index of that power.

Thus 1 is the index of the first power; 2 is the index of the 2d power; 3 of the third power, &c. Also $\frac{1}{2}$ is the index of the square root; $\frac{1}{3}$, the index of the cube root, &c.

52. Any number may be considered as a power of some other number.

Thus 64 may be taken as the 2d power of 8, and the third power of 4, &c.

53. The root of a given number considered as a power, is a number which being raised to the index of that power, will either be equal to the given number, or approach very near to it.

54. *To extract the Square Root of a given number.*

RULE. 1st. Begin at the unit's place, put a point over it, and also over every next figure but one, reckoning to the left for integers, and to the right for fractions; and there will be as many integer places in the root, as there are points over the integers in the given number.

The figure under a point, with its left hand place, is called a period. 2d. Under the left hand period write the greatest square contained in it, and set the root thereof in the Quotient; subtract the square, and to the remainder bring down the next period, as in Division.

3d. On the left of this Remainder write the double of the Root or Quotient for a Divisor; seek how often this may be had in the Remainder, except the right hand place; write what ariseth both in the Root, and on the right of the Divisor.

4th. Multiply this increased Divisor by the last Quotient-figure; subtract, and to the Remainder bring down the next period; double the Root for a Divisor, and proceed as before.

55. Fractional places will arise in the Root, by annexing to the Remainders, periods of two cyphers each, and renewing the operation.

Ex.

Ex. I. *What is the Square Root of 1444?*

Put a point over the units place 4, and also over the place of 100s.

Now the number consists of 2 periods, and will have 2 integer places in the Root: Then the greatest Square in 14, the left hand period, is 9 and its Root is 3; write 9 under the period, and 3 in the Root; now 9 from 14 leaves 5, to which annex the next period 44; the Root 3 doubled makes 6, which in 54 is contained 8-times, annex 8 to the 3 in the Quotient, and to the Divisor 6, makes the Root 38 and the Divisor 68; then 8 times 68 is 544; and there remaining 0, on subtraction, it may be concluded, that 38 is the true Root.

$$\begin{array}{r} 1444 \text{ (38 Root.} \\ 9 \end{array}$$

$$\begin{array}{r} 68) 544 \\ 544 \\ \hline \end{array}$$
Ex. II. *What is the Square Root of 36372961?*

$$\begin{array}{r} 36372961 \text{ (6031 Root.} \\ 36 \end{array}$$

$$\begin{array}{r} 1203) 3729 \\ 3609 \\ \hline \end{array}$$

$$\begin{array}{r} 12061) 12061 \\ 12061 \\ \hline \end{array}$$
Ex. IV. *What is the Square Root of 24681024?*

Answer 4968.

Ex. VI. *What is the Square Root of 76395820?*

$$\begin{array}{r} 76395820 \text{ (8740,4702} \\ 64 \end{array}$$

$$\begin{array}{r} 167) 1239 \\ 1169 \\ \hline \end{array}$$

$$\begin{array}{r} 1744) 7058 \\ 6976 \\ \hline \end{array}$$

$$\begin{array}{r} 174804) 822000 \\ 699216 \\ \hline \end{array}$$

$$\begin{array}{r} 1748087) 12278400 \\ 12236609 \\ \hline \end{array}$$

$$\begin{array}{r} 174809402) 417910000 \\ 349618804 \\ \hline 68291196 \end{array}$$
Ex. III. *What is the Square Root of 1,0609?*

$$\begin{array}{r} 1,0609 \text{ (1,03 Root.} \\ 1 \end{array}$$

$$\begin{array}{r} 203) 0609 \\ 609 \\ \hline \end{array}$$
Ex. V. *What is the Square Root of 911236798,794365?*

Answer 30186,699, &c.

$$\begin{array}{r} 911236798,794366 \text{ (30186,6} \\ 601) 1123 \\ 6028) 52267 \\ 60366) 404308 \\ 603726) 4220279 \\ 597923 \\ \hline \end{array}$$

Here the products are omitted, the multiplication and subtraction being made in the mind.

In the VIth example, after all the periods given were brought down, there remained 8220, to which a period of two cyphers was annexed, and the operation renewed, and continued until 4 decimal places were obtained in the Root; every period brought down giving one place.

56. 70

Ex. II. *What is the Cube Root of 518749442875?*

$$\begin{array}{r}
 518749442875 \cdot (8 \\
 \underline{512} \\
 8,000) \underline{6749442,875} \\
 \quad 843680 \\
 \quad \underline{192000000} \\
 192843680 (6749442875 (035 \\
 \quad 578531040 \\
 \quad \underline{964132475} \\
 \quad 964218400
 \end{array}$$

In this example, because 3 periods were remaining, and consequently 3 places more to be found; therefore in the last division a point is put over the 3d place from the right hand, and the Divisor is first to be tried in the Dividend as far as this point, in which as it cannot be taken, 0 is put in the Quotient, &c. here the last figure 5 is too much, but it is much nearer to 5 than to 4; then 035 annexed to the first Root 8, makes 8035 for the Root.

Ex. III. *What is the Cube of 114604290,028?*

$$\begin{array}{r}
 114604290,028 \\
 \underline{64} \\
 4,00) \underline{50604290} \\
 \quad 126510 \\
 \quad \underline{480000} \\
 \quad 606510) \underline{50604290 (8} \\
 \quad \quad 48 \\
 \quad \quad \underline{48} \\
 \quad \quad 384 \\
 \quad \quad \underline{192} \\
 \quad \quad 2304 \\
 \quad \quad \underline{48} \\
 \quad \quad 18432 \\
 \quad \quad \underline{9216} \\
 \quad \quad 110592
 \end{array}$$

Here 480 is taken for the Root at the first operation.

Then 114604290,028 (48
110592

480) 4012290 The work of the Division is supposed to be done on a waste paper.

8358,9 the Quotient.

691200 = triple the Square of 480, viz. 230400 × 3.

Divisor 699558,9) 4012290,028 (5736
... 34977945

To 480 the first Root
Add 5,736
Sum 485,736

248043
209867

38176

57. Here, instead of bringing down the figures of the Dividend to the Remainders, the Divisor is lessened each time, by pointing off a place on the right; but regard is to be had to the carriage which will arise from the places thus omitted.

SECTION

SECTION IX. OF NUMERAL SERIES.

58. A rank of three or more numbers that increase or decrease by an uniform progression, is called a *Numeral Series*.

59. If the Progression is made by equal differences, that is, by the constant addition or subtraction of the same number; the series is called an *Arithmetic Progression*.

Thus $\left\{ \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 & 27 \\ 49 & 43 & 37 & 31 & 25 & 19 & 13 & 7 & 1 \end{array} \right.$ $\begin{array}{l} \text{Etc. increasing by adding 1,} \\ \text{Etc. increasing by adding 3,} \\ \text{Etc. decreasing by subtracting 6,} \end{array}$

are ranks of numbers in Arithmetic Progression: and of such ranks there may be an infinite variety.

60. If the Progression is made by a constant multiplication or division with the same number, the series is called a *Geometric Progression*.

Thus $\left\{ \begin{array}{cccccccc} 1 & 2 & 4 & 8 & 16 & 32 & 64 & \text{Etc. increasing by 2,} \\ 1 & 5 & 25 & 125 & 625 & 3125 & 15625 & \text{Etc. increasing by 5,} \\ 6561 & 2187 & 729 & 243 & 81 & 27 & 9 & \text{Etc. decreasing by 3,} \\ 16384 & 4096 & 1024 & 256 & 64 & 16 & 4 & \text{Etc. decreasing by 4,} \end{array} \right.$

are ranks of numbers in Geometric Progression; and of such ranks there may be an infinite variety.

61. The common Multiplier or Divisor is called the *ratio*.

Thus 2 is the ratio in the 1st rank, 5 in the 2d rank, 3 is the ratio in the 3d rank, and 4 in the 4th rank.

62. In any series of terms in Arithmetic Progression, the sum of any two terms, considered as extremes, is equal to the sum of any two terms taken as means equally distant from the extremes.

Thus in 3 terms (where the 1st and 3d are extremes, and the other the mean) viz. $6 \cdot 9 \cdot 12$, then $6 + 12 = 9 + 9 = 18$.

And in 4 terms, viz. $13 \cdot 19 \cdot 25 \cdot 31$.

Then $13 + 31 = 19 + 25 = 44$.

Also in the terms $49 \cdot 43 \cdot 37 \cdot 31 \cdot 25 \cdot 19 \cdot 13 \cdot 7 \cdot 1$.

Then $49 + 1 = 43 + 7 = 37 + 13 = 31 + 19 = 25 + 25 = 50$.

63. In a series of terms in Geometric Progression, the Product of any two terms considered as extremes, is equal to the Product of any two intermediate equidistant terms considered as means.

Thus in 3 terms, viz. $5 \cdot 25 \cdot 125$. Or $3 \cdot 9 \cdot 27$.

Then $5 \times 125 = 25 \times 25 = 625$. Also $3 \times 27 = 9 \times 9 = 81$.

And in 4 terms $4 \cdot 8 \cdot 16 \cdot 32$.

Then $32 \times 4 = 16 \times 8 = 128$.

Also in the terms $1 \cdot 4 \cdot 16 \cdot 64 \cdot 256 \cdot 1024 \cdot 4096 \cdot 16384$.

Then $16384 \times 1 = 4096 \times 4 = 1024 \times 16 = 256 \times 64 = 16384$.

64. In

64. In any Arithmetic Progression, the sum of any two terms lessened by the first term; or their difference increased by the first term, will be a term also in that progression.

Thus in the Progression $1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13 \cdot 15 \cdot 17 \cdot 19 \cdot 21 \text{ \&c.}$

Then $7 + 11 = 18$, and $18 - 1 = 17$ is a term of the Progression.

Also $11 - 7 = 4$, and $4 + 1 = 5$ is a term of the Progression.

65. In any Geometric Progression, the product of any two terms divided by the first term; or the Quotient of any two terms-multiplied by the first term, will give a term also in that series.

Thus in the Progression $3, 6, 12 \cdot 24 \cdot 48 \cdot 96 \cdot 192 \cdot 384 \cdot 768 \text{ \&c.}$

Then $\frac{12 \times 96}{3} = 384$; and $\frac{192}{12} \times 3 = 48$, are terms in the Progression.

66. If over a series of terms in Geometric Progression, be wrote a series of terms in Arithmetic Progression, whose first term is 0, and common difference is 1, term for term; then any term in the Arithmetic Series, will shew how far its corresponding term in the Geometric Series is distant from the first term.

Thus $\begin{Bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{Bmatrix} \text{ \&c. Arithmetic Series.}$

$\begin{Bmatrix} 1 & 3 & 9 & 27 & 81 & 243 & 729 \end{Bmatrix} \text{ \&c. Geometric Series.}$

Here 729 is distant from the 1st term, 6 terms; 243 is distant 5 terms, 81 is distant 4 terms.

67. The terms of the Arithmetical Series are called indices to the terms of the Geometric Series.

Thus 5 is the index to 243; 3 is the index to 27; 1 is the index to 3; \&c.

68. PROBLEM I. In an Arithmetic Progression: Given the first term, the common difference, and the number of terms.
Required the last term.

RULE. Multiply the number of terms less one by the common difference, to the Product add the first term, and the sum will be the last term.

Ex. I. Suppose 1 and 9 to be the first and second terms, of an Arithmetic Progression of 1074 terms: What is the last term?

Here $9 - 1 = 8$ is the com. diff. Now $1074 - 1 = 1073$.
And $1073 \times 8 = 8584$. Then $8584 + 1 = 8585 = \text{last term.}$

Ex. II. A person agrees to discharge a certain debt in a year, by weekly payments, viz. the first week 5s. the 2d week 8s. &c. constantly increasing each week by 3s.: How much was the last payment?

$5 = 1^{\text{st.}}$ term. Now $52 - 1 = 51$.

$3 = \text{com diff.}$ And $51 \times 3 = 153$.

$52 = \text{N}^{\circ}$ of terms. Then $153 + 5 = 158s. = 7\text{ } \text{ } 18s. = \text{last payment.}$

69. PRO-

69. PROBLEM II. In an Arithmetick Progression: Given the first term, last term, and the number of terms.

Required the sum of all the terms.

RULE. Add the first and last terms together, the sum multiplied by half the number of terms, gives the sum of all the terms.

Ex. I. Required the sum of the first 1000 numbers in their natural order.

Here 1 = 1st term, 1 = com. diff.

1000 = N^o of terms, its $\frac{1}{2}$ is 500.

Now $1000 + 1 = 1001$.

Then $1001 \times 500 = 500500$ is the sum required.

Ex. II. A debt is to be discharged in a year by weekly payments equally increasing; the 1st to be 5^l. and the last 7^l. 18^s. How much was the debt?

Here 7^l. 18^s. = 158^l. = last term.

52 = N^o of terms, its $\frac{1}{2}$ is 26.

Now $158 + 5 = 163$.

Then $163 \times 26 = 4238$. = 211^l. 18^s. is the sum of the terms, or debt.

Ex. III. Suppose a basket and 500 stones were placed in a straight line, a yard distant from one another: Required in what time a man could bring them one by one to the basket, allowing him to walk at the rate of 3 miles an hour?

Between the basket and stones are 500 spaces, which is the number of terms. Now $500 + 1 = 501$. Then $501 \times 250 = 125250$ = sum of the terms.

But as he goes backwards and forwards, he walks 250500 yards.

Which divided by 1760 (the yards in 1 mile) gives 142,329 miles.

Which at 3 miles an hour, will take 47 h. 26 min. 34 seconds.

70. PROBLEM III. In a Geometric Progression: Given the first term, the ratio and the last term.

Required the sum of all the terms.

RULE. Multiply the last term by the common ratio, from the Product subtract the first term for a Dividend.

Subtract 1 from the ratio for a Divisor; then divide, and the Quotient will be the sum of all the terms.

Ex. I. Suppose the first term of a series to be 3, the ratio 3, and the last term 6561: Required the sum of all the terms.

Now 6561 = last term. And 3 = ratio.

Mult. by 3 = ratio. Sub. 1

Subtr. $\frac{19683}{3}$ = Product. Rem. 2 = Divisor.

$\frac{19680}{19680}$ = Dividend.

Then $2 \times 19680 (9840)$ is the sum of all the terms.

Ex. II. Let the first term be 2, the second term 10, and the last term 156250: Required the sum of all the terms.

Here 2) 10 (5 is the common ratio.

Now $156250 \times 5 = 781250$. And $781250 - 2 = 781248$ = Dividend.

Also $5 - 1 = 4$ the Divisor.

Then $4) 781248 (195312$ is the sum of all the terms. 71. PRO.

71. PROBLEM IV. In a Geometric Progression: Given the first term, the ratio and the number of terms.

Required the last term.

RULE. 1st. Write down 6 or 7 of the leading terms in the Geometric Series, and over them their Indices.

2d. Add together the most convenient indices to make an index less by unit than the number expressing the place of the term sought.

3d. Multiply together the terms of the Geometric Series, belonging to those indices which were added, make the product a dividend.

4th. Raise the first term to a power whose index is equal to the number, less one, of the terms multiplied; make the result a Divisor to the former Dividend, and the Quotient will be the term sought.

Ex. I. What is the 12th term of a Geometric Series whose first term is 3 and second term is 6?

Now $\frac{6}{3} = 2$ is the common ratio.

And $\begin{cases} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{cases}$ Indices.

Then $3 \cdot 6 \cdot 12 \cdot 24 \cdot 48 \cdot 96 \cdot 192$ &c. Geometric terms.

And $6+5=11$, is the index to the 12th term.

And $192 \times 96 = 18432$, is the Dividend.

The number of terms multiplied is 2; and $2-1=1$, the power to which the first term 3 is to be raised; but the 1st power of 3 is 3.

Then $\frac{18432}{3} = 6144$ is the 12th term of the given series.

Ex. II. A Person, being asked to dispose of a fine horse, said he would sell him on condition of having one farthing for the 1st nail in his shoes, two farthings for the 2d nail; one penny for the 3d nail; two-pence for the 4th; four-pence for the 5th; 8d. for the 6th, &c.; doubling the price of every nail to 32, the number of nails in the four shoes: How much would that horse be sold for at that rate?

Here the 1st term is 1, the ratio 2, and the number of terms 32.

First. To find the last term.

Now $\begin{cases} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{cases}$ Indices.

And $1 \cdot 2 \cdot 4 \cdot 8 \cdot 16 \cdot 32 \cdot 64 \cdot 128 \cdot 256$ &c. Geometric terms.

Then $8+8=16$; $16+8=24$; $24+7=31$.

The 1st term being 1, any power thereof is 1; so the 4th article of the rule is useless in this question.

Now $256 \times 256 = 65536$ is the 17th term.

$65536 \times 256 = 16777216$ is the 25th term.

$16777216 \times 128 = 2147483648$ is the 32d term.

Then 2147483648

$$\begin{array}{r} 4294967296 \\ 2 \\ \hline 8589934592 \\ 20 \\ \hline 17179869184 \\ 20 \\ \hline 34359738368 \\ 20 \\ \hline 68719476736 \\ 20 \\ \hline 137438953472 \\ 20 \\ \hline 274877906944 \\ 20 \\ \hline 549755813888 \\ 20 \\ \hline 1099511627776 \\ 20 \\ \hline 2199023255552 \\ 20 \\ \hline 4398046511104 \\ 20 \\ \hline 8796093022208 \\ 20 \\ \hline 17592186044416 \\ 20 \\ \hline 35184372088832 \\ 20 \\ \hline 70368744177664 \\ 20 \\ \hline 140737488355328 \\ 20 \\ \hline 281474976710656 \\ 20 \\ \hline 562949953421312 \\ 20 \\ \hline 1125899906842624 \\ 20 \\ \hline 2251799813685248 \\ 20 \\ \hline 4503599627370496 \\ 20 \\ \hline 9007199254740992 \\ 20 \\ \hline 18014398509481984 \\ 20 \\ \hline 36028797018963968 \\ 20 \\ \hline 72057594037927936 \\ 20 \\ \hline 144115188075855872 \\ 20 \\ \hline 288230376151711744 \\ 20 \\ \hline 576460752303423488 \\ 20 \\ \hline 1152921504606846976 \\ 20 \\ \hline 2305843009213693952 \\ 20 \\ \hline 4611686018427387904 \\ 20 \\ \hline 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SECTION X. OF LOGARITHMS.

72. LOGARITHMS are a Series of numbers so contrived, that by them, the work of multiplication may be performed by addition; and the operation of division may be done by subtraction.

73. Or, Logarithms are the Indices to a series of numbers in Geometrical Progression.

Thus $\left\{ \begin{array}{l} 0 \ 1 \ 2 \ 3 \ 4 \ 5 \\ 1 \ 2 \ 4 \ 8 \ 16 \ 32 \end{array} \right\}$ 6 Indices or Logarithms.
 or $\left\{ \begin{array}{l} 0 \ 1 \ 2 \ 3 \ 4 \\ 1 \ 3 \ 9 \ 27 \ 81 \end{array} \right\}$ 6 Geometric Progression.
 or $\left\{ \begin{array}{l} 0 \ 1 \ 2 \ 3 \ 4 \\ 1 \ 10 \ 100 \ 1000 \ 10000 \end{array} \right\}$ 5 Indices or Logarithms.
 or $\left\{ \begin{array}{l} 0 \ 1 \ 2 \ 3 \ 4 \\ 1 \ 10000 \ 100000 \ 1000000 \end{array} \right\}$ 5 Geometric Series.

Where the same Indices serve equally for any Geometric Series.

74. Hence it is evident, there may be as many kinds of Indices or Logarithms, as there can be taken kinds of Geometric Series.

But the Logarithms most convenient for common uses, are those adapted to a Geometric Series increasing in a tenfold Progression, as in the last of the examples above.

75. In the Geometric Series $1 \cdot 10 \cdot 100 \cdot 1000 \cdot 10000 \cdot 100000$ between the terms 1 and 10, if the numbers $2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9$ were interposed, to them might Indices be also adapted in an Arithmetic Progression, suited to the terms interposed between 1 and 10, considered as a Geometric Progression: Also proper Indices may be found to all the numbers that can be interposed between any two terms of the Geometric Series.

But it is evident that all the Indices to the numbers under 10 must be less than 1; that is, are fractions: Those to the numbers between 10 and 100 must fall between 1 and 2; that is, are mixed numbers consisting of 1 and some fraction: And so the indices to the numbers between 100 and 1000 will fall between 2 and 3; that is, are mixed numbers consisting of 2 and some fraction: And so of the other Indices.

76. Hereafter, the integral part only of these indices will be called the Index; and the fractional part will be called the Logarithm: And the computing of those fractional parts is called the making of Logarithms; the most troublesome part of this work is to make the Logarithms of the prime numbers; that is, of such numbers which cannot be divided by any other number than by itself and unity.

77. To find the Logarithms of prime numbers

RULE. 1st. Let the sum of the proposed number and its next less number be called A.

2d. Divide 0,868588963 * by A, reserve the Quotient.

* The number 0,868588963 is the Quotient of 2 divided by 2,302585093 which is the Logarithm of 10, according to the first form of the Lord Napier who was the inventor of Logarithms. The manner whereby *Napier's Log.* of 10 is found, may be seen in many books of Algebra; but is here omitted, because this treatise does not contain the elements of that science: However, those who have not opportunity to enter thoroughly into this subject, had better grant the truth of one number, and thereby be enabled to try the accuracy of any Logarithm in the tables, than to receive those tables as truly computed, without any means of examining the certainty thereof.

3d. Divide

3d. Divide the reserved Quotient by the Square of A, reserve this Quotient.

4th. Divide the last reserved Quotient by the Square of A, reserving the Quotient; and thus proceed as long as division can be made.

5th. Write the reserved Quotients orderly under one another, the first being uppermost.

6th. Divide these Quotients respectively by the odd numbers 1. 3. 5.

7. 9. 11, &c. that is, divide the first reserved Quotient by 1, the 2d by 3, the 3d by 5, the 4th by 7, &c. let these Quotients be wrote orderly under one another, add them together, and their sum will be a Logarithm.

7th. To this Logarithm, add the Logarithm of the next less number, and the sum will be the Logarithm of the number proposed.

Ex. I. Required the Logarithm of the number 2.

Here the next less number is 1, and $2 + 1 = 3 = A$.
And the Square of A is 9. Then;

$$\begin{array}{rcl}
 \frac{0,868,88963}{3} = ,289529654. & \text{And} & \frac{,289529654}{1} = ,289529654. \\
 \frac{0,289529654}{9} = ,032169962. & \& & \frac{,032169962}{3} = ,010723321 \\
 \frac{0,032169962}{9} = ,003574440. & \& & \frac{,003574440}{5} = ,000714888 \\
 \frac{0,003574440}{9} = ,000397160. & \& & \frac{,000397160}{7} = ,000056737 \\
 \frac{0,000397160}{9} = ,000044129. & \& & \frac{,000044129}{9} = ,000004903 \\
 \frac{0,000044129}{9} = ,000004903. & \& & \frac{,000004903}{11} = ,000000445 \\
 \frac{0,000004903}{9} = ,000000545. & \& & \frac{,000000545}{13} = ,000000042 \\
 \frac{0,000000545}{9} = ,000000060. & \& & \frac{,000000060}{15} = ,000000004
 \end{array}$$

To this Log.

Add the Log. of

$$\begin{array}{r}
 0,301029994 \\
 1 = 0,000000000
 \end{array}$$

Their sum is the Log. of 2 = 0,301029994

This process needs no other explanation than comparing it with the rule.

That the manner of computing these Logarithms may be familiar to the Reader, the operations of making several of them are here subjoined.

D 2

Ex. II.

Ex. II. Required the Logarithm of the number 3.

Here the next less number is 2; and $3 + 2 = 5 = A$, whose Square is 25.

$$\frac{0,86858963}{5} = 173717792. \text{ And } \frac{173717792}{1} = 173717792$$

$$\frac{0,173717792}{25} = 6948712. \text{ \& } \frac{6948712}{3} = 2316237$$

$$\frac{0,006948712}{25} = 277948. \text{ \& } \frac{277948}{5} = 55590$$

$$\frac{0,000277948}{25} = 11118. \text{ \& } \frac{11118}{7} = 1588$$

$$\frac{0,000011118}{25} = 445. \text{ \& } \frac{445}{9} = 49$$

$$\frac{0,000000445}{25} = 18. \text{ \& } \frac{18}{11} = 2$$

To this Logarithm

Add the Log. of 2

The sum is the Logarithm of 3

$$0,176091258$$

$$0,301029992$$

$$0,477121250$$

78. Since the Logarithms are the Indices of numbers considered in a Geometric Progression; therefore the sums, or differences of these Indices, will be Indices or Logarithms belonging to the Products, or Quotients, of such terms in the Geometric Progression as correspond to those Logarithms as were added or subtracted (71).

Ex. III. Required the Log. of 4.

Now $4 = 2 \times 2$.

Then to the Log. of 2 $0,301029992$

Add the Log. of 2 $0,301029992$

Sum is the Log. of 4 $0,602059984$

Ex. V. Required the Log. of 10.

In the original Series, 1 is assumed for the Logarithm of 10.

per 502 ad. 5. 100

Ex. VII. Required the Log. of 8.

Now $8 = 2 \times 2 \times 2$.

Therefore the Log. of 2 taken thrice

gives the Log. of the number 8.

The Log. of 2 is $0,301029992$

Which multiplied by 3

Gives the Log. of 8 $0,903089976$

Ex. IV. Required the Log. of 6.

Now $3 \times 2 = 6$.

Then to the Log. of 3 $0,477121250$

Add the Log. of 2 $0,301029992$

Sum is the Log. of 6 $0,770151242$

Ex. VI. Required the Log. of 5.

Now 10 divided by 2 gives 5.

Then from Log. of 10 $1,000000000$

Take Log. of 2 $0,301029992$

Leaves the Log. of 5 $0,698970008$

Ex. VIII. Required the Log. of 9.

Now $9 = 3 \times 3$.

Therefore the Log. of 3 doubled gives

the Log. of 9.

The Log. of 3 is $0,477121250$

Which multiplied by 2

Gives the Log. of 9 $0,954242500$

Ex. IX.

Ex. IX. Required the Logarithm of 7.

Here 6 is the next lesser. Then $7 + 6 = 13 = A$; and $169 = \text{Square of } A$.

$$\frac{0,968588061}{13} = ,066814536. \text{ And } \frac{,066814536}{1} = ,066814536$$

$$\frac{0,066814536}{169} = \frac{395352}{3} = 131784$$

$$\frac{0,000395352}{169} = \frac{2339}{5} = 468$$

$$\frac{0,00002339}{169} = \frac{14}{7} = 2$$

To this Logarithm

Add the Logarithm of 6

$$\begin{array}{r} 0,066946790 \\ 0,778151242 \\ \hline \end{array}$$

The sum is the Logarithm of 7

$$0,845098032$$

The Log. of 12 is equal to the sum of the Logs. of 3 & 4, or of 2 & 6.
The Log. of 14 is equal to the sum of the Logs. of 7 & 2.

of 15

of 16

of 18

of 20

of 3 & 5.

of 4 & 4, or of 8 & 2.

of 3 & 6, or of 9 & 2.

of 4 & 5, or of 10 & 2.

79. The Logs. of the prime numbers 11, 13, 17, 19, are to be found as in the examples I. II. IX. and in like manner is the Log. of any other prime number to be found; but it may be observed, that the operation is shorter in the larger prime numbers; for any number exceeding 400, the first Quotient added to the Logarithm of its next lesser number, will give the Logarithm sought, true to 8 or 9 places; and therefore it will be very easy to examine any suspected Logarithm in the tables.

80. The manner of disposing the Logarithms, when made, into tables, is various: But in this treatise they are ordered as follows.

Any number under 100, or not exceeding two places, and its Logarithm, is found in the first page of the table, where they are placed in adjoining columns; and distinguished by the title Num. for the common numbers; and by Log. for the Logarithms.

These tables are at the end of Book IX.

A number of three or four places being given, its Logarithm is thus found.

Seek for a page in which the given number shall be contained between the two numbers marked at the top, annexed to the letter N: Then right against the three first figures of the given number, found in the column signed Num, and in the column signed by the fourth, stands the Logarithm belonging to that number of four places.

If the number consisted of 3 places only; then these places found as before directed, the Logarithm stands against them in the column signed 0.

Thus, to find the *Logarithm* of 5738. Seek for a page wherein stands at top N^o 5500 to 6000; then in the column signed N^o find 573, right against which in the column signed 8 at top or bottom stands, 75876, which is the *Logarithm* to 5738, exclusive of its *Index*.

81. A *Logarithm* being given, its number is thus found.

Seek for a page wherein the three first figures of the given *Logarithm* is found at top annexed to the letter L; then in one of the columns signed with the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, find a number the nearest to the given *Logarithm*; against this number in the column signed N^o stand three figures, to the right of these annex the figure with which the column was signed at top or bottom, and this will be the number corresponding to the given *Logarithm*, not regarding the *Index*.

82. All numbers consisting of the same figures, whether they be integral, fractional, or mixed, have the fractional parts of their *Logarithms* the same.

If the following examples be well attended to, there will be no difficulty in finding the *Logarithm* to a proposed number, or the number to a proposed *Logarithm*, within the limits of the table of *Logarithms* here used.

Num.	Logarithms.	Logarithms.	Numbers.
5874	3,76893	0,37205	2360
587.4	2,76893	1,28631	19,33
58.74	1,76893	2,51947	330,7
5.874	0,76893	3,75062	5632,
0,5874	1,76893	3,18397	0,001527
0,05874	2,76893	2,43020	0,02693
0,005874	3,76893	1,85962	0,7238

83. A general rule to find the *Index* to the *Log.* of a given number. To the left of the *Logarithm*, write that figure (or figures) which expresses the distance from unity, of the highest-place digit in the given number, reckoning the unit's place 0, the next place 1, the next place 2, the next place 3, &c.

When there are integers in the given number, the *Index* is always affirmative; but when there are no integers, the *Index* is negative, and is to be marked by a little line drawn above it: thus $\bar{2}$.

Thus a number having $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$ &c. integer places.

The index of its *Logarithm* is $0 \cdot 1 \cdot 2 \cdot 3 \cdot 4$ &c.

And a fraction having a digit in the place of Primes, Seconds, Thirds, Fourths, &c.

Then the *Index* of its *Logarithms* will be $\bar{1} \cdot \bar{2} \cdot \bar{3} \cdot \bar{4}$ &c.

By the above rule, the place of the fractional comma, or mark of distinction, in the number answering to a given *Logarithm*, will be always known.

84. The more places the *Logarithms* consist of, the more accurate, in general, will be the result of any operation performed with them: But for the purposes of Navigation, as five places, exclusive of the *Index*, are sufficient, therefore the logarithmic tables in this treatise are not extended any farther.

85. MULTIPLICATION BY LOGARITHMS;

Or, Two or more numbers being given, to find their Product by Logarithms.

RULE. Add together the Logarithms of the Factors, and the sum is a Logarithm, whose corresponding number is the Product required.

Observing to add what is carried from the Logarithm to the sum of the affirmative Indices.

And that the difference between the affirmative and negative Indices are to be taken for the Index to the Logarithm of the Product.

Ex. I. Multiply 86,25 by 6,48.

86,25 its Log. is 1,93576
6,48 its Log. is 0,81157

Product 558,9 2,74733

Ex. III. Multiply 3,768 by 2,053
and by 0,007693.

3,768 its Log. is 0,57611
2,053 0,31239
0,007693 3,88610

Product 0,05951 2,77460

The 1 carried from the left hand column of the Logs. being affirmative, reduces 3 to 2.

86. DIVISION BY LOGARITHMS;

Or, Two numbers being given, to find how often the one will contain the other, by Logarithms.

RULE. For the Log. of the Dividend, subtract the Log. of the Divisor; then the number agreeing to the Remainder, will be the Quotient required.

But observe, to change the Index of the Divisor from negative to affirmative, or from affirmative to negative: And then let the difference of the affirm. and neg. Indices be taken for the Index to the Log. of the Quotient.

When an unit is borrowed in the left hand place of the Logarithm, add it to the Index of the Divisor, if affirmative; but subtract it if negative; and let the Index arising be changed and worked with as before.

Ex. I. Divide 558,9 by 6,48.

Log. of Divid. 558,9 is 2,74733
Log. of Divisor 6,48 is 0,81157

The Quotient is 86,25 1,93576

Ex. III. Divide 0,05951 by 0,007693.

Log. of Divid. 0,05951 is 2,77459
Log. of Divisor 0,007693 is 3,88610

The Quotient is 7,735 0,88849

Ex. II. Multiply 46,75 by 0,3275.

46,75 its Log. is 1,66978
0,3275 its Log. is 1,51521

Product 15,31 1,18499

Ex. IV. Multiply 27,63 by 1,859
and by 0,7258 and by 0,03591.

27,63 its Log. is 1,44138
1,859 0,26928
0,7258 1,86082
0,03591 2,55521

Product 1,339 0,12669

Here 2 being carried to the Index 1, makes 3; which takes off the 1 and 2.

87. OF PROPORTION.

1st. State the terms of the question (by 46) and let them be wrote orderly under one another, prefixing to the first term the word *As*, to the second *To*, to the third *So*, and under them set the word *To*.

2d. Against the first term, write the arithmetical complement of its Logarithm. See Art. 88.

3d. Against the second and third terms, write their Logarithms.

4th The sum of those three Logarithms, abating 10 in the Index, will be the Logarithm of the fourth term; which sought in the tables, the number answering thereto, is the answer or term sought.

88. The arithmetical complement of a Logarithm is thus found. Beginning at the Index, write down what each figure wants of 9, except the last, or right hand figure, which take from 10.

But if the Index is negative, add it to 9; and proceed with the rest as before.

Ex. I. Find a fourth proportional
number to 98,45 and 1,969 and 347,2.

As	98,45	its	* Ar. Co. Log.	8,00678
To	1,969			0,29425
So	347,2			2,54058
To	6,944			0,84161

Ex. II. Find a third proportional
number to 9,642 and 4,821.

As	9,642	its	Ar. Co. Log.	9,01583
To	4,821			0,68314
So	4,821			0,68314
To	2,411			0,38211

Ex. III. What will a gunner's pay
amount to in a year at 2*£*. 12*s*. 6*d*.
a month of 28 days?

As	28	days	its	Ar. Co. Log.	8,55286
To	365	days			2,56229
So	2 <i>£</i> . 12 <i>s</i> . 6 <i>d</i> .		$\equiv$ 2,625 <i>£</i> .	0,41913	
To	34 <i>£</i> . 4 <i>s</i> . 5 <i>d</i> .		$\equiv$ 34,22 <i>£</i> .	1,53428	

Ex. IV. If $\frac{2}{3}$ of a yard of cloth
cost $\frac{2}{3}$ of a guinea: How many ells
English for 3*£*. 10*s*.?

As	$\frac{2}{3}$	Guin.	$\equiv$ 14 <i>s</i> .	Ar. Co.	8,85387
To	3 <i>£</i> . 10 <i>s</i> .		$\equiv$ 70 <i>s</i> .		1,84510
So	$\frac{1}{3}$ ell		$\equiv$ c,6		1,77815
To	3 ells				0,47712

Ex. V. What number will have the
same proportion to 0,8538 as 0,3275
has to 0,0131?

As	0,0131	its	Ar. Co. Log.	11,88273
To	0,3275			1,51521
So	0,8538			1,93125
To	21,34			1,32919

Ex. VI. How many yards of shal-
loon of $\frac{3}{4}$ ell wide will be enough to
line a coat containing $3\frac{1}{2}$ ells of $1\frac{1}{4}$
yards wide?

As	$\frac{3}{4} \times \frac{1}{4}$	yd. w.	$\equiv$ 0,9375	10,02803
To	$1\frac{1}{4}$	yd. w.	$\equiv$ 1,75	0,24304
So	$5\frac{1}{2} \times \frac{1}{4}$	yd. l.	$\equiv$ 4,375	0,64098
To	$8\frac{1}{2}$ yd. long		$\equiv$ 8,167	0,91205

where w. stands for wide, l. for long.

* Ar. Co. Log. stands for the Arithmetical Complement of the Logarithm.

OF

OF POWERS AND THEIR ROOTS.

89. *A number being given, to find any proposed power thereof.*

RULE. 1st. Seek the Logarithm of the given number.

2^d. Multiply this Logarithm by the Index of the proposed power.

3^d. Find the number corresponding to the Product, and it will be the power required.

90. In multiplying a Logarithm having a negative Index, the Product of that Index is negative.

But the carriage from the Logarithm is affirmative.

Therefore the difference will be the Index of the Product.

And is to be of the same kind with the greater, or that made the minuend.

Ex. I. *What is the second power of the number 3,874?*

To 3,874 its Log. is 0,58816
The Index is 2

The power sought is 15,01 1,17632

Ex. III. *What is the 12th power of the number 1,539?*

1,539 its Log. is 0,18724
The Index is 12

The power sought is 176,6 2,24688

In the IVth Ex. the Index of the Product being 109, shews that the required power will consist of 110 integer places; of which no more than 4 places are found in these tables; therefore the number sought may be thus expressed 7515 [106]: That is, 7515 with 106 cyphers annexed.

Ex. V. *What is the 2d power of the number 0,2857?*

To 0,2857, its Log. is 1,45591
The Index of 2d power 2

The power 0,08162 2,91182

Here, there being no carriage from the Product of the Log. the whole Product of the negative Index is negative, viz. 2.

Ex. II. *What is the 3d power of the number 2,768?*

The N^o 2,768 its Log. is 0,44217
The Index is 3

The power sought is 21,21 1,32651

Ex IV. *What is the 365th power of the number 2?*

2 Its Log. is 0,30103
The Index is 365

150515
180618
90309

109,87595

Ex. VI. *What is the 3d power of the number 0,7916?*

To 0,7916, its Log. is 1,89851
The Index of 3d power 3

The power is 0,496 1,69553

Here the carriage from the Product of the Log. is 2; then the Product of the negative Index 1, viz. 3, being lessened by 2, leaves 1, the Index of the Product.

91. *A number being given, to find any proposed Root thereof.*

RULE. 1st. Seek the Logarithm of the given number.

2^d. Divide this Logarithm by the denominator of the Index of the proposed root.

3^d. The number corresponding to the Quotient will be the root.

When the Index to the Logarithm to be divided is negative, and less than the Divisor, or Denominator of the root. Then

Increase the negative Index by as many units, borrowed, as to be equal to the Divisor, and the Quotient will give $\bar{1}$, for the Index.

Carry the units borrowed as tens to the left-hand place of the Logarithm, and then divide that Logarithm as in whole numbers.

Ex. I. *What is the Square Root of the number 1501?*

$$\begin{array}{r} 2)3,17638 \\ \hline 1,58819 \end{array}$$

Root sought is 38,74

Ex. II. *What is the Cube Root of the number 2121?*

$$\begin{array}{r} 3)3,32634 \\ \hline 1,10885 \end{array}$$

Root sought is 12,85

Ex. III. *What is the Root, whereof 176,6 is the 12th power?*

$$\begin{array}{r} 12)2,24686 \\ \hline 0,18724 \end{array}$$

Root sought is 1,539

Ex. IV. *What is the Root, whereof 2 is the 365th power?*

$$\begin{array}{r} 365)0,30103 \\ \hline 0,00082 \end{array}$$

Root sought is 1,002

Ex. V. *What is the Square Root of the number 0,08162?*

$$\begin{array}{r} \text{Log. of } 0,08162, \text{ div. by } 2)1,91180 \\ \hline \text{Gives Root } 0,2857 \text{ to } \bar{1},45590 \end{array}$$

Here the Divisor 2 can be taken in the Index 2, and gives for the Quotient $\bar{1}$.

Ex. VI. *What is the Cube Root of the number 0,496?*

$$\begin{array}{r} \text{Log. of } 0,496, \text{ div. by } 3)1,69548 \\ \hline \text{Gives Root } 0,7916 \text{ to } \bar{1},89849 \end{array}$$

Here the Divisor 3 cannot be taken in the Index $\bar{1}$; then $\bar{2}$ borrowed makes with $\bar{1}$, $\bar{3}$; in which the Divisor 3 will go $\bar{1}$: the 2 borrowed carried to the 6, &c. makes 269548, which divided by 3, gives 89849.

END OF BOOK I.

THE



THE ELEMENTS OF NAVIGATION.

BOOK II. OF GEOMETRY.

SECTION I.

Definitions and Principles.

1. **G**EOMETRY is a science which treats of the descriptions, properties and relations of magnitudes in general: Or of such things where length, or where length and breadth, or length, breadth, and thickness, are considered.

2. A **POINT** is that which is without parts or dimensions.

3. A **LINE** is length without breadth: It is called a **A** ——— **B** **RIGHT LINE** when it is the shortest distance between two points (as **AB**): Or a **CURVED LINE** when it is not the shortest distance (as **CD**).

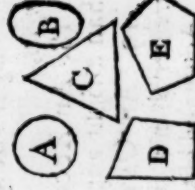
*A line is usually denoted by two letters, viz. one at each end (as **AB**, **CD**).*

4. A **SUPERFICIES** or **SURFACE** is that magnitude which has only length and breadth, and is bounded by lines: as **FG**.

5. A **SOLID** is that magnitude which has length, breadth, and thickness.

6. A **FIGURE** is a bounded space, whose limits or bounds may be either lines or surfaces.

7. A **PLANE**, or a **PLANE FIGURE**, is a superficies which lies evenly, or perfectly flat, between its limits; and may be bounded by one curve line; but not with less than three right lines, as **A**, **F**, **C**, **I**, or **E**, &c.



8. A

8. A CIRCLE is a plain figure, bounded by an uniformly curved line, called the Circumference (as ABD), which is every where equally distant from one point (C) within the figure, called the CENTER.



9. A RADIUS is a right line drawn from the center to the circumference (as CA , CD , CE).

All the radii of the same circle are equal.



10. An ARC is any part of the circumference.
As the arc AB , or the arc AD .

11. A CHORD is a right line joining the ends of an arc (as AB), and is said to subtend that arc; it divides the circle into two parts, called SEGMENTS.

12. A DIAMETER is a chord passing through the center (as DE), and divides the circle into two equal parts, called SEMICIRCLES.

13. The CIRCUMFERENCE of every circle is supposed to be divided into 360 equal parts, called DEGREES; each degree into 60 equal parts, called MINUTES; each minute into 60 equal parts, called SECONDS, &c.

14. A PLANE ANGLE, is the inclination of two lines on the same plane meeting in a point (as ACB).

A right lined angle is formed by two right lines. The point where the lines meet is called the *angular point* (as C).



The lines which form the angle are called *legs*.

Thus CA and CB are the legs of the angle ACB .

An Angle is usually marked by three letters, viz. one at the angular point, and one at the other end of each leg; but that at the angular point is always to be read the middle letter (as ACB , or BCA).



15. The measure of a right lined angle is an arc (BA) contained between the legs (CB , CA) including the angle, the angular point (C) being the center of that arc.

16. A RIGHT ANGLE is that whose measure is a fourth part of the circumference of a circle, or ninety degrees. *Thus the angle ACB is a right angle.*

17. A PERPENDICULAR is that right line which cuts another at right angles; or which makes equal angles on both sides. *Thus DC is perpendicular to AB , when the angles DCA and DCB are equal, or are right angles.*



18. An ACCUTE ANGLE (ACB) is that which is less than a right angle (DCB).

19. An OBTUSE ANGLE (EFG) is that which is greater than a right angle (HFG).

Acute and obtuse angles are called OBLIQUE ANGLES.



20. PARALLEL LINES, are such right lines in the same plane, which do not incline to one another. (As AB, CD).



21. A TRIANGLE is a plane figure bounded by three lines.



22. An EQUILATERAL TRIANGLE is that whose three lines, or sides, are equal (as A).



23. An ISOSCELES TRIANGLE is that which has only two equal sides, (as B or C).



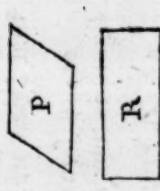
24. A RIGHT ANGLED TRIANGLE (ADC) is that which has one right angle (B).



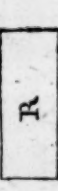
25. An OBTUSE ANGLED TRIANGLE (DEF) has one obtuse angle (E).



26. An ACUTE ANGLED TRIANGLE (G) has all its angles acute.



27. A QUADRANGLE, or QUADRILATERAL, is a plane figure bounded by four right lines, or sides.



37. An **ANGLE** (BAC) in a Segment (CADB) is, when the angular point is in the circumference of the segment, and the legs including the angle pass through the ends (B , C) of the chord of the segment.

Such an angle is said to be in a circumference; and to stand on the arc (BC) included between the legs (AB , AC) of the angle.



38. Right lined figures having more than four sides, are called *Polygons*; and have their names from the number of their angles, or sides; as those of five sides are called Pentagons; of six sides Hexagons; of seven sides Heptagons; of eight sides Octagons, &c.

39. A Regular Polygon is a figure with equal sides, and equal angles.

40. A figure is said to be inscribed in a circle, when all the angles of that figure are in the circumference of the circle.

41. A figure is said to circumscribe a circle, when every side of the figure is touched by the circumference of the circle.

42. A Proposition is something proposed to be considered; and requires either a solution or answer, or that something be made out or proved.

A Problem is a practical proposition, in which something is proposed to be done, or effected.

A Theorem is a speculative proposition, or rule, in which something is affirmed to be true.

A Corollary is some conclusion gained from a preceding proposition.

A Scholium is a remark on some proposition; or an exemplification of the matter which it contains.

An Axiom is a self-evident truth, or principle, that every one assents to upon hearing it proposed.

A Postulate is a principle, or condition, requested; the simplicity or reasonableness of which cannot be denied.

In Mathematics, the following Postulates and Axioms, are some of the principles that are generally taken for granted.

When a proposition, from supposed premises, asserts such and such consequences; and subjoins, *And the contrary*: it is to be understood, that if the consequences be assumed as premises; then what were first taken as premises, would become consequences.

Thus in Article 95, it is premised, *that if two parallel right lines are cut by another right line*, there results this consequence; *The alternate angles are equal*. And the contrary, means; *that where equal alternate angles are made by a right line cutting two other right lines; the right lines, so cut, are parallel lines*.

Postulates

Postulates.

43. I. That a right line may be drawn from any given point to another given point.
44. II. That a given right line may be continued, or lengthened at pleasure.
45. III. That from a given point, and with any radius, a circle may be described.

Axioms.

46. I. Things equal to the same thing, are equal to one another.
47. II. If equal things are added to equal things, the sums or wholes will be equal. But if unequals be added, the sums are unequal.
48. III. If equal things are taken from equal things, the remainders or differences are equal : But are unequal, when unequals are taken.
49. IV. Things are equal which are double, triple, quadruple, &c. or half, third part, &c. of one and the same thing, or of equal things.
50. V. Things which have equal measures, are equal. And the contrary.
51. VI. Equal circles have equal radii.
52. VII. Equal arcs in equal circles have equal chords, and are the measures of equal angles. And the contrary.
53. VIII. Parallel right lines, have each the same inclination to a right line cutting them.

In what follows, it is to be understood, that right lines (*viz.* straight lines) are drawn by the edge of a straight ruler : circles, or arcs, are described with one foot of a pair of compasses, the other foot resting on the point which is taken for the center ; and the distance of the feet, or points, of the compasses is taken as the radius : also, that the point marked out by a letter, is to be understood, when the reference is made to that letter.

54. It is also taken for granted, that a line or distance can be taken between the compasses, and may be transferred or applied from one place to another. Also, that one figure can be applied to, or laid upon another, or conceived to be so applied.

In any problem, when a line, angle or figure is said to be given ; that line, angle or figure must be made, before any part of the operation is performed.

SECTION II.

Geometrical Problems.

55.

PROBLEM I.

To bisect, or divide into two equal parts, a given line AB.

OPERATION. 1st. From the ends A and B*, with one and the same radius, greater than half AB, describe arcs cutting in c and d. (45)

2d. A ruler laid by c and d, gives E, the middle of AB, as required.

The proof of this operation depends on articles 101, 99.

56.

PROBLEM II.

To bisect a given right lined angle ABC.

OPERATION. 1st. From B, describe an arc AC.

2d. From A and c*, with one and the same radius, describe arcs cutting in d. (45)

3d. A right line drawn through B and d will divide the angle into two equal parts, as required.

The proof depends on article 101.

57.

PROBLEM III.

From a given point B, in a given right line AF, to draw a right line perpendicular to the given line.

When B is near the middle of the line.

OPERATION. 1st. On each side of B, take the equal distances BC, and BE. (54)

2d. On c and e* describe, with one radius, arcs cutting in d. (45)

3d. A right line drawn through B and d will be the perpendicular required. (43)

The proof depends on article 103.

58.

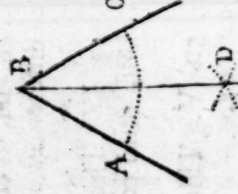
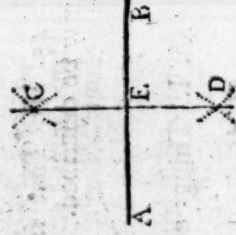
When B is at, or near the end of the given line.

OPERATION. 1st. On any, convenient, point c, taken at pleasure, with the distance, or radius, CB, describe an arc DBE, cutting AF in d, B. (45)

2d. A ruler laid by d and c will cut this arc in e.

3d. A right line drawn through B and e will be the perpendicular required.

This depends on article 130.



* That is, first describe an arc from one point; then describe an arc from the other point, with the same opening of the compasses.

PROBLEM IV.

59.

To draw a line perpendicular to a given right line AB, from a point C without that line.

When the point C is nearly opposite to the middle of the given line.

OPERATION. 1st. On c with one radius cut AB in D and E. (45)

2d. On D and E with one radius describe arcs A cutting in F. (45)

A ruler laid by C and F gives G; then draw CG, and that will be the perpendicular required.

This depends on articles 101, 99.



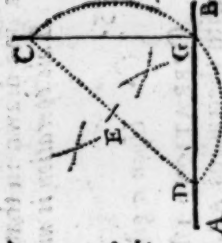
60. When C is nearly opposite to one end of the given line AB.

OPERATION. 1st. To any point D in AB, draw the line CD. (43)

2d. Bisect the line CD in E. (55)

3d. On E, with the radius EC, cut AB in G. Then CG being drawn, will be the perpendicular required.

This depends on article 130.



61.

PROBLEM V.

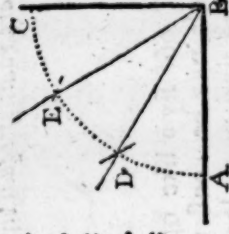
To trisect, or divide into three equal parts, a right angle ABC.

OPERATION. 1st. From B, with any radius BA, describe the arc AC, cutting the legs BA, BC, in A, C.

2d. From A, with the radius AB, cut the arc AC in E, and from C, with the same radius, cut AC in D.

3d. Draw BE, BD, and the angle ABC will be divided into three equal parts.

This depends on article 193.



62.

PROBLEM VI.

At a given point D, to make a right lined angle equal to a given right lined angle ABC.

OPERATION. 1st. From D and B with one radius describe the arcs EF, and AC, cutting the legs of the B given angle in the points A, C.

2d. Transfer the distance AC to the arc EF, from F to E. (54)

3d. Lines drawn from D, through E and F, will form the angle EDF equal to the angle ABC.

This depends on article 101.



63

PROBLEM VII.

To draw a line parallel to a given right line AB.

When the parallel line is to pass thro' a given point c.

OPERATION. 1st. From c with any convenient radius describe an arc DF, cutting AB in D.

2d. Apply the radius CD from D to E; and from E with the same radius cut the arc DF in F.

3d. A line drawn through F and c will be parallel to AB.



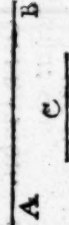
This depends on article 101, 95.

64. When the parallel line is to be at the given distance c from AB.

OPERATION. 1st. From the points A and B, with the radius c, describe arcs D and E.

2d. Lay a ruler to touch the arcs D and E, and a line drawn in that position is the parallel required.

This operation is mechanical.



65.

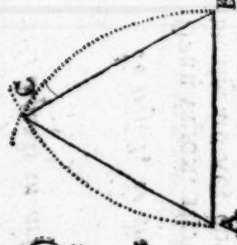
PROBLEM VIII.

Upon a given line AB, to make an equilateral triangle.

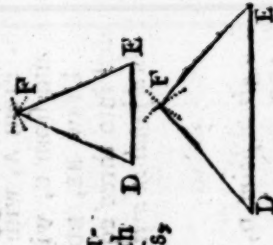
OPERATION. 1st. From the points A and B, with the radius AB, describe arcs cutting in c. (45)

2d. Draw CA, CB, and the figure ABC is the triangle required. (43)

The truth of this operation is evident; for the sides are radii of equal circles.



66. By a like operation, may an Isosceles triangle DEF be constructed on a given base DE, with the given equal legs DE, EF, either greater, or less, than the base DE.



67.

PROBLEM IX.

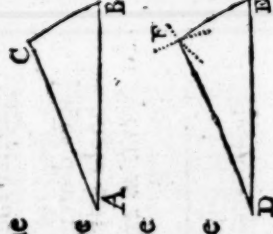
To make a right lined triangle, whose sides shall be respectively equal, either to those of a given triangle ABC, or to three given lines, provided any two of them taken together are greater than the third.

OPERATION. 1st. Draw a line DE equal to the line AB. (54)

2d. On D, with a radius equal to AC, describe an arc F. (45)

3d. On E, with a radius equal to BC, describe another arc, cutting the former arc in F. (45)

4th. Draw FD, FE, and the triangle DFE will be that required.



68. PRO.

68.

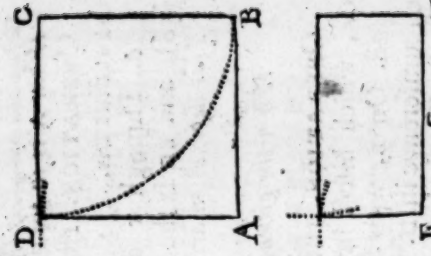
PROBLEM X.

Upon a given line AB, to describe a square.
 OPERATION. 1st. Draw BC perpendicular, and equal to AB. (58)

2d. On A and C, with the radius AB, describe arcs cutting in D. (45)

3d. Draw DC, DA; and the figure ABCD is the square required.

This depends on articles 101, 104, 95, 30.



69. By a like operation a rectangle may be described, whose length shall be equal to the given line EF, and the breadth equal to the given line G. Or thus, drawing from E and F lines perpendicular to EF, and making each of those lines equal to the line G.

70.

PROBLEM XI.

To find the center of a circle.

OPERATION. 1st. Draw any chord AB.

2d. Bisect AB with the chord CD. (55)

3d. Bisect CD with the chord EF, and their intersection G will be the center required.

This depends on article 125.



71.

PROBLEM XII.

To divide the circumference of a circle into two, four, eight, sixteen, thirty-two, &c. equal parts.

OPERATION. 1st. A diameter AB divides the circle into two equal parts. (12)

2d. A diameter DE, perpendicular to AB, divides the circumference into four equal parts.

3d. On A, D, B, describe arcs cutting in a, b; A then by the intersections a, b, and the center, the diameters FG, HI, being drawn, divide the circumference into eight equal parts.

4th. On A, H, D, F, B, describe arcs cutting in g, f, e, d; by these and the center, diameters being drawn, divide it into sixteen equal parts; and so on by a continual bisection.

For at each operation, the intercepted arcs are bisected, and the parts doubled

PROBLEM XIII.

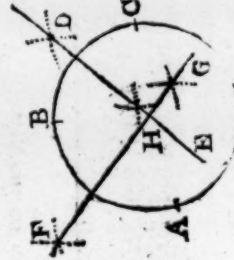
To describe a circle, whose circumference shall pass through three given points A, B, C, provided they do not lie in one right line.

OPERATION. 1st. Bisect the distance CB with the line DE. (55)

2d. Bisect the distance AB with the line FG.

3d. On H, the intersection of these lines, with the distance to either of the given points, describe the circle required.

This depends on article 125.



E 2

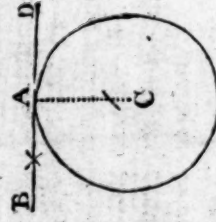
73. PRO-

73.

PROBLEM XIV.

To draw a tangent to a given circle, that shall pass through a given point A, When A is in the circumference of the circle.

OPERATION. 1st. From the center C, draw the radius CA. (43)
2d. Through A draw BD perpendicular to CA (58), and BD is the tangent required.
This depends on article 126.



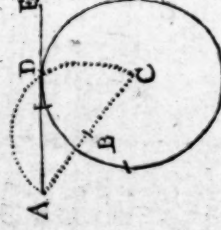
74. When the given point A, is without the given circle.

OPERATION. 1st. From the center C, draw CA, which bisect in B. (55)

2d. On B, with the radius BA, cut the given circumference in D.

3d. Through D, the line AE being drawn, will be the tangent required.

This depends on articles 130, 126.



75.

PROBLEM XV.

To two given right lines A, B, to find D a third proportional.

OPERATION. 1st. Draw two right lines making any angle, and meeting in a.

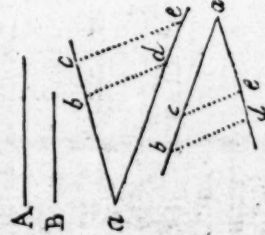
2d. In these lines, take $ab =$ first term, and B ac, ad , each equal to the second term

3d. Draw bd , and through c draw ce parallel to bd ; then ae is the third proportional sought.

And $ab : ac :: ad : ae$.*

Or $A : B :: B : D$. And $B : A :: A : D$.

This depends on article 165.



76.

PROBLEM XVI.

To three given right lines A, B, C, to find D a fourth proportional.

OPERATION. 1st. Draw two right lines making any angle, and meeting in a.

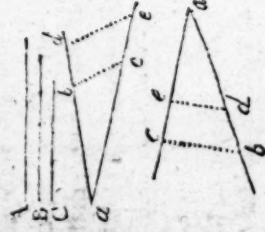
2d. In these lines take $ab =$ first term, $ac =$ second term, and $ad =$ third term.

3d. Draw bc , and parallel thereto through d draw de ; then ae is the fourth proportional required.

And $ab : ac :: ad : ae$.

Or $A : B :: C : D$. And $C : B :: A : D$.

This depends on article 165.



* And is thus read : ab is to ac , so is ad to ae .
The first : stands for $is\ to$, the last : for is , and : : for $so\ is$.

PROBLEM XVII.

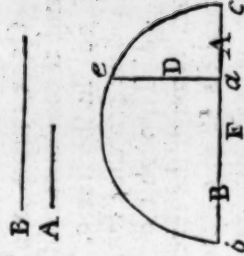
77. *Between two given right lines A, B, to find D, a mean proportional.*

OPERATION. 1st. Draw a right line, in which take $ac = A$, $ab = B$.

2d. Bisect bc in F (55); and on F , with the radius Fb , describe a semicircle bce .

3d. From a draw ae perpendicular to bc (57): then ae is the mean proportional required.

And $ac : ae :: ae : ab$. Or $A : D :: D : B$.
This depends on article 171.



PROBLEM XVIII.

78. *To divide a given line BC in the same proportion as a given line A is divided.*

OPERATION. 1st. From one end B of BC, draw BD making any angle with BC.

2d. In BD apply from B the several divisions of A; so BD will be equal to A, and alike divided.

3d. Draw CD; then lines drawn parallel to CD through the several divisions of BD, will divide the line BC in the manner required.

This depends on article 165.



PROBLEM XIX.

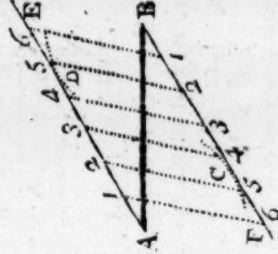
79. *To divide a given right line AB into a proposed number of equal parts (suppose 7).*

OPERATION. 1st. From the ends A, B, with the same radius, describe the arcs C, D; and from A and B lines drawn to touch these arcs, as AE, BF, will be parallel. (64)

2d. In each of the lines AE, BF, beginning at A and B, take of any length, as many equal parts less one as AB is to be divided into, viz. 1, 2, 3, 4, 5, 6.

3d. Lines drawn from 1 to 6, 2 to 5, 3 to 4, &c. will divide AB as was required.

This depends on article 165.

80. *Another Method.*

OPERATION. 1st. Through one end A, draw a line AC nearly perpendicular to AB.

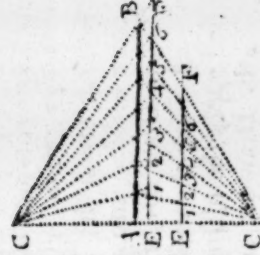
2d. Draw EF parallel to A B, at any convenient distance.

3d. In EF take of any length, as many equal parts as AB is to be divided into; as 1, 2, 3, 4, 5, 6, to F.

4th. Through B and F, where the divisions terminate, draw BC.

5th. Lines drawn from c through the several divisions of EF, will cut AB, into the equal parts required.

Note, If the sum of the divisions from E to F chance to be less than AB, the point c, and the line EF, will be on the same side of AB: But if greater, c falls on the contrary side.



81.

PROBLEM XX.

To make Scales of equal parts.

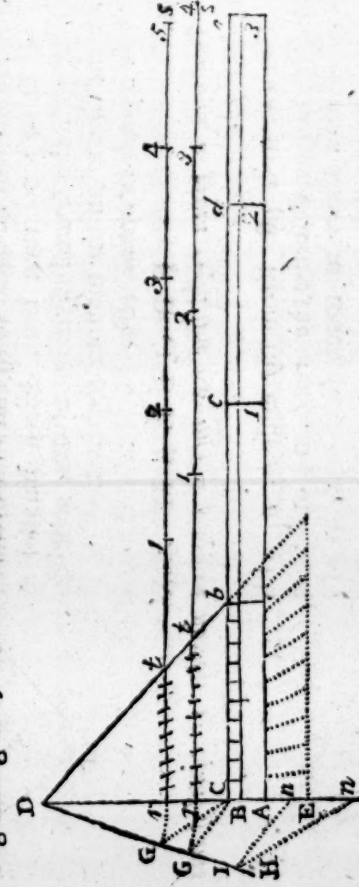
OPERATION. 1st. Draw three lines, A, B, C, parallel to one another, and at convenient distances, such as are here expressed.

2^d. In the line C, take the equal parts *cb, bc, cd, da, &c.* each equal to some proposed length.

3^d. Through c draw the line DE perpendicular to *ca*; and parallel to DE, through the several points *b, c, d, a, &c.* draw lines across the three parallels A, B, C; then are the spaces or distances *cb, bc, cd, &c.* called the primary divisions.

4th. Divide the left hand space into 10 equal parts (80), and through these points let lines parallel to DE be drawn across the parallels AC; and the primary division *cb* will be parted into 10 equal spaces, called subdivisions.

5th. Number the primary divisions from the left towards the right, beginning at *c*, and the Scale is constructed.



Such Scales are useful for constructing of figures whose sides are expressed in measures of length; as feet, yards, miles, &c. Or to find the measures of the length of lines in a given figure.

Thus a line of 36, or 360, feet, yards, &c. is taken, by setting one point of the compasses on the third primary division, and extending the other point to the sixth subdivision: and so of others.

In these Scales it is usually supposed, that an inch is divided into some number of such parts, as are expressed by the subdivisions.

82. *To find the divisions of a Scale of equal parts, when any given number of them are to make an inch.*

OPERATION. 1st. Make a Scale *ca*, where the primary divisions shall be each one inch; let DC be at right angles to *ca*, and the left hand space *cb* be divided into 10 equal parts, and draw *db* from any point D.

2^d. Draw DH, making with DC any angle; and make *DI = cb*.

3^d. Take the number of parts proposed in an inch, from the Scale *ca*, and apply them from D to *n*, in the line DC, continued if necessary.

4th. Draw *ni*, and *CG* parallel to *ni*; and make *DR = DG*.

5th. Through *r* draw *rs* parallel to *ca*, cutting *db* in *t*; then *rt* will be one of the primary divisions containing 10 of the parts the inch was to be divided into.

6th. Lines drawn from *p* to the divisions of *cb*, divides *rt* into 10 equal parts.

83. PRO-

PROBLEM XXI.

83.

To divide the circumference of a circle into degrees; and thence to make a Scale of chords.

OPERATION. 1st. Describe the semicircle ABD, whose center is c and draw CD at right angles to AB.

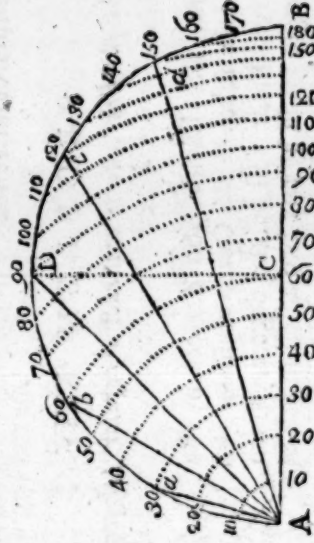
2d. Trisect the angles ACD, DCB, in the points *a, b; c, d* (61): then by trials, divide the arcs *Aa, ab, bD, Dc, cd, dB*, each into three equal parts, and the semicircumference will be divided into 18 equal parts, of 10 degrees each.

3d. These arcs being bisected, will give arcs of 5 degrees.

4th. And these arcs being divided into 5 equal parts, by trials, will give arcs of 1 degree each.

The like may be done with the whole circumference.

5th. The chords of the arcs *Ad, Ac, AD, Ab, Aa*, and of every other arc, being applied from A on the diameter AB, will divide AB into a Scale of chords; which are to be numbered from A towards B.



A Scale of chords is useful for constructing an angle of a given number of degrees: And for finding the measure of a given circle.

84.

PROBLEM XXII.

To make an angle of a proposed number of degrees.

OPERATION. 1st. Take the first 60 deg. from the Scale of chords, and with this radius describe an arc BC, whose center is A.

2d. Take the chord of the proposed number of degrees from the Scale, reckoning from its beginning, and apply this distance to the arc BC, from B to C.

3d. Lines drawn from A, through the points B and C, will form an angle BAC, whose measure is the degrees proposed.

With Scales of chords not exceeding 90 degrees, such as are generally put on rulers, angles greater than 90 degrees, suppose of 138 deg. are to be taken at twice, viz. 69 and 69 ($\frac{1}{2}$ of 138), or 90 and 48.

When a given angle BAC, is to be measured.

From the angular point A, with the chord of 60 degrees, describe the arc BC, cutting the legs in the points B and C.

Then the distance BC applied to the chords, from the beginning, will shew the degrees which measure the angle BAC.

E 4

85. PRO-

85.

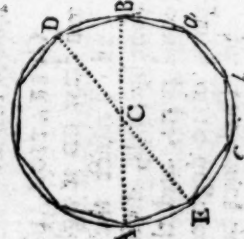
PROBLEM XXIII.

In a given circle to inscribe a regular polygon, the number of sides being given.

OPERATION. 1st. Divide 360 degrees by the number of sides, and the Quotient will be the degrees which measure the angle at the center of the circle, subtended by a side of the polygon.*

2d. Draw the radius CB, make an angle BCD equal to those degrees (84), and draw the chord DB; then will DB be the side of the polygon required; which applied to the circumference from B to *a*, *a* to *b*, *b* to *c*, &c. will give the points to which the sides of the polygon are to be drawn.

If the polygon has an even number of sides; draw the diameter AB; and divide half the circumference, as before; then lines drawn from these points through the center, as ED, will give the remaining points, in the other semicircumference.



86.

PROBLEM XXIV.

On a given right line AB, to construct a regular polygon of any assigned number of sides.

OPERATION. 1st. Divide 360 degrees by the number of sides; subtract the Quotient from 180 degrees, the remainder will be the degrees which measure the angle made by any two adjoining sides of that polygon, and is called the angle of the polygon †.

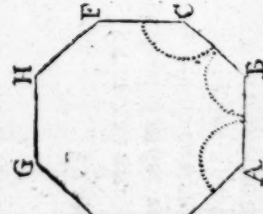
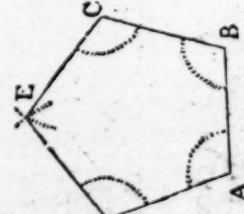
2. At the ends A, B, of the line AB, make angles ABC, BAD, equal to the angle of the polygon.

3d. Make AD, BC, each equal to AB.

4th. At the points C, D, make angles equal to that of the polygon as before; and let the sides including those angles be each equal to AB; and thus proceed until the polygon is constructed.

In figures of any number of sides, the two last sides DE, CE; or EG, HG; are readiest found by describing arcs from C and D, or from E and H, with the radius AB, intersecting in E, or in G.

In figures of an even number of sides, having drawn half the number, AD, AB, BC, CF, by means of the angles; the remaining sides may be found, by drawing through the points D and F the lines DE, FH, parallel and equal to their opposite sides CF, AD; and so of the rest.



* For there will be as many equal angles at the center as there are equal sides.
 And the whole circumference measures the sum of all the angles at the center.
 † For each side of the Polygon with radii drawn to its ends, form an Isosceles triangle.
 Then the angle of the Polygon is derived from articles 85, 96, 104.

87. PROBLEM XXV.

About a given Regular Polygon, to circumscribe a circle: Or, within that Polygon to inscribe a circle.

OPERATION. 1st. Bisect any two angles FAB, CBA, with the lines AG, BG, (56) and the point G, where they intersect one another will be the center of the polygon.

2d. A circle described from G, with the radius FG, will circumscribe the given polygon.

This depends on articles 85, 96, 104.

88. Again. 1st. Bisect any two sides FE, ED, in the points H and I (55); and draw HG, IG, at right angles to FE, ED (57); then the point G, where they intersect each other, will be the center of the Polygon.

2d. A circle described from G, with the radius GH, will be inscribed in the given polygon.

This depends on article 126.

89. PROBLEM XXVI.

On a given right line (AB), to describe a segment of a circle, that shall contain an angle equal to a given right lined angle (C).

OPERATION. 1st. Make an angle BAF equal to the given angle C. (62)

2d. From H, the middle of AB, draw HI at right angles to AB (57), and from A draw AI at right angles to AF (58), cutting HI in I.

3d. From I, with the radius IA, describe a circle.

Then will the segment AGB contain an angle AGB equal to the given angle C.

This depends on articles 125, 126, 132.

90. PROBLEM XXVII.

To divide a right line in continued proportion, in the ratio of two given right lines AB, AC.

OPERATION. 1st. From B with the radius AB (the antecedent) describe an arc Ac.

2d. In that arc apply (the consequent) AC from A to c; draw Ac, and apply cA from c to D.

3d. Apply the following lines in the order directed, viz. AD from A to d, and from d to E; AE from A to e, and from e to F; AF from A to f, and from f to G; AG from A to g, and from g to H, &c. Then will the proportional lines be AB, AC, AD, AE, AF, AG, &c. And

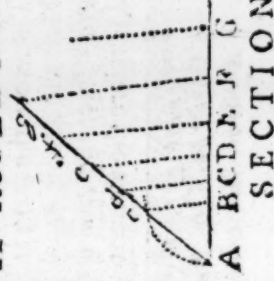
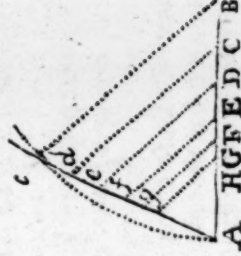
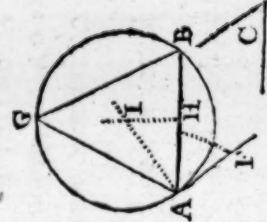
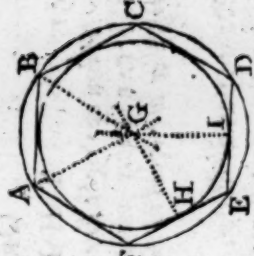
$$AB : AC :: (Ac =) AC : (Ad =) AD.$$

$$AB : AC :: (Ad =) AD : (Ae =) AE.$$

$$AB : AC :: (Ae =) AE : (Af =) AF.$$

&c.

This depends on articles 104, 95, 165.



SECTION

SECTION III.

Geometrical Theorems.

OF RIGHT LINES AND PLANES.

THEOREM I.

91. *When one right line (CD) stands upon another right line (AB), they make two angles (BCD, ACD) which together are equal to two right angles.*



DEMONSTRATION. For describing a semicircle ADB, on C. (45)
Then the arc DB measures the angle BCD. (15)
And the arc DA measures the angle ACD. (15)

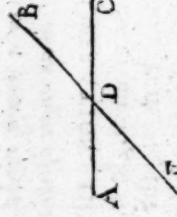
But the arcs DB and AD together measure two right angles. (13, 16)
Therefore BCD and ACD together, are equal to two right angles. (50)

92. COROLLARY. Hence if ever so many right lines (cd) stand on one point (c), on the same side of another right line (AB); the sum of all the angles are equal to two right angles; or are measured by 180 degrees.

THEOREM II.

93. *If two right lines (AC, EB) intersect each other (in D), the opposite angles are equal, viz.*

* $\angle CDB = \angle ADE$, and $\angle CDE = \angle ADB$.



† DEM. For the angles ADE and ADB together make two right angles. (91)

And the angles CDB and ADB together make two right angles. (91)

Therefore the sum of ADE and ADB = sum of CDB and ADB. (46)

Consequently the $\angle CDB$ is equal to the $\angle ADE$. (48)

† 94 COROL. Hence if ever so many right lines cross each other in one point, the sum of all the angles which they make about that point, are equal to four right angles; or are measured by 360 degrees.

THEOREM III.

95. *If a right line (FE) cut two parallel right lines (AB, CD); then is the outward angle (a) || equal to the inward and opposite angle (d); and the alternate angles (c, d) are equal: and the contrary.*



DEM. Because CD and AB are parallel by supposition:

Then FE has the same inclination to CD and AB. (53)

And this inclination is expressed by the $\angle a$ or $\angle d$: (14)

Therefore the outward $\angle a$ is equal to the inward and opposite $\angle d$. (93)

Now the $\angle a$ is equal to the $\angle c$,

And since the $\angle a$ is equal to the $\angle d$;

Therefore the alternate angles c and d are equal. (46)

* The mark $\angle$ stands for the word angle.

† DEM. stands for Demonstration.

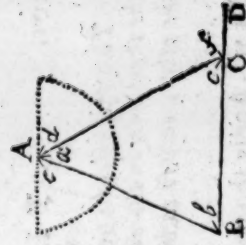
‡ COROL. stands for Corollary.

§ Angles are sometimes marked by a single letter. Thus, angle a is used for angle a .

THEOREM IV.

96.

In any right lined triangle (ABC), the sum of the three angles (a, b, c,) is equal to two right angles: And if one side (BC) be continued, the outward angle (F) is equal to the sum of the two inward and opposite angles (a, b).



DEM. Through A, draw a right line parallel to BC, (63) making with AB the $\angle e$, and with AC the angle d .

Now $\angle e = \angle b$; and $\angle d = \angle c$, being alternates (95)

And two right angles measure the $\angle e + \angle a + \angle d$. (92.)

Therefore $\angle b + \angle a + \angle c = \angle e + \angle a + \angle d$. (47)

Consequently $\angle b + \angle a + \angle c =$ two right angles. (46)

Moreover $\angle c + \angle f =$ two right angles. (91)

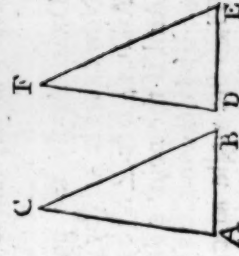
Therefore $\angle b + \angle a = \angle f$. (46)

97. Hence, if one angle is right or obtuse, each of the other's is acute.

98. If two angles of one triangle are equal to two angles of another triangle, the remaining angles are equal. And if one angle in one triangle is equal to one angle in another triangle, then is the sum of the remaining angles in one, equal to the sum of the remaining angles in the other.

THEOREM V.

99. *If two sides (AB, AC) and the included angle (A) in one triangle (ABC), are respectively equal to two sides (DE, DF) and the included angle (D) of another (DEF), each to each; then are those triangles congruous.*



DEM. Apply the point D to the point A, and the line DE to AB. (54)

Now as $DE = AB$ (by supposition); therefore the point E falls on B.

But $\angle D = \angle A$ (by sup.); therefore DF will fall on AC.

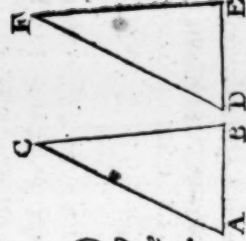
And since $DF = AC$ (by sup.); therefore the point F falls on c.

Consequently FE will fall on CB.

Therefore the triangles ACB, DFE, are congruous; since every part agrees.

100. THEOREM VI.

If two triangles (ABC, DEF) have two angles (A, B) and the included side (AB) in one, respectively equal to two angles (D, E) and the included side (DE) in the other, each to each; then are those triangles congruous.



DEM. Apply the point D to A, and the line DE to AB. (54)

Now as $DE = AB$ (by sup.); therefore the point E falls on B.

And as $\angle D = \angle A$ (by sup.) therefore the line DF falls on AC.

Now if the line AC is less or greater than the line DF;

Then the line FE not falling on CB, makes the $\angle B$ less or greater than $\angle F$.

But $\angle B = \angle E$ (by sup.); therefore AC is neither less nor greater than DF.

Or the line $AC = DF$; consequently $FE = CB$.

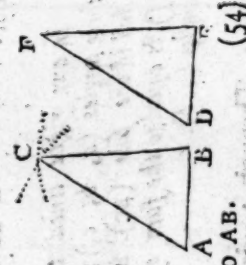
Therefore the triangles are congruous.

101. THEO-

101.

THEOREM VII.

Two triangles (ABC, DEF) are congruous, when the three sides in the one, are equal to the three sides in the other, each to each.



(54)

DEM. Apply the point D to A, and the line DE to AB.

Now as $DE = AB$ (by sup.); therefore the point E falls on B.

On A, with the radius AC describe an arc.

Then as $DF = AC$, the point F will fall in that arc.

Also on B with the radius BC describe another arc, cutting the former in C.

And since $EF = BC$, the point F will fall in this arc.

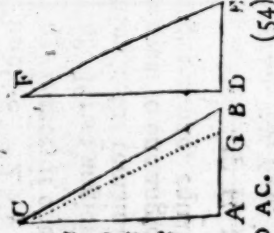
But the point F can be only where these arcs intersect, as in C.

Consequently the triangles are congruous.

102.

THEOREM VIII.

Two triangles (ABC, DEF) are congruous, when two angles (A, B) and a side (AC) opposite to one of them, in one triangle, are respectively equal to two angles (D, E) and a side (DF) opposite to a like angle in the other triangle, each to each.



(54)

DEM. Apply the point D to A, and the line DE to AC.

Now as $DF = AC$ (by sup.), therefore the point F falls on C.

And as $\angle D = \angle A$ (by sup.), the line DE will fall on the line AB.

And if the point E does not fall on B, suppose it falls on any other point G, and draw CG.

Then the angle AGC is equal to the angle DEF.

And the $\angle ABC = (\angle DEF =) \angle AGC$, which is not true.

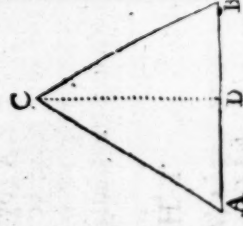
Therefore the point E can fall no where but on the point B.

Consequently the triangles are congruous.

103.

THEOREM IX.

In the Isosceles, or Equilateral triangle (ACB); a line drawn from the vertex (C) to the middle of the base (AB), is perpendicular to the base, and bisects the vertical angle: and the contrary.*



DEM. The triangles ADC, BDC, are congruous.

Since $CA = CB$ (23); $CD = CD$, and $AD = DB$ by supposition:

Therefore $\angle A = \angle B$, $\angle ACD = \angle BCD$, $\angle ADC = \angle BDC$.

Consequently CD is at right angles to AB.

104. COROL. Hence in any right lined triangle where there are equal sides, or angles:

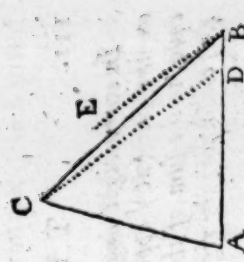
The angles (A, B.) opposite to equal sides (BC, AC) are equal.

And the sides (BC, AC.) opposite to equal angles (A, B.) are equal.

* That is, If a line drawn perpendicular to the base of a triangle, bisects the vertical angle; then that triangle must be Isosceles, and the perpendicular is drawn from the middle of the base.

105. THEOREM X.

In every right lined triangle (ABC), the greatest angle (c) is opposite to the greatest side (AB).



DEM. In the greatest side AB, take $AD=AC$, draw CD, and through B draw BE parallel to CD. (63)

Then the angles ADC, ACD, are equal. (104)

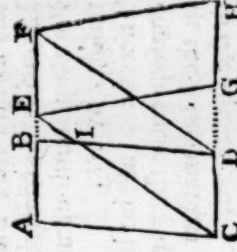
And $\angle ADC = \angle ABE$. (95)

Therefore $\angle ACD = \angle ABE$. (46)

That is, a part of the angle ACB is greater than the angle ABC.

Consequently the $\angle c$ is greater than the $\angle B$; and in the same manner, it will be found to be greater than the $\angle A$.

106. COROL. Hence in every right lined triangle, the greatest side is opposite to the greatest angle.



107. THEOREM XI.

Parallelograms (ACDB, ECDF, EGHF) standing on the same base (CD), or on equal bases (CD, GH, and between the same parallels, (CH, AF), are equal.

DEM. The triangles ACE, BDF, are congruous.

For $AC=BD$, $CE=DF$ (28); and $AE=(AB+BE)=BF$ ($=EF+BE$).

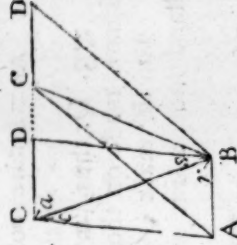
Now if from each of the triangles ACE and BDF, be taken the triangle BIE, the remaining trapeziums ABIC and FEID are equal. (48)

Then if to each of the trapeziums ABIC, FEID, be added the triangle CID, their sum will be the parallelograms AD and DE, which are equal. (47)

And in like manner it may be shewn, that the parallelogram EH is equal to the parallelogram ED $= AD$.

108. THEOREM XII.

A triangle (ABC) is the half of a parallelogram (AD), when they stand on the same base (AB), and are between the same parallels (AB, CD).



DEM. Since CB is a diagonal to the parallelogram AD, and cuts the parallels AB, CD, and CA, DB.

Then $\angle DCB = \angle ABC$, and $\angle ACB = \angle DBC$, being alternates. (95)

And $\angle ABC$, CB, $\angle ACB$, are respectively equal to $\angle DCB$, CB, $\angle DBC$;

Therefore the triangles ABC and BCD are congruous. (100)

Consequently the triangle ABC is half the parallelogram AD.

109. COROL. I. Hence every parallelogram is bisected by its diagonal.

110. COROL. II. Also, triangles standing on the same base, or on equal bases, and between the same parallels, are equal.

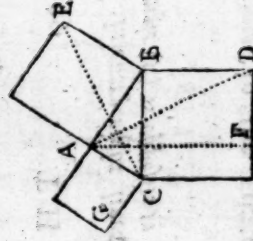
They being the halves of equal parallelograms under like circumstances

III. THEO.

III.

THEOREM XIII.

In every right angled triangle (BAC), the square on the side (BC) opposite to the right angle (A) is equal to the sum of the squares of the two sides (AB, AC) containing the right angle.



DEM. On the sides AB, AC, BC, construct the squares AG, AE, CD (68): draw AD, CE; and draw AF parallel to BD. (63)
Then the triangles ABD, EBC, are congruous. (99)
For the $\angle ABE = \angle CBD$, being right angles. (30)
To each add the angle ABC, then $\angle EBC$ and $\angle ABD$ are equal. (47)
Therefore EB, BC, $\angle EBC$ are respectively equal to AB, BD, $\angle ABD$.
Also the triangle EBC is half the parallelogram AE. (108)
For they stand upon the same base EB, and are between the same parallels EB and AC; BA making right angles with BE and CA continued. (49)
Likewise the triangle ABD is half the parallelogram BF. (108)
For they stand upon the same base BD, and are between the same parallels BD, AF.
Therefore, as the halves of the parallelograms EA and BF are equal, consequently the parallelogram BF is equal to the square AE.
In the same manner may it be shewn, that the parallelogram CF is equal to the square AG.
But the parallelograms BF and CF together, make the square CD.
Therefore the square CD is equal to the squares EA and AG.

II2. COROL. I. Hence if any two sides of a right angled triangle are known, the other side is also known.

For BC = square root of the sum of the squares of AC and AB.

AC = square root of the difference of the squares of BC and AB.

AB = square root of the difference of the squares of BC and AC.

II3. Or thus, making the quantities $\overline{BC}^2$, $\overline{AB}^2$, $\overline{AC}^2$, to stand for the squares made on those lines.

And the mark $\sqrt{}$ to stand for the square root of such quantities as stand under a line joined to the top of this mark.

$$\text{Then } BC = \sqrt{\overline{AC}^2 + \overline{AB}^2};$$

$$AC = \sqrt{\overline{BC}^2 - \overline{AB}^2};$$

$$AB = \sqrt{\overline{BC}^2 - \overline{AC}^2}.$$

Scholium. The lines of the lengths 5, 4, 3, (or their doubles, triples, &c.) will form a right angled triangle.

$$\text{For } \sqrt{5^2} = 4^2 + 3^2. \text{ Or } 25 = 16 + 9.$$

II4. COROL.

114. COROL. II. Of all the lines drawn from a given point to a given line, the perpendicular is the shortest.

115. COROL. III. The shortest distance between two parallel right lines, is a right line drawn from one to the other perpendicular to both.

116. COROL. IV. Parallel right lines are equidistant; and the contrary. For two opposite sides of a rectangular parallelogram are equal (28); and each is the shortest distance between the other sides.

THEOREM XIV.

If a right line (AB) be divided into any two parts (AC, CB); then will the square on the whole line be equal to the sum of the squares on the parts, together with two rectangles under the two parts.

That is $\overline{AB}^2 = \overline{AC}^2 + \overline{CB}^2 + 2 \times AC \times CB$.*

DEM. Let AD, AF, be squares on AB, AC. (68)

Then will FG, and GD be each equal to CB. (48)

Hence FD is a square on a line equal to CB. (30)

Also FB and FE are rectangles on lines equal to AC, CB.

But the squares AF, FD, and the rectangles FB, FE, E fill up the square AD, or are equal to AD.



118. COROL. I. Hence the square of (AC) the difference between two lines (AB, CB), is equal to the square of the greater (AB), lessened by the square of the less (CB) and by two rectangles under the lesser line (CB) and the said difference.

That is $\overline{AB} - \overline{BC}^2 = \overline{AB}^2 - \overline{BC}^2 - 2BC \times AC$.

For $\overline{AB} - \overline{BC} = \overline{AC}$.

Then $\overline{AC}^2 = \overline{AB}^2 - \overline{BC}^2 - 2BC \times AC$. (48)

119. COROL. II. The difference between the squares on two lines (AB, AC) is equal to the rectangle under the sum (AB + BC) and difference (AB - BC) of those lines.

That is $\overline{AB}^2 - \overline{AC}^2 = \overline{AB + BC} \times \overline{AB - AC}$.

For $\overline{AB}^2 - \overline{AC}^2 = \overline{CB}^2 + 2AC \times CB$.

$= \overline{CB + AC + AC} \times CB$.

$= \overline{AB + AC} \times (CB =) \overline{AB - AC}$.

(117, 48)

* The rectangle under two lines is generally expressed by 3 letters; the first two letters stand for one line, and the last two for the other line.

Thus for $AC \times CB$, is wrote ACB .

for $AB \times BC$, is wrote ABC .

And for $2 \times AC \times CB$, is wrote $2ACB$.

120.

THEOREM XV.

In every triangle, (ADC) the square of a side (CD) subtending an acute angle (A), is equal to the squares of the sides (AD, AC) about that angle, lessened by two rectangles under one of those sides (AC) and that part thereof (AB) contained between the acute angle and the perpendicular (DB) drawn to that side (AC) from its opposite angle (D).

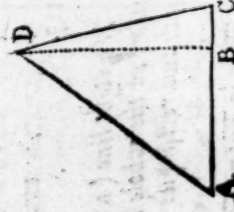
That is, $\overline{DC}^2 = \overline{AD}^2 + \overline{AC}^2 - 2CAB$.

DEM. $\overline{DC}^2 - \overline{BC}^2 = (\overline{DB}^2 =) \overline{AD}^2 - \overline{AB}^2$. (111)

And $\frac{\overline{AC}^2}{\overline{AB}^2} = \frac{2ABC + \overline{BC}^2 + \overline{AB}^2}{\overline{AB}^2}$. (117)

Then $\overline{DC}^2 - \overline{BC}^2 - \overline{AC}^2 = \overline{AD}^2 - \overline{BC}^2 - 2\overline{AB}^2 - 2ABC$.

(48)



And $\overline{DC}^2 = \overline{AD}^2 + \overline{AC}^2 - 2\overline{AB}^2 - 2ABC$, by adding $\overline{BC}^2 + \overline{AC}^2$. (47)

$$(-\overline{AB} \times 2AB - \overline{BC} \times 2AB)$$

$$(-\overline{AB} + \overline{BC} \times 2AB)$$

Then $\overline{DC}^2 = \overline{AD}^2 + \overline{AC}^2 - (\overline{AC} \times 2AB =) 2CAB$.

121. COROL. Hence $AB = \frac{\overline{AD}^2 + \overline{AC}^2 - \overline{DC}^2}{2CA}$.

122.

THEOREM XVI.

In an obtuse angled triangle (ACD), the square of the side (AD) opposite to the obtuse angle (C) is equal to the sum of the squares of the sides (AC, CD) about the obtuse angle; together with two rectangles under one side (AC), and the continuation (CB) of that side to meet a perpendicular (DB) drawn to it from the opposite angle (D).

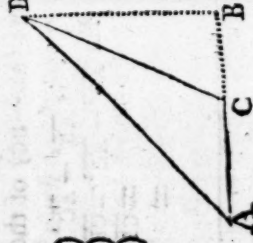
That is, $\overline{AD}^2 = \overline{AC}^2 + \overline{CD}^2 + 2AC \times CB$.

DEM. For $\overline{AD}^2 - \overline{AB}^2 = (\overline{DB}^2 =) \overline{CD}^2 - \overline{CB}^2$. (111)

And $\frac{\overline{AB}^2}{\overline{AD}^2} = \frac{2ACB + \overline{AC}^2 + \overline{CB}^2}{\overline{AC}^2 + \overline{CD}^2 + 2ACB}$. (117)

Then $\overline{AD}^2 = \overline{AC}^2 + \overline{CD}^2 + 2ACB$. (47)

123. COROL. Hence $CB = \frac{\overline{AD}^2 - \overline{AC}^2 - \overline{CD}^2}{2AC}$.



124.

THEOREM XVII.

In any circle, a diameter (AB) drawn perpendicular to a chord (DE), bisects that chord and its subtended arc (DBE).

DEM. From the center C, draw the radii CB, CE, to the extremities of the chord DE.

Then the triangle CFE, CFD, are congruous. (102).

For CF being at right angles to DE, the $\angle CFD = \angle CFE$.

And the triangle CDE being isosceles (23), the $\angle D = \angle E$ (104). Also CF is common. (17)

Therefore $DF = FE$: And the arc $DB = BE$.

For those arcs measure the equal angles FCE, FCD. (15)

125. COROL. Hence, in a circle, a right line drawn through the middle of a chord at right angles to it, passes through the center of that circle; and the contrary.

126.

THEOREM XVIII.

A tangent (AB) to a circle, is perpendicular to a diameter (DC) drawn to the point of contact (C).

DEM. If it be denied that DC is perpendicular to AB.

Then from the center D, let some other line DB, cutting the circle in E, be drawn perpendicular to AB.

Now the angle DBC being right, the angle DCB is acute.

And $DC = DE$, is greater than DB (106), which is absurd.

Therefore no other line passing through the center can be perpendicular to the tangent, but that which meets it at the point of contact. (97)



127.

THEOREM XIX.

An Angle (BCD) at the center of a circle, is double to the angle (BAD) at the circumference, when those angles stand on the same arc (BD).

DEM. Through the point A draw the diameter AE.

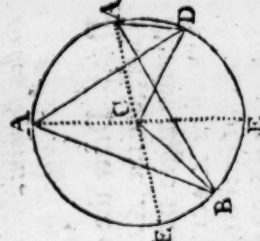
Then the angle $ECD = \angle CAD + \angle CDA$. (96)

But the $\angle CAD = \angle CDA$. (104)

Therefore the $\angle ECD$ is equal to twice the angle CAD.

In the same manner it may be shewn, that the angle BCE is equal to twice the angle BAE.

Consequently the angle $BCD (= \angle ECD + \angle BCE)$ is equal to twice the angle BAD $(= \angle EAD + \angle BAE)$. (47, 48)



128. COROL. I. Hence an angle (BAD) at the circumference is measured by half the arc (BD) on which it stands.

For the angle at the center (BCD) is measured by the arc BD. (15)
Consequently the angle $BAD =$ half the angle BCD, is measured by half the arc BD.

129. COROL. II. All angles in the circumference, and standing on the same arc, are equal.

* The mark $\frac{1}{2}$, signifies the sum or difference.

F

130. THEO-

130.

THEOREM XX.

An angle (BAC) in a semicircle, is a right one.

An angle (DAC) in a segment less than a semicircle, is obtuse.

An angle (EAC) in a segment greater than a semicircle is acute.



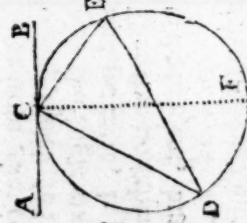
DEM. For the angle BAC is measured by half the semicircular arc BEC, or is measured by half of 180 degrees; that is, by 90 degrees. (128, 16) And $\angle DAC$ is measured by half the arc DEC, greater than 180°. $\dagger$ Also $\angle EAC$ is measured by half the arc EC, less than 180°. Therefore these angles are respectively equal to, greater, or less than 90 degrees.

131. COROL. Hence in a right angled triangle (BAC); the angular point (A) of the right angle, and the ends (B, C) of the opposite side, are equally distant from (F) the middle of that side; that is, a circle will always pass through the right angle, and the ends of its opposite side taken as a diameter.

132.

THEOREM XXI.

The angle (ACD) formed by a tangent (AB) to a circle (CDE), and a chord (CD) drawn from the point of contact (C), is equal to an angle (CED) in the alternate segment; and is measured by half the arc (DC) of the included segment.



DEM. Draw the diameter CF; then $\angle ACF$ is right. Therefore the $\angle ACF$ is measured by half the arc CDF. But the angle DCF is measured by half the arc DF. Therefore $\angle ACD (= \angle ACE - DCF)$ is measured by half the arc DC ($= CDF - DF$) of the included segment. And $\angle CED$ is measured by half the arc CD. Therefore $\angle ACD = \angle CED$.

(126)

(76)

(128)

(128)

(50)

$\dagger$ A small ° put above any figure, signifies degrees.

133.

THEOREM XXII.

Between a circular arc (AHF) and its tangent (AE), no right line can be drawn from the point of contact (A).

DEMON. For if any other right line can be drawn, let it be the right line AB.

From D the center of AHF, draw DG perpendicular to AB, cutting AB in G, and the arc in H.

Now as $\angle DGA$ is right; therefore DA is greater than DG . (106)
But $DA = DH$ (9). Therefore DH is greater than DG , which is absurd. Consequently no right line can be drawn between the tangent AE and the arc AHF .

134. COROL. I. Hence the angle DAH , contained between the radius DA , and an arc AH , is greater than any right lined acute angle.

For a right line AB must be drawn from A , between the tangent AE and radius AD , to make an acute angle.

But no such right line can be drawn between AE and the arc AH . (133)

135. COROL. II. Hence the angle EAH , between the tangent EA and arc AH , is less than any right lined acute angle.

136. COROL. III. Hence it follows, that at the point of contact, the arc has the same direction as the tangent, and is at right angles to the radius drawn to that point.

137. THEOREM XXIII.

If two right lines (AB, CD) intersect any bow (in E) within a circle, their inclination (AED, or CEB) is measured by half the sum of the intercepted arcs (AD, CB).

DEM. For drawing DB ;

The $\angle AED = \angle EDB + \angle EBD$. (43)

But the $\angle EDB$ is measured by $\frac{1}{2}$ arc CB . (96)

And the $\angle EBD$ is measured by $\frac{1}{2}$ arc AD . (128)

Consequently the $\angle AED$ is measured by half the arc CB together with half the arc AD . (128)

(50)

SECTION IV.

Of Proportion.

DEFINITIONS AND PRINCIPLES.

138. One quantity A, is said to be measured or divided by another quantity B, when A contains B some number of times.

Thus if $A=20$, and $B=5$; then A contains B four times. A is called a multiple of B; and B is said to be part of A.

139. If a quantity A ($=20$) contains another B ($=5$) as many times as a quantity C ($=24$) contains another D ($=6$); then A and C, are called like multiples of B and D.

B and D, are called like parts of A and C: And A is said to have the same relation to B, as C has to D.

Or, like-multiples of quantities are produced, by taking their Rectangle, or Product, by the same quantity, or by equal quantities.

The Rectangle or Product of quantities (A and B) is expressed by writing this mark $\times$ between them. Thus $A \times B$, or $B \times A$.

140. When two quantities of a like kind are compared together, the relation which one of them has to the other, in respect to quantity, is called Ratio.

The first term of a ratio, or the quantity compared, is called the Antecedent; and the second term, or the quantity compared to, is called the Consequent.

A ratio is usually denoted by setting the antecedent above the consequent with a line drawn between them.

Thus $\frac{A}{B}$ signifies, and is thus to be read, the ratio of A to B.

The multiple of a ratio $\frac{A}{B}$, is the product of each of its terms by the same

quantity, or by equal quantities. Thus $\frac{A \times C}{B \times C}$ is the ratio $\frac{A}{B}$ taken c times.

The product of two, or more ratios, $\frac{A}{B}, \frac{C}{D}$, is expressed by taking the product of the antecedents for a new antecedent, and the product of the consequents for a new consequent. Thus $\frac{A \times C}{B \times D} = \frac{A}{B} \times \frac{C}{D}$.

141. Equal ratios are those where the antecedents are like multiples or parts of their respective consequents.

Thus in the quantities A, B, C, D: Or 20, 5, 24, 6.

In the ratio of A to B, or of 20 to 5, the antecedent is a multiple of its consequent four times.

And in the ratio of C to D, or 24 to 6, the antecedent is a multiple of its consequent four times.

That is, the ratio of A to B is the same as the ratio of C to D.

And this equality of ratios is thus expressed, $\frac{A}{B} = \frac{C}{D}$.

142. *Ratio of equality is, when the antecedent is equal to the consequent.*

Thus when $A=B$, then $\frac{A}{B}$, or $\frac{A}{A}$, or $\frac{B}{B}$, is a ratio of equality.

143. *Four quantities are said to be proportional, which, when compared together by two and two, are found to have equal ratios.*

Thus, let the quantities to be compared be A, B, C, D : Or 20, 5, 24, 6.
Now in the ratio of A to B, or of 20 to 5; A contains B four times.
And in the ratio of C to D, or of 24 to 6; C contains D four times.

Then the ratios of A to B, and of C to D, are equal: Or $\frac{A}{B} = \frac{C}{D}$. (141)

And their proportionality is thus expressed, $A : B :: C : D$. (75)

Also in the ratio of A to C, or of 20 to 24; C contains A, once and $\frac{2}{3}$.

And in the ratio of B to D, or of 5 to 6; D contains B, once and $\frac{1}{3}$.

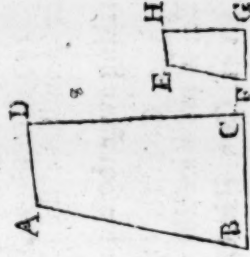
Where the ratios are likewise equal, viz. $\frac{A}{C} = \frac{B}{D}$.

And these are also proportional, $A : C :: B : D$.

144. *So that when four quantities of the same kind are proportional, the ratio between the first and second, is equal to the ratio between the third and fourth; and this proportionality is called Direct.*

145. *Also the ratio between the first and third, is equal to the ratio between the second and fourth; and this proportionality is called Alternate.*

146. *Similar, or like, right lined figures, are such which are equiangular, (that is, whose several angles are equal one to the other;) and also, the sides about the equal angles proportional.*



Thus if the figures AC and EG are equiangular,
And $AB : BC :: EF : FG$; Or $BC : CD :: FG : GH$;
Then are those figures called similar, or like figures.
And the like in triangles, or other figures.

147. *Like arcs, chords, or tangents, in different circles, are those which subtend, or are opposite to, equal angles at the center.*

Let F be the center of two concentric arcs AEB, C
aeb, terminated by the radii FAA, FBB, produced;
AB, ab, their chords, and CD, cd, their tangents.
Then as the angle CFD is measured either by the
arc AEB, or aeb, those arcs are said to be alike,
or similar; that is, the arc aeb is the same part
of its whole circumference, as the arc AEB is of
its whole circumference.

148.

T H E O R E M XXIV.

Quantities, and their like multiples, have the same ratio.

That is, the ratio of A to B, is equal to the ratio of twice A to twice B, of thrice A to thrice B, &c. Or thus $\frac{A}{B} = \frac{2A}{2B} = \frac{3A}{3B}$, &c. $= \frac{cA}{cB}$; that is, equal to the ratio of c times A to c times B.

DEM. For the ratio of A to B, must either be equal to the ratio of like multiples of A and B, or to the ratio of unlike multiples of them.

Now suppose the ratio of A to B is equal to the ratio of their unlike multiples, c times A, d times B; that is, $\frac{A}{B} = \frac{cA}{dB}$.

Then $A : B :: cA : dB$ (143). And $A : cA :: B : dB$. (145)

Therefore $\frac{A}{cA} = \frac{B}{dB}$ (144). Where the consequents are unequal multiples of their antecedents, by supposition.

But $\frac{A}{cA}$ is not equal to $\frac{B}{dB}$. (141)

Then $A : cA :: B : dB$ is not true. Also $A : B :: cA : dB$ is not true.

Consequently $\frac{A}{B}$ is unequal to $\frac{cA}{dB}$.

Therefore the ratio of unlike multiples of two quantities, is not equal to the ratio of those quantities.

Consequently the ratio of two quantities, and the ratio of their like multiples, are the same. Or $\frac{A}{B} = \frac{cA}{cB}$.

149. COROL. I. In any ratio, if both terms contain the same quantity or quantities; the value of the ratio will not be altered by omitting, or taking away those quantities. For $\frac{cA}{c \times B} = \frac{A}{B}$, by taking away c.

150. COR. II. Quantities, and their like parts, have equal ratios. For A and B are like parts of cA and cB.

151. COR. III. Quantities, and their like multiples, or like parts, are proportional. For $A : B :: cA : cB$. And $cA : cA :: cB : cB$. (148)

152. COR. IV. If quantities are equal, their like multiples, or like parts, are also equal. For if $A = B$; and $\frac{A}{B} = \frac{cA}{cB}$;

Then are the antecedents and consequents in a ratio of equality. (141)

153. COR. V. If the parts of one quantity, are proportional to the parts of another quantity, they are like parts of their respective quantities. For only like parts are proportional to their wholes. (151)

154. COR. VI. Ratios, which are equal to the same ratio, are equal to one another. For the ratio of $\frac{A}{B} = \frac{cA}{cB} = \frac{dA}{dB}$, &c. (148)

155. COR.

155. COR. VII. Proportions, which are the same to the same proportion, are the same to one another.

If $A : B :: C : D$; and $A : B :: E : F$; Then $C : D :: E : F$.

For $\frac{A}{B} = \frac{C}{D}$; and $\frac{A}{B} = \frac{E}{F}$ (144.) Then $\frac{C}{D} = \frac{E}{F}$. (46)

156. COR. VIII. If two ratios or products are equal, their like multiples, either by the same or by equal quantities, or by equal ratios, are also equal.

That is, if $\frac{A}{B} = \frac{C}{D}$: Then $\frac{A \times E}{B \times E} = \frac{C \times E}{D \times E}$.

And if $E = F$: Then $\frac{A \times E}{B \times E} = \frac{C \times F}{D \times F}$.

And if $\frac{E}{F} = \frac{G}{H}$: Then $\frac{A \times E}{B \times F} = \frac{C \times G}{D \times H}$.

For in either case, the ratios may be considered as quantities.

157. THEOREM XXV.

Equal quantities (A and B) have the same ratio or proportion to another quantity (C). And any quantity has the same ratio to equal quantities.

That is, if $A = B$: Then $A : C :: B : C$. And $C : A :: C : B$.

DEM. Since $A = B$; then C is the like multiple, or part of B, as it is of A. And $A : B :: C : C$ (151). Therefore $A : C :: B : C$. (145). Also $C : C :: A : B$ (151). Therefore $C : A :: C : B$. (145).

158. COROL. I. Hence, when the antecedents are equal, the consequents are equal; and the contrary.

159. COR. II. Quantities are equal, which have the same ratio to another quantity; or to like multiples or parts of another quantity. Thus, if $A : C :: B : C$. Then $A = B$.

160. COR. III. Since $A : C :: B : C$; and $C : A :: C : B$. Therefore, when four quantities are in proportion, As antecedent is to consequent, so is antecedent to consequent: Then shall the first consequent be to its antecedent, as the second consequent to its antecedent: And this is called the *inversion of ratios*.

161.

THEOREM XXVI.

In two, or more, sets of proportional quantities, the rectangles under the like terms are proportional.

That is, if $A : B :: C : D$; and $E : F :: G : H$.

Then $A \times E : B \times F :: C \times G : D \times H$.

DEM. Since $\frac{A}{B} = \frac{C}{D}$; and $\frac{E}{F} = \frac{G}{H}$.

Therefore $\frac{A \times E}{B \times F} = \frac{C \times G}{D \times H}$.

Consequently $A \times E : B \times F :: C \times G : D \times H$.

F 4

162. THEO.

162.

THEOREM XXVII.

In four proportional quantities $A : B :: C : D$. Then the Rectangle or Product of the two extremes, is equal to the Rectangle or Product of the two means. That is, $A \times D = B \times C$.

DEM. Since $A : B :: C : D$ by supposition. Therefore $\frac{A}{B} = \frac{C}{D}$. (144)

And $\frac{A}{B} = \frac{A \times D}{B \times D}$ (156). Also $\frac{C}{D} = \frac{C \times B}{D \times B}$. (156)

Therefore $\frac{A \times D}{B \times D} = \frac{C \times B}{D \times B}$ (46), where the consequents are equal.

Consequently $A \times D = C \times B$. (158)

163. Hence, if the Rectangle or Product of two quantities is equal to the Rectangle or Product of other two quantities; those four quantities are proportional.

Thus, suppose the two Rectangles, X , Z , are equal;

Where A , C , are their lengths, and B , D , their breadths.

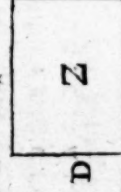


Then $A \times B = C \times D$ by supposition.

Therefore $A : C :: D : B$.

That is, As the length of X is to the length of Z .

So the breadth of Z is to the breadth of X .



In such cases, the lengths are said to be one another reciprocally, as their breadths.

Or that proportion $A : C :: D : B$ is reciprocal, when $A \times B = C \times D$.

164.

THEOREM XXVIII.

If four quantities are proportional; then will either of the extremes, and the ratio of the product of the means to the other extreme, be in a ratio of equality: And either mean, and the ratio of the product of the extremes to the other mean, will be also in a ratio of equality.

That is, if $A : B :: C : D$. Then $A = \frac{B \times C}{D}$. And $B = \frac{A \times D}{C}$.

DEM. Since $A : B :: C : D$ by supposition.

Therefore $A \times D = B \times C$.

And $\frac{A \times D}{C} = \frac{B \times C}{C}$ (157). Also $\frac{B \times C}{D} = \frac{A \times D}{D}$. (162)

But $B = \frac{B \times C}{C}$ (149). And $A = \frac{A \times D}{D}$. (149)

Therefore $\frac{A \times D}{C} = B$. And $\frac{B \times C}{D} = A$. (46)

165. THEO.

165. THEOREM XXIX.

In any plane triangle (ABC) any two adjoining sides (AB, AC) are cut proportionally by a line (DE) drawn parallel to the other side (BC) , viz.

$$AD : DB :: AE : EC.$$

DEM. Through B and C draw Bb, Cc at right angles to BC , meeting ba drawn through A parallel to BC : Through p, q , the middles of Ab, Aa , draw pp, qq , parallel to Bb on Ca , meeting AB, AC , in the points d, e , and join da . Now the triangles Adp, Bdp , and Aeq, Ceq , are congruous. Therefore $Ad = Bd$, $Ae = Ce$, $pd = pd$, $qe = qe$. But $pp = qq$ (116): Therefore $pd = qe$. And de is parallel to BC .

In the same manner it may be shewn, that lines parallel to Bb , drawn through the middles of Ap, pb ; Aq, qa ; will also bisect Ad, Bd ; Ae, Ce ; and that lines joining these points of bisection will also be parallel to BC : And the same may be proved at any other bisections of the segments of the lines AB, AC : Also the like may be readily inferred at any other divisions of the lines Ab, Aa .

Therefore lines parallel to BC , cut off like parts from the lines AB, AC . Then $AB : AC :: AD : AE$. And $AB : AC :: BD : CE$. (151)

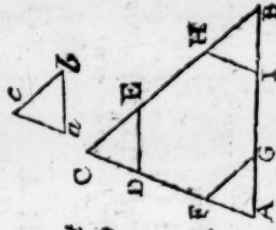
Therefore $AD : AE :: BD : CE$. (155)

And by Alternation $AD : BD :: AE : CE$. (145)

166. COROL. Hence, when the sides (AB, AC) of a triangle are cut proportionally. (in D, E) the segments $(AD, AE; DB, EC)$ of those sides are proportional to the sides: And the line (DE) drawn to those sections, is parallel to the other side (BC) .

167. THEOREM XXX.

In equiangular triangles (ABC, abc) , the sides about the equal angles are proportional; and the sides opposite to equal angles are also proportional.



DEM. In CA, CB , take $CD = ca$, $CE = cb$; and F draw DE .

Then the triangles CDE, cab , being congruous. (99) Therefore DE is parallel to AB . The $\angle CDE = (\angle a =) \angle A$. (95)

In the same manner, taking $AF = ac$, $AG = ab$; also $BH = bc$, $BI = ba$; and drawing FG, HI , the triangles AGF, IBH, abc , are congruous; therefore FG is parallel to CB , and HI is parallel to CA .

Then $(CD =) ca : CA :: (CE =) cb : CB$.

$$(AF =) ca : CA :: (AG =) ab : AB.$$

$$(BH =) bc : BC :: (BI =) ab : AB.$$

(165)

168. COROL. Hence, Triangles having one angle in each equal, and the sides about those equal angles proportional, those triangles are equiangular and similar.

169. THEO-

169. THEOREM XXXI.

In a right lined triangle (ABC), if a line (BD) be drawn from the right angle (B) perpendicular to the opposite side (AC); then will the triangles (ABD, BCD) on each side the perpendicular be similar to the whole (ABC), and to one another.



DEM. For in the triangles ABC, ADB, the $\angle A$ is common; And the right angle $ABC = \text{right angle } ADB$.

Therefore the remaining $\angle C = \angle ABD$.

In the same manner it will appear, that the triangles ABC, BDC, are like. (98)
Therefore the triangles ABD, BCD, are also similar.

$$\begin{aligned} 170. \text{ COROL. I. Hence, } & AC : AB :: AB : AD. \\ & AC : BC :: BC : DC. \\ & AD : DB :: DB : DC. \end{aligned} \quad \left. \begin{array}{l} (162) \\ (131) \end{array} \right\}$$

171. COR. II. Hence a right line (BD) drawn from the circumference of a circle perpendicular to the diameter (AC) is a mean proportional between the segments (AD, DC) of the diameter.

And $AD \times DC = DB^2$. (162)

For a circle whose diameter is AC, will pass through A, B, C. (131)

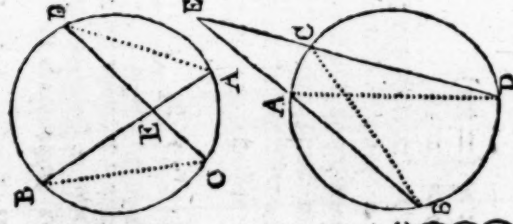
Scholium. This corollary includes what is usually called one of the chief properties of the circle, namely.

The Square of the Ordinate is equal to the rectangle under the two Abscissas.

Here, the ordinate is the perpendicular BD; and the two Abscissas are the two segments AD, DC, of the diameter AC.

172. THEOREM XXXII.

In a circle, if two chords (AB, CD) intersect each other (in E), either within the circle, or without by prolonging them; then the Rectangle under the segments, terminated by the circumference and their intersection, will be equal.



That is, $AE \times EB = CE \times ED$.

DEM. Draw the lines BC, DA.

Then the triangles DEA, BEC, are similar.

For the angle at E is equal (93), or common.

And the $\angle D = \angle B$, as standing on the same arc

AC (129): Then the other angles are equal. (98)

Therefore $AE : CE :: ED : EB$. (167)

Consequently $AE \times EB = CE \times ED$. (162)

173.

THEOREM XXXIII.

If with the least side AB of a given triangle ABC, a semicircle be described from the angular point A; meeting the side AC, produced, in the points D, E; and from B, the lines BE, BD, be drawn and also BG perpendicular to DE: Then the values of the several lines AG, CG, GE, GD, BE, BD, EG, may be expressed in terms of the sides of the triangle ABC. as follow.



$$174. \quad AG = \frac{BC^2 - AC^2 - AB^2}{2AC}. \quad (123)$$

$$\text{Or } AG = \frac{AC^2 + AB^2 - CB^2}{2AC}. \quad (121)$$

$$175. \quad CG = \frac{AC^2 - AB^2 + BC^2}{2AC}. \quad \text{For } CG = AC + AG, \text{ or to } AC - AG.$$

$$\text{And } AC \pm AG = AC \pm \frac{BC^2 - AC^2 - AB^2}{2AC}. \quad (174)$$

$$= \frac{2AC^2 + BC^2 - AC^2 - AB^2}{2AC}. \quad (149)$$

$$176. \quad GE = \frac{2H \times H - CB}{2AC} : \text{ Here } 2H = AC + AB + BC.$$

$$\text{For } GE = AB \pm AG = AB + \frac{AC^2 + AB^2 - CB^2}{2AC}. \quad (174)$$

$$= \frac{2AC \times AB + AC^2 + AB^2 - CB^2}{2AC}. \quad (149)$$

$$\text{But } 2AC \times AB + AC^2 + AB^2 = (AC + AB)^2 = CE^2. \quad (117)$$

$$\text{Then } GE \left(= \frac{CE^2 - BC^2}{2AC} \right) = \frac{CE + BC \times CE - BC^2}{2AC}. \quad (119)$$

$$= \frac{CA + AB + BC \times CA + AB - BC^2}{2AC}.$$

$$= \frac{2H \times 2H - 2BC^2}{2AC} = \frac{2H \times H - BC^2}{2AC}.$$

$$177. \quad GD =$$



$$177. \quad GD = \frac{H - AC \times \frac{2H - 2AB}{AC}}{AC}$$

Here $2H = AC + AB + BG$.

$$\text{For } GD = AD \mp AG = AB \mp AG = AB - \frac{AC^2 - AB^2 + BC^2}{2AC} \quad (174)$$

$$= \frac{2AC \times AB - AC^2 - AB^2 + BC^2}{2AC}$$

$$= \frac{AC - AB^2 + BC^2}{2AC} = \frac{BC^2 - CD^2}{2AC} \quad (118)$$

$$= \frac{BC - CD \times BC + CD}{2AC} \quad (119)$$

$$= \frac{BC + AB - AC \times BC + AC - AB}{2AC}$$

$$= \frac{2H - 2AC \times \frac{2H - 2AB}{2AC}}{2AC}$$

$$178. \quad BE = \sqrt{\frac{AB}{AC} \times 2H \times H - CB}$$

For $BE^2 = DE \times GE$. (170)

$$= 2AB \times \frac{CE^2 - BC^2}{2AC} \quad (176)$$

$$\text{Therefore } BE = \sqrt{\frac{2AB}{2AC} \times CE^2 - BC^2} = \sqrt{\frac{AB}{AC} \times 2H \times H - CB} \quad (176)$$

$$179. \quad BD = \sqrt{\frac{AB}{AC} \times H - AC \times \frac{2H - 2AC}{2AC}}$$

For $BD^2 = DE \times GD$. (170)

$$= 2AB \times \frac{BC^2 - CD^2}{2AC} \quad (177)$$

$$\text{Therefore } BD = \sqrt{\frac{2AB}{2AC} \times BC^2 - CD^2} \quad (177)$$

$$180. \quad BG = \frac{2}{AC} \times \sqrt{H \times H - CB \times H - AC \times H - AB}$$

For $BG^2 = GE \times GD$. Therefore $BG = \sqrt{GE \times GD}$. (170)

$$\text{And } GE = \frac{2}{AC} \times H \times H - CB. \quad (176) \quad GD = \frac{2}{AC} \times H - AC \times H - AB. \quad (177)$$

181. COROL. Hence is derived the Rule usually given for finding the area, or superficial content, of a Triangle, the three sides being known.
Rule. 1. From half the sum of the three sides, subtract each side severally, noting the three remainders.

2d. Multiply the said half sum and the three noted remainders continually.

3. The square root of the product is the area of the Triangle.

182. THEO.

182. THEOREM XXXIV.

If a regular polygon (ABCDEF) be inscribed in a circle; and parallel to these sides if tangents to the circle be drawn, meeting one another (in the points a, b, c, d, e, f ;) then shall the figure, formed by these tangents, circumscribe the circle, and be similar to the inscribed figure.



DEM. Since the circle touches every side of the figure $abcdef$, by construction; therefore the circle is circumscribed by that figure. (41) Through A and B, draw the radii SA, SB, prolonged till they meet the tangent ab , in a, b .

Then the triangles ASB, asb , are equiangular.

For the $\angle s$ is common; and the other angles are equal, because AE and ab are parallel, by supposition.

Also $sa = sb$: For the triangles ASB, asb , are Isosceles. (104)

And the same may be proved of the other triangles; and also, that they are equal to one another.

Therefore the figure $abcdef$ has equal sides, and is equiangular to the figure ABCDEF.

Now: SA : sa :: AB : ab; and SA : sa :: AF : af. (167)

Therefore AB : ab :: AF : af. And the like of the other sides. (155)

Consequently the figures ABCDEF, $abcdef$, are similar. (145)

183. COROL. I. If two figures are composed of like sets of similar triangles, those figures are similar. ..

184. COR. II. Hence, if from the angles ($a, b,$) of a regular polygon circumscribing a circle, lines (as, bs) be drawn to the center (s); the chords (AB) of the intercepted arcs, will be the sides of a similar polygon, inscribed in the circle: and the sides (AB, ab) of the inscribed and circumscribing polygons will be parallel.

185. COR. III. The chords or tangents of like arcs in different circles. are in the same proportion as the radii of those circles.

For if a circle circumscribe the polygon $abcdef$; then the sides of the polygons $abcdef$, ABCDEF, are chords of like arcs in their respective circumscribing circles.

And if a circle be inscribed in the polygon ABCDEF, the sides AB, ab , &c. are tangents of like arcs also: And these have been shewn to be proportional to their radii SA, sa.

186. COR. IV. The Perimeters of like polygons (or the sum of their sides) are to one another, as the radii of their inscribed or circumscribed circles.

For SA : sa :: AB : ab. (182)

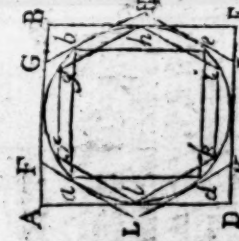
And AB, ab , are like parts of the perimeters of their polygons.

Therefore SA : sa :: perimeter ABCDEF : perimeter $abcdef$. (151)

187.

THEOREM XXXV.

If there be two regular and like polygons applied to the same circle, the one inscribed and the other circumscribed: Then will the circumference of that circle, and half the sum of the perimeters of these polygons, approach nearer to equality, as the number of sides in the polygons increase.



DEM. It is evident at sight, that the circumscribing hexagon FGHKL is less than the circumscribing square ABED.

And also that the inscribed hexagon *fgbiki* is greater than the inscribed square *abed*.

And in both cases, the difference between the hexagon and the circle, is less than the difference between the circle and the square.

Therefore the polygon, whether inscribed or circumscribed, differs less from the circle, as the number of its sides is increased.

And when the number of sides in both is very great, the perimeters of the polygons will nearly coincide with the circumference of the circle; for then the difference of the polygonal perimeters becomes so very small, that they may be esteemed as equal.

And yet so long as there is any difference between these polygons, though ever so small, the circle is greater than the inscribed, and less than the circumscribed polygons: Therefore half their sums may be taken for the circumference of the circle, when the number of those sides is very great.

188. Hence, the circumferences of circles are in proportion to one another, as the radii of those circles, or as their diameters.

For the perimeters of the inscribed and circumscribing polygons are to one another, as the radii of the circles. (186)

And these perimeters and circumferences continually approach to equality.

189. THEO.

189.

THEOREM XXXVI.

In a circle (AFB), if lines (BA, DA, FA) be drawn from the extremities of two equal arcs (BD, DF) to meet in that point (A) of the circumference determined by one of them (BA) passing through the center; then shall the middle line (AD) be a mean proportional between the sum $(AB + AF)$ of the extreme lines, and the radius (BC) of that circle.



DEM. On D with the distance DA, cut AF produced in E.

Then drawing DE, DF, DB, the triangles ADB, EDF, are congruous. (102)

For $\angle EFD = (\angle FDA + \angle FAD \text{ (96)}) = \angle DEA$ (128). Because the arc DFA = DF + FA.

And $\angle E = \angle FAD$ (104) = $\angle DAB$, by construction; and $DE = DA$. Therefore $EF = AB$; and $AE = AB + AF$.

Draw CD; then the triangles ACD, ADE, are similar.

For they are Isoscelar and equiangular.

Therefore $AC : AD :: AD : (AE =) AB + AF$. (167)

190. Hence, when the radius of the circle is expressed by 1, and one of the extreme lines, or chords, passes through the center; then if the number 2 be added to the other extreme chord, the square root of that sum will be equal to the length of the mean chord.

For since $AC : AD :: AD : AB + AF$ (189). Th. $AD^2 = AC \times AB + AF$. (162) Now if $AC = 1$. then $AB = 2$; and $AD^2 = 2 + AF$; because multiplying by 1 is useless here. Therefore $AD = \sqrt{2 + AF}$.

As the arcs BD and DFA make a semicircle, they are called the supplements of one another: Therefore if the arc BD is any part (as $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, &c.) of the semicircumference; then is the line DA called the supplemental chord of that part.

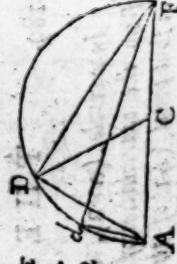
191. REMARK. In the posthumous works of the Marquis de l'Hospital, (page 319, English edition) this principle is applied to the doctrine of angular sections; that is, to the dividing a given arc into any proposed number of equal parts: Or to find the chord of any proposed arc.

For if BF was any assumed arc, whose chord had a known ratio to the given radius BC; then as BFA is a right angled triangle (130), the side $AF = \sqrt{AB^2 - BF^2}$ (113) will also be known: And by this Theorem, the mean chord AD will be known; and also DB ($= \sqrt{AB^2 - AD^2}$) the chord of half the arc BF will also be known.

And by bisecting the arc DB in G, and drawing AG, GB, the mean chord AG is known (189); and GB ($= \sqrt{AB^2 - AG^2}$) is also given.

And in this manner by a continual bisection, the chord of a very small arc may be obtained; the practice whereof is facilitated by article (190), deduced from page 330 of the said work.

192. EXAMPLE. Required the chord of the $\frac{1}{1312}$ part of the circumference of a circle whose radius is 1. Or, required the side of a regular polygon of 3072 sides, inscribed in a circle whose diameter is 2.



Let ADF be a semicircle, whose diameter AF=2, and center is C. Take the arc AD= $\frac{1}{1312}$ of the semicircumference, or equal to 60 degrees; and draw DC, DA, DF.

Let d represent the point where the arc is bisected; df the supplemental chord to that bisection: and let the marks d' , d'' , d''' , d^{iv} , &c. express the bisected points agreeable to the number of bisections.

193. Now since $\angle ACD=60^\circ$.

Therefore $\angle CAD + \angle ADC = (180^\circ - 60^\circ =) 120^\circ$.

But $\angle CAD = \angle ADC$ (104); then $\angle CAD = (\frac{120^\circ}{2} =) 60^\circ$.

Therefore DA=(DC=AC=) 1.

And as the triangle ADF is right angled at D.

(130)

Then DF= $(\sqrt{AF^2 - AD^2}) = \sqrt{4 - 1} = \sqrt{3} = 1,7320508075688773$

Therefore the supplemental chord of the arc AD, or of

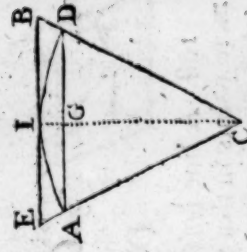
$\frac{1}{2}$ of the semicircumference is FD	$= \sqrt{3}$	$= 1,7320508075688773$
$\frac{1}{6}$ of the same (190)	$= \sqrt{2 + FD}$	$= 1,9318516525781366$
$\frac{1}{12}$	$= \sqrt{2 + FD^I}$	$= 1,9828897227476208$
$\frac{1}{24}$	$= \sqrt{2 + FD^{II}}$	$= 1,9957178464772070$
$\frac{1}{48}$	$= \sqrt{2 + FD^{III}}$	$= 1,9989291749527313$
$\frac{1}{96}$	$= \sqrt{2 + FD^{IV}}$	$= 1,9997322758191236$
$\frac{1}{192}$	$= \sqrt{2 + FD^V}$	$= 1,9999330678348022$
$\frac{1}{384}$	$= \sqrt{2 + FD^{VI}}$	$= 1,9999832668887013$
$\frac{1}{768}$	$= \sqrt{2 + FD^{VII}}$	$= 1,9999958167178004$
$\frac{1}{1536}$	$= \sqrt{2 + FD^{VIII}}$	$= 1,9999989541791767$

Now FD^{IX} the supplemental chord of $\frac{1}{1536}$ being known, the chord Ad^{IX} of $\frac{1}{1536}$ part of the semicircumference, or of $\frac{1}{3072}$ part of the whole circumference, is also known.

That is $Ad^{IX} = (\sqrt{AF^2 - FD^{IX^2}} =) \sqrt{4 - 2 + FD^{VIII}} = 0,0020453073606764$

194. Consequently, the side of a regular polygon of 3072 sides inscribed in a circle whose diameter is 2, is 0,0020453073606764

195. *The side of a similar polygon circumscribing the same circle, whose center is C, may be thus found.*



Let BE be the side of the circumscribed polygon, and draw BC, EC, cutting the circle in D and A.

Draw DA, and it will be the side of the inscribed polygon, and is parallel to BE. (184)

Draw CI bisecting the angle BCE, and it will bisect BE and DA at right angles (103). And $DG = (\frac{1}{2}DA =) \frac{1}{2}Ad^{\frac{1}{2}}$.

Then $CG = \sqrt{CD^2 - DG^2} = \sqrt{1 - \frac{1}{4}Ad^{\frac{1}{2}}}$.

But $\frac{1}{4}Ad^{\frac{1}{2}} = 0,001022653680338$. And its square is 0,0000104582055

0,9999895417945

0,99999947708959

Which subtracted from 1 leaves

Whose square root, or CG, is equal to

Now the triangles CNI, CDG, are similar.

Then $CG : CI :: 2DG : 2BI$.

(167, 151)

Therefore $(2BI =) BE = (\frac{2DG \times CI}{CG} =) \frac{DA}{\frac{1}{2}CG}$: For $IC = I$.

Or $BE = \frac{0,0020453073606764}{0,9999994770895883}$

0,0020453084301895

which is the side of a regular polygon of 3072 sides circumscribing a circle whose diameter is 2.

196. **SCHOLIUM.** The side of a regular polygon of 3072 sides, inscribed in a circle whose diameter is 2, is 0,0020453073606764. (194) Which multiplied by 3072, will give the perimeter of that polygon, which is

6,2831842119979622.

The side of a similar polygon, circumscribing the same circle is

0,0020453084301895. (195)

Which multiplied by 3072, will give for the perimeter of that polygon

6,2831874973420925.

The sum of these perimeters is

12,5663717093400547.

The half sum is

6,28318585 &c.

Which is very nearly equal to the circumference of a circle whose diameter is 2 (187), the difference between it

{ and the inscribed polygon being only 0,00000164, &c.

{ and the circumscribed polygon being only 0,00000164, &c.

197. Now the circumferences of circles being in the same proportion, as their diameters.

Therefore the diameter of a circle being 1,

The circumference will be

3,141592 &c. which agrees with

the circumference as found by other methods.

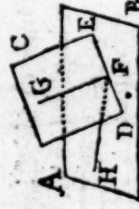
SECTION V.

Of Planes and Solids.

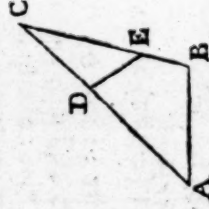
DEFINITIONS and PRINCIPLES.

198. A line is said to be in a plane, when it passes through two or more points in that plane.

199. The inclination of two meeting planes (AB), (CD), is measured by an acute angle (GFH), made by two right lines (FG, FH), one in each plane, and both drawn perpendicular to their common section (DE), from (F) some point therein.



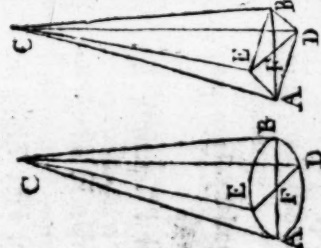
200. A right line (DE) intersecting two sides (AC, BC) of a triangle (ABC), so as to make angles (CDE, CED) within the figure, equal to the angles (CBA, CAB) at the base (AB), but with contrary sides of the triangle, is said to be in a subcontrary position to the base.



201. If a circle in an oblique position be viewed, it will appear of an oval form (as ABCD); that is A it will seem to be longer one way (as AC) than another (as DB); nevertheless the radius's (EA, EB) are to be esteemed as equal: And the same must be understood in viewing of any regular figure, when placed obliquely to the eye.



202. If a line be fixed to any point (C) above the plane of a circle (ADBE), and this line while stretched be moved round the circle, so as always to touch it; then a solid which would fill the space passed over by the line, between the circle and the point C, is called a CONE.



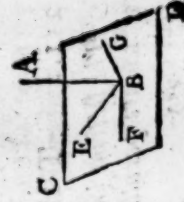
203. If the figure ADBE had been a polygon, and the stretched line had moved along its sides, the figure which would then have been described, is called a PYRAMID.

So that Cones and Pyramids are solids which regularly taper from a circle, or polygon, to a point.

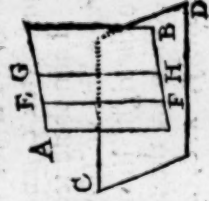
The circle or polygon is called the *Base*; and the point C the *Vertex*. When the vertex is perpendicularly over the middle or center of the base, then the solid is called a RIGHT CONE, or a RIGHT PYRAMID; otherwise an OBLIQUE CONE, or OBLIQUE PYRAMID.

204. If a Cone or Pyramid be cut by a plane passing through the vertex C, and center of the base F, the section ABC, or EDC, is a triangle.

205. A right line (AB) is perpendicular to a plane (CD) when it makes right angles (ABE, ABF, ABG) with all the right lines (BE, BF, BG) drawn in that plane to touch the said right line (AB).



206. So that from the same point (B) in a plane, only one perpendicular can be drawn to that plane on the same side.

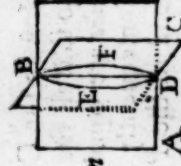


207. A plane (AB) is perpendicular to a plane (CD), when the right lines (EF, GH) drawn in one plane (AB) at right angles to (FB), the common section of the two planes, are also at right angles to the other plane (CD).

208. So that a line (EF) perpendicular to a plane (CD) is in another plane (AB), and at right angles to (FB) the section of the two planes

209. THEOREM XXXVII.

If two planes (AB, CD) cut each other, their common section (BD) will be a right line.



DEM. For if it be not, draw a right line DEB in the plane AB, from the point D to the point B; also draw a right line DFB in the plane BC.

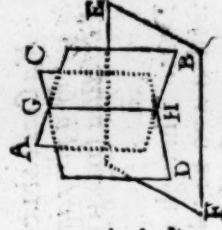
Then two right lines DEB, DFB, have the same terms, and include a space or figure, which is absurd. (7)

Therefore DEB and DFB are not right lines: Neither can any other lines drawn from D to B, besides BD, be right lines.

Consequently the line DB, the section of the planes, is a right line.

210. THEOREM XXXVIII.

If two planes (AB, CD), which are both perpendicular to a third plane (EF), do cut one another; their intersection (HG) is at right angles to that third plane (EF).



DEM. For the common section of AB and CD is a right line (GH); (209) Also HB, HD, are the sections of AB, CD, with the plane EF.

Now from the point H, a line HG drawn perpendicular to the plane EF, must be at right angles to HB, HD. (205)

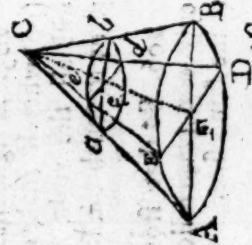
But HG must be in both planes AB, CD. (208)

Therefore it must be in the sections of those planes.

Consequently the section HG of the planes AB, CD, is at right angles to the plane EF.

211. T H E O R E M XXXIX.

The sections (aebd) of a Cone or Pyramid (CAEBD), which are parallel to the base (AEBD), are similar to that base.



DEM. For let AFBC, DFEC, be sections through the vertex C, and center F of the base.

Then these sections will cut one another in the right line FC (209), and the transverse section *abde* in the right lines *ab*, and *ed*, intersecting in *f*.

Then are the following sets of triangles similar; namely, AFC, afc; BFC, bfc; DFC, dfc; EFC, efc.

Wherefore $FC : fc :: FA : fa$
 $:: FE : fb$ } And the like in any other sections
 $:: FD : fd$ } through c and F. (165)
 $:: FE : fe$

Now in the Cone, $FA = FB = FD = FE$; therefore $fa = fb = fd = fe$. (152)
 So that all the right lines drawn from *f* to the circumference of the figure *abde* are equal to one another.

Consequently the figure *abde* is a circle.

And in the Pyramid, $FC : fc :: DC : dc :: DB : db :: DA : da$. (9)

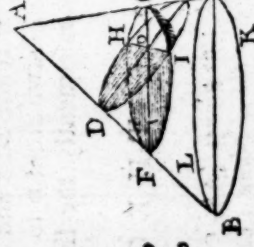
$FC : fc :: EC : ec :: EA : ea :: EB : eb$.

Therefore in each pair of corresponding triangles in the base and transverse section, the sides are respectively proportional.

Consequently, as the base and transverse section are composed of like sets of similar triangles; therefore they are also similar. (183)

212. T H E O R E M XL.

If a Cone (ABLCK), whose base is a circle (CBLCK), be cut by a plane in a subcontrary position to the base, the section (DIEH) will be a circle.



DEM. Through the vertex A, and center of the base, let the triangular section ABC be taken, so as to be at right angles to the planes of the base BCKL, of the subcontrary section DIEH, and of the section FIGH taken parallel to the base and cutting the subcontrary section in the line IOH. Therefore IOH is perpendicular to DE and FG (210) cutting one another in O.

Now the section FIGH is a circle (211). Therefore $FO \times OG = \overline{OI}^2$. (171)
 Again the triangles GOE, FOD, are similar.

For $\angle GEO = \angle DFO = \angle ABC$ by constr. And $\angle GOE = \angle DOF$. (93)

Therefore $EO : OG :: FO : DO$. (167). And $EO \times DO = FO \times OG$ (162) $= \overline{OI}^2$.

So that OI is a mean proportional, either between FO and OG, or DO and EO. But as the same would happen wherever FG cuts DE; therefore all the lines OI, both in the sections FIGH and DIEH, are lines in a circle. Consequently the section DIEH is a circle.

213. If

213. If the section cut both sides of the cone not in a subcontrary position to BC the diameter of the base, then the section (suppose it still) DIH , is called an **ELLIPSE** SECTION, which though not a circle, will be a bounded curve, longer one way than the other; and, like a circle, return into itself.

The curve DIH is called an **Ellipsis**.

214. The line DE , the **TRANSVERSE DIAMETER** OF **AXIS**.

The line OH or OI , is called an **ORDINATE**.

215. The Ordinate through the middle of DE , is called the **CONJUGATE AXIS**.

The intersection of the **Transverse** and **Conjugate Axes**, is called the **CENTER** of the **Ellipsis**.

216. If a circular arc be described, with a radius equal to half the **Transverse Axis**, from one end of the **conjugate Axis**, its intersections with the **Transverse Axis**, are called **Focus's**, one on each side of the center of the **Ellipsis**.

217. Every right line passing through the center of the **Ellipsis** and terminated at each end by the curve, is called a **DIAMETER**.

218. The radius that would describe a circular arc of the same curvature with the ellipsis at any point thereof, is called the **RADIUS OF CURVATURE**.

A **TANGENT** to any point in the **Ellipsis**, is a right line perpendicular to the radius of curvature at that point.

219. Two **Diameters** being so drawn, that one is parallel to a tangent, and the other passes through the point of contact; those two **Diameters** are said to be, **CONJUGATE DIAMETERS**; and have certain relations to their **Ordinates**, **Tangents**, **Radii** of curvature and other lines belonging to the **Ellipsis**.

220. A third proportional to any two **Conjugate Diameters**, is called the **PARAMETER**.

SECTION VI.

Of the Spiral.

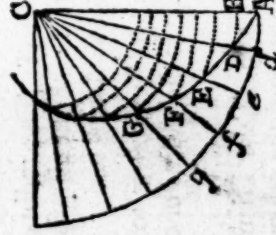
221. Suppose the radius of a circle to revolve with an uniform motion round its center, and while it is revolving, a point is moving along the radius; then will the successive places of that point be in a curve, which is called a spiral.

This will be readily conceived by imagining a fly to move along the spoke of a wheel, while the wheel is turning round.

If while the radius revolves once, the point has moved the length of the radius; then the spiral will have revolved but once round the center, or pole; consequently the motion in the circumference is to the motion in the radius, as the circumference is to the radius: And if the wheel revolves twice, thrice, or in any proportion to the motion in the radius; then the spiral will make so many turns, or parts of a turn, round the center.

222. Now suppose while the radius revolves equably, if a point from the circumference moves towards the center, with a motion decreasing in a geometric progression; then will a spiral be generated, which is called a proportional spiral.

Let the radius CA be divided in any continued decreasing geometric progression (90), as of 10 to 8; then the series of terms will be 10; 8; 6,4; 5,12; 4,096; 3,2768; 2,62144, &c. Also let the circumference be divided into any number of equal parts, in the points $d, e, f, g, \&c.$ Then if the several divisions of the radius CA , be successively transferred from the center C , cutting the other radii in the points $D, E, F, G, \&c.$ and a curved line be evenly drawn through those points, it will be a spiral of the kind proposed.



223. From the nature of a decreasing geometric progression, it is easy to conceive that the radius CA may be continually divided; and although each successive division becomes shorter than the next preceding one, yet there must be an indefinite number of divisions, or terms, before the last of them becomes of no finite magnitude.

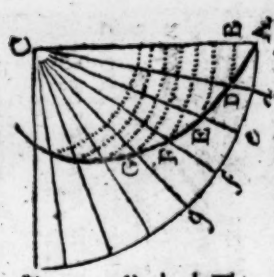
224. Hence it follows, that this spiral winds continually round the center, and does not fall into it till after an indefinite number of revolutions.

Also, that the number of revolutions decrease, as the number of the equal parts, into which the circumference is divided, increases.

225.

THEOREM XLII.

Any proportional spiral cuts the intercepted radii at equal angles.



DEM. If the divisions Ad , de , ef , fg , gc . of the circumference were very small, then would the several radii be so close to one another, that the intercepted parts AD , DE , EF , FG , GC . of the spiral might be taken as right lines.

And the triangles CAD , CDE , CEF , EGC . would be similar, having equal angles at the point C , and the sides about those angles proportional.

Therefore the angles at A , D , E , F , G , C . being equal, the spiral must necessarily cut the radii at equal angles.

226.

THEOREM XLII.

If the radii of any proportional spiral be taken as numbers, then will the corresponding arcs of the circle, reckoned from their commencement, be as the logarithms of those numbers.

DEM. As the lines CA , CD , CE , CF , CG , GC . are a series of terms in geometric progression; and the arcs Ad , Ae , Af , Ag , GC . are a series of terms in arithmetic progression; therefore these arcs may serve (I. 66) as the indices to the geometric terms, and be thus placed;

Radii of the spiral CA , CD , CE , CF , CG , GC . *Geometric terms.*

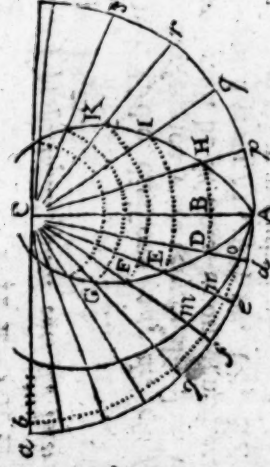
Corresponding arcs O , Ad , Ae , Af , Ag , GC . *Arith. terms; or indices.*

In this disposition, the first term CA is not distant from itself, therefore its index is represented by O .

Then if the distance of the second term CD from the first term CA be expressed by the arc Ad ; the distance of the third term CE , from CA , will be expressed by the arc Ae ; and so of the rest.

Consequently, if the terms in the geometric series be represented by numbers, taken as parts of the radius, then the numbers of the same kind, expressing the measures of the arcs, or indices, will be as the logarithms of the geometric terms. (I. 73)

227. COROL. If the difference between CA and CB was indefinitely small, or CA and CB were nearly in a ratio of equality; then might the number of proportional lines, into which CA could be divided, be so many, that any proposed number might be found among the terms of this series; and if the number of parts in the circumference was increased in like manner, then would every term of the proportional division of the radius CA , have its corresponding index among the equal divisions of the circumference; and consequently would exhibit the logarithms of all numbers.



There may be an almost infinite variety of proportional spirals, and as many different kinds of logarithms.

DEM. For with the same equal divisions $Ad, de, ef, \xi f, \xi g$. of the circumference, every variation in the ratio of CA to CB , as of ca to cb , will produce a different spiral $Aomm$.

And with the same divisions of the radius CA , and different sets of equal parts $Ad, de, ef, \xi f$. and $Ap, pq, qr, \xi r$. of the circumference, may be formed different spirals $ADEF, AHK$.

All, varying at the same time both the divisions of the radius and circumference, different spirals will be produced.

But the variations in these three cases may be almost infinite: Therefore the number of such spirals are almost infinite.

Now 'tis evident, that there is a peculiar relation between the rays of any spiral, and the corresponding arcs of the circle; that is, between the terms of a geometric progression and its indices: Therefore there may be as many kinds of logarithms, as there are proportional spirals.

229.

THEOREM XLIV.

That proportional spiral which intersects equidistant rays at an angle of 45 degrees, produces logarithms that are of Napier's kind.

DEM. Suppose AB , the difference between CA and CB , the first and second terms of the geometric progression, to be indefinitely small, and take Ap , the logarithm of CB , equal to AB ; then may the figure $ABHp$ be taken as a square, whose diagonal AH would be part of the spiral $AHIK$, and the angle BAH would be half a right one, or 45 degrees.

Therefore that spiral which cuts its rays $CA, CH, \xi c$. at angles of 45 degrees, has a kind of logarithms belonging to it, so related to their corresponding numbers, that the smallest variation between the first and second numbers is equal to the logarithm of the second number.

But of this kind were the first logarithms made by Lord Napier.

Therefore the logarithms to the spiral which cuts its equidistant rays at an angle of 45 degrees, are of the *Nepierian* kind.

END OF BOOK II.

THE



THE
ELEMENTS
OF
NAVIGATION.

BOOK III.
OF PLANE TRIGONOMETRY.

SECTION I.

Definitions and Principles.

1. PLANE TRIGONOMETRY is an art which shews how to find the measures of the sides and angles of plane Triangles, some of them being already known.

It will be proper for the learner, before he reads the following Articles, to turn to the definitions relative to a circle and angle, contained in the Articles 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, and 36, of Book II.

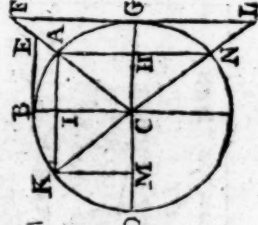
2. A Triangle consists of six parts; namely, three sides and three angles.

The sides of plane triangles are denoted or estimated by measures of length; such as Feet, Yards, Fathoms, Furlongs, Miles, Leagues, &c.

The angles of triangles are estimated by circular measures, that is, by arcs containing Degrées, Minutes, Seconds, &c. (II. 15); and for convenience these circular measures are represented by right lines, called right sines, tangents, secants, and versed sines.

3. The RIGHT SINE of an arc, is a right line drawn from one end of the arc perpendicular to a radius drawn to the other end : Or it is half the chord of the double of that arc.

Thus AH is the right sine of the arc AG ; and also of the arc DBA .



4. The TANGENT of an arc, is a right line touching one end of the arc, and continued till it meets a right line drawn from the center through the other end of that arc.

Thus GF is the tangent of the arc GA .

5. The SECANT of an arc, is a right line drawn through the center and one end of the arc, and produced till it meets the tangent drawn from the other end.

Thus CF is the secant of the arc AG .

6. The **VERSED SINE** of an arc, is that part of the radius intercepted between the arc and its right sine.

Thus HG is the versed sine of the arc AG.

7. The COMPLEMENT of an arc, is what that arc wants of 90 degrees.

Thus if the arc $GB = 90^\circ$. Then AB is called the complement of AG ; and AG is the complement of AB .

8. The SUPPLEMENT of an arc, is what that arc wants of 180 degrees.

Thus the arc ABD is the supplement of AG ; and AG of ABD .

9. The Co-sine of an arc, is the right line of the complement of that arc.

The Co-Tangent of an arc, is the tangent of that arc's complement.

The Co-SECANT of an arc, is the secant of its complement.

The Co-versed SINE of an arc, is the verfed sine of its complement.

Thus AI , BE , CE , BI , being respectively the fine, tangent, secant and verfed line of the arc AB , which is the complement of AG ; therefore AI is called the co-fine, BE the co-tangent, CE the co-secant, BI the co-verfed line, of the arc AG .

The right lines, called *lines*, *tangents*, *secants*, and *versed-lines*, are used as well for the measures of angles, as for the arcs which measure these angles: And it is as common to say the *line*, *tangent*, *sec.* of an angle, as the *line*, *tangent*, *sec.* of an arc.

10. The greatest right fine, is the fine of 90° ; and the fines to arcs less than 90° , serve equally for arcs as much greater than 90° .

Thus the fines of 80° and 100° ; of 60° and 120° ; of 40° and 140° , &c. are respectively equal.

11. The same tangent and secant will serve to arcs equally distant from 90 degrees; that is, to any arc and its supplement.

Thus if the arc $BAG = 90^\circ$, and $BK = BA$; then the arcs GN , GA , DK , are equal; and the arcs GAK and GN , or DK , are supplements to one another: Then the fine KM , the tangent GL , the secant CL , of the arc GBK , are respectively equal to the fine AH , the tangent GF , the secant CA of the arc GA .

12. When an arc is greater than 90° ; the fine, tangent, secant, of the supplement is to be used.

13. The chord of an arc, is equal to twice the cosine of half the supplemental arc.

Thus AN , the chord of the arc AGN , $= 2CI$, the cosine of the arc AB , and AB is half of the arc ABK , the supplement of AGN .

14. The versed fine and co-fine together ($HG + CH$ of any arc AG), is equal to the radius. CH being equal to AI .

15. The fines, tangents, secants, or versed fines of similar arcs in different circles, are in the same proportion to one another, as the radii of those circles. (II. 185)

16. The angles of two triangles may be respectively equal, although their sides may be unequal.

Therefore in a triangle among the things given, in order to find the rest, one of them must be a side.

In Trigonometry, the three things given in a triangle must be either, 1st. Two sides and an angle opposite to one of them.

2d. Two angles and a side opposite to one of them.

3d. Two sides and the included angle.

4th. The three sides.

In either case, the other three things may be found by the help of a few Theorems, and a *Triangular Canon*, which is a table wherein are orderly inserted every degree and minute in a quadrant or arc of 90 degrees; and against them, the measures of the lengths of their corresponding fines, tangents, and secants, estimated in parts of the Radius, which is usually supposed to be divided into a number of equal parts, as 10 , 100 , 1000 , 10000 , 100000 , &c.

SECTION II.

Of the Triangular Canon.

PROPOSITION I.

17. *To find the lengths of the Chords, Sines, Tangents, and Secants to arcs of a circle of a given radius.*

CONSTRUCTION. Through each end of the given radius CD , and at right angles to it (II. 60) draw the lines CF , DG : On c with the radius CD , describe the quadrant arc DA , and draw the chord DA .

18. FOR THE CHORDS. Trisect the arc AD (II. 61), and (by trials) trisect each part; then the arc AD will be divided into 9 equal parts of 10 degrees each; if these arcs are divided each into 10 equal parts, the quadrant will be divided into 90 degrees: But, in this small figure, the divisions to every 10 degrees only are retained, as in (II. 83).

From D as a center, with the radius to each division, cut the right line DA ; and it will contain the chords of the several arcs into which the quadrant arc AD was divided.

For the distances from D to the several divisions of the right line DA , are hereby made respectively equal to the distances of chords of the several arcs reckoned from D .

19. FOR THE SINES. Through each of the divisions of the arc AD , draw right lines parallel to the radius AC ; these parallel lines will be the right sines of their respective arcs, and CD will be divided into a line of sines, which are to be numbered from c to D , for the right sines; and from D to c , for the versed sines.

For the distances from c to the several divisions of the right line CD , are respectively equal to the sines of the several arcs beginning from A .

20. FOR THE TANGENTS. A ruler on c , and the several divisions of the arc AD , will intersect the line DG ; and the distances from D to the several divisions of DG , will be the lengths of the several tangents.

21. FOR THE SECANTS. From the center C , with radii to the divisions of the tangents DG , cut the line CF ; and the distances from c to the several divisions of CF , will be the lengths of the secants to the several arcs.

For these lengths are made respectively equal to the secants reckoned from c to the several divisions of the tangent DG .

22. If



22. If the figure was so large, that the quadrantal arc could contain every degree and minute of the quadrant, or 5400 equal parts; then the chord, sine, tangent and secant to each of them could be drawn. Now a scale of equal parts being constructed (II. 81), 1000 of which parts are equal to the radius CD ; then the lengths of the several sines, tangents and secants may be measured from that scale, and entered in a table called the triangular canon, or the table of sines, tangents and secants.

But as these measures cannot be taken with sufficient accuracy to serve for the computations to which such tables are applicable; therefore the several lengths have been calculated for a radius divided into a much greater number of equal parts; as is shewn in the following articles.

23. PROP. II.

In any circle the chord of 60 degrees, is equal to the radius: and the sine of 30 degrees, is equal to half the radius.

DEM. Let the arc CB , or $\angle CAB = 60$ degrees; and draw the chord CB .

Now since the radii AC and AB are equal; (II. 9)

Therefore $\angle C = \angle B$. (II. 104)

And the $\angle C + \angle B = (180^\circ - (\angle A =) 60^\circ =) 120^\circ$. (II. 96)

Therefore $\angle C$, or $\angle B = (\text{half } 120^\circ; \text{ or } =) 60^\circ = \angle A$.

Consequently $CB = AB = AC$.

From A , draw the radius AE perpendicular to CB .

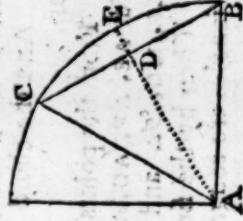
Then AE bisects the arc CB , and its chord.

And $CD = \text{fine of (the arc } CE = \text{half of } 60^\circ =) 30^\circ$.

Consequently CD is equal to half the radius AB .

24. Hence, *Twice the co-sine of 60 degrees is equal to the radius.*

For 30° is the complement of 60° , and twice the sine of 30° is equal to the radius.



(II. 124)
(3)

25. PROP III.

To find the sine of one minute of a degree.

It is evident (II. 187), that the less the arc is, the less is the difference between the arc and its sine, or half chord; so that a very small arc, such as that of one minute, may be reckoned to differ from its sine, by so small a quantity, that they may be esteemed as equal; and consequently may be expressed by the same number of such equal parts of which the radius is supposed to contain 1,00000 &c. which is readily found by the following proportion.

As the circumference of the circle in minutes 21600

To the circumf. in equal parts of the radius (II. 196) 6,283185

So is the arc of one minute 1,

To the corresponding parts of the radius

So that for the sine of one minute, may be taken 0,0002908882

26. PROP.

26.

PROP. IV.

In a series of arcs in arithmetic progression, the sine of any one of them, taken as a mean, and the sum of the sines of any other two, taken as equidistant extremes, are ever in a constant ratio, of radius to twice the co-sine of the common difference of those arcs.

DEM. For in a circumference, whose center is C, and diameter AB, let there be taken a series of arcs, ARB, ARD, ARE, ARF, ARG, ARH, &c. whose common difference is the arc BD.

Then drawing the chords AB, AD, AE, AF, AG, AH, &c. their halves will be the sines of half the arcs ARB, ARD, &c. (3)

Also half the arc BD, is the common difference of half the arcs ARB, ARD, ARE, &c.

And the chord AD is twice the co-sine of half the supplemental arc BD. (11. 150)

From the points D, E, F, G, &c. with the radii DA, EA, FA, GA, &c. cut AE, AF, AG, AH, &c. produced, in I, K, L, M, &c. draw ID, KE, LF, MG, &c. and BD, DE, EF, FG, GH, &c.

Then by the first part of the demonstration (II. 189), the following triangles are congruous, namely,

ABD, IED; ADE, KFE; AEF, LGF; AFG, MHG, &c.
Therefore IE=AB; KF=AD; LG=AE; MH=AF, &c.

Also the triangles IDA, KEA, LFA, MGA, &c. being each of them Isosceles, and their angles respectively equal, are similar to DCA. (II. 167)

Therefore CA : AD :: (AI=) AB + AE :: $\frac{1}{2}$ AD : $\frac{1}{2}$ AB + $\frac{1}{2}$ AE.
:: (AE : (AK=) AD + AF ::) $\frac{1}{2}$ AE : $\frac{1}{2}$ AD + $\frac{1}{2}$ AF.
:: (AF : (AL=) AE + AG ::) $\frac{1}{2}$ AF : $\frac{1}{2}$ AE + $\frac{1}{2}$ AG.
&c.

The halves being in the same ratio as the wholes.

(II. 150)

27. Consequently, in a series of arcs in arithmetic progression, viz. $\frac{1}{2}$ ARB, $\frac{1}{2}$ ARD, $\frac{1}{2}$ ARE, &c. whose common difference is half the arc BD, it will be,

As (AC) Radius,

To (AD) twice the co-sine of the common difference;

So is the sine of either arc taken as a mean,

To the sum of the sines of two equidistant extremes.

28. Hence; *The sine of either extreme, subtracted from the product of the sine of the mean by twice the co-sine of the common difference, will give the sine of the other extreme.* (II. 164)

29. When

38. The principles by which the lengths of the sines, tangents, secants, &c. may be constructed, being delivered, the following examples are annexed to illustrate this doctrine.

Required the co-sine of one minute.

The sine of 1 minute being

0,0002908882 (25)

Its square is

0,0000008461594

Which subtracted from the square of radius

1,

Leaves

0,99999991538406

Whole square root

0,9999999577 is the

co-sine of 1 minute; or the sine of $89^{\circ} 59'$.

Now having the sine and cosine of 1 minute, the other sines may be found in the following manner. (28)

twice the cos. 1 min. $\times$ sine of 1 m. = sum of the sines of $0'$ & $2'$.

twice the cos. 1 min. $\times$ sine of 2 m. = sum of the sines of $1'$ & $3'$.

twice the cos. 1 min. $\times$ sine of 3 m. = sum of the sines of $2'$ & $4'$.

twice the cos. 1 min. $\times$ sine of 4 m. = sum of the sines of $3'$ & $5'$.

twice the cos. 1 min. $\times$ sine of 5 m. = sum of the sines of $4'$ & $6'$.

Proceeding thus in a progressive order from each sine to its next, all the sines may be found.

But as twice the co-sine of 1 minute, *viz.* 1,9999999154 is concerned in each operation, therefore if a table be made of the products of this number by the nine digits, as here annexed, the computations of the sines may be performed by addition only.

For the products by the digits in the given multiplier, being taken from the table, and wrote in their proper order, will prevent the trouble of multiplication.

And even this operation may be very much shortened, by setting under the right hand place (*viz.* 4) of the double co-sine of one minute, the unit place of the sine used as a multiplier, and reversing, or placing in a contrary order, all its other figures; then the right hand figure of each line arising by the multiplication, is to be set under one another; and in these lines, the first figure to be set down, is what arises from the figure standing over the present multiplying one; observing to add what would be carried from the places omitted.

Now if the products of the figures in the multiplier, thus inverted, be taken from the above table of products; it is necessary to remark what number of places will arise from each digit used in the multiplier; then in the products of those digits in the table, take only the like number of places, observing to add 1 to the right hand place, if the next of the omitted figures exceed 5.

Multipliers	Products.
1	1,9999999154
2	3,9999998308
3	5,9999997462
4	7,9999996616
5	9,9999995770
6	11,9999994924
7	13,9999994078
8	15,9999993232
9	17,9999992386

Required

Required the sine of two minutes.

The sine of 1 min. placed in an inverted order under the double cof. of 1 min. as in the margin; the right hand figure 2 stands under the 9 in the 6th decimal place, therefore the first 6 decimal places of the product against 2 in the table, are to be used; but 1 being added, because the 7th place 8, exceeds 5; makes the product 400000: Also, for 9 the next figure in the multiplier, standing under the 5th decimal place, take 17,99999 from the table of products, and 1 being added to the 5th place, because the 6th exceeds 5, make it 18,00000:

In like manner, the product by 8, adding 1, is 16000 &c. and the sum of these products 0,0005817764 is the sine of 2 minutes, as required.

This kind of operation will be very easily conceived without farther illustration, by comparing the process in this and the following operations, with what has been already said.

1,9999999154	
288892000,0	
4000000	
1800000	
16000	
1600	
160	
4	
0,0005817764	

Required the sines of 3', 4', 5', and 6 minutes.

For 3 min.	For 4 min.	For 5 min.	For 6 min.
1,9999999154	1,9999999154	1,9999999154	1,9999999154
4677185000,0	6255361100,0	5044454100,0	
9999999	16000000	19999999	19999999
1600000	1400000	2000000	8000000
20000	40000	1200000	1000000
14000	12000	60000	80000
1400	1200	10000	8000
120	80	1000	800
8	10	40	10
0,0011635527	0,0017453290	0,0023271051	0,0029088809
0,0002008882	0,0005817764	0,0008726645	0,0011635526
0,0008726645	0,0011635526	0,0014544406	0,0017453283

In each example, the sine of 2 minutes less, (28) is subtracted.

The sines being made, the tangents, secants, &c. are to be constructed as before shewn. (33, 34)

39. There are many methods by which the triangular canon may be made; but that which is here delivered was chosen as the most easy, the best adapted to this work, and what would give the learner a sufficient notion how these numbers are to be found: For at this time, there is no occasion to construct new tables of sines, and rarely to examine those already extant; they having passed through the hands of a great many careful examiners, and for a long time have been received by the learned as a work sufficiently correct.

These lines were first introduced into mathematical computations by *Hipparchus* and *Menelaus*, whose methods of performance were contracted by *Ptolomy*, and afterwards perfected by *Regiomontanus*; and since his time *Rheticus*, *Cleavius*, *Petiscus*, and many other eminent men, have treated largely on this subject, and greatly exemplified the use of this triangular Canon, or Tables; which are now, by way of distinction, called Tables of *natural* fines, tangents, &c. But the greatest improvement ever made in this kind of mathematical learning, was by the Lord *Nepier*; Baron of *Merchiston* in *Scotland*; who being very fond of such studies, wherein calculations by the fines, tangents, &c. did frequently occur, judged it would be of vast advantage could these long multiplications and divisions be avoided; and this he effected by his happy invention of computing, by certain numbers considered as the indices of others (I. 73), which he called logarithms; this was about the year 1614.

The tables now chiefly used in Trigonometrical computations, are the logarithms of those numbers which express the lengths of the fines, tangents, &c. and therefore to distinguish them from the *natural* ones, they are called *Logarithmic* fines, tangents, &c. (or by some artificial fines, &c.) Only those of the logarithmic fines and tangents are annexed to this treatise, because the business of Navigation may be performed by them; neither are these tables carried to more than five places beside the index, that being sufficiently exact for all nautical purposes: But it must be allowed that, for general use, such tables are the most esteemed, as consist of most places.

40. These tables are at the end of Book IX. and are so disposed, that each opening of the book contains eight degrees; four of which are numbered at top, and four at the bottom of the page; and those at top proceed from left to right, or forwards, from 0 degrees to 45; and those at bottom, from right to left, or backwards, from 45 to 90 degrees: To each degree there are four columns, titled fines, co-fines, tangents, co-tangents; and the minutes are in the marginal column of each page, signed with M; those on the left side belong to the degrees at top, and those on the right hand side of the page, to the degrees at bottom.

41. Now a sine, tangent, co-sine, co-tangent, to a given number of degrees, is found as follows:

For an arc less than 45 degrees,

Seek the degree at top, and the minutes in the column signed M at top; against which, in the column signed at top with the proposed name, stands the sine, or tangent, &c. required.

But when the arc is greater than 45 degrees,

Seek the degree at bottom, the minute in the column with M at bottom, and the proposed name at bottom.

EXAMPLE

EXAMPLE I. *Required the logarithmic sine of $28^{\circ} 37'$.*

Find 28° deg. at the top of the page; and in the side column, marked with M at top, find 37; against which, in the column signed at top with the word sine, stands 9,68029, the log. sine of $28^{\circ} 37'$, as required.

EXAMPLE II. *Required the logarithmic tangent of $67^{\circ} 45'$.*

Find 67° deg. at the bottom of the page; and in the side column, titled M at the bottom, find 45; then against this, in the column marked tangent at bottom, stands 10,38816, which is the log. tangent required.

42. But when a logarithm sine or tangent is proposed, to find the degrees and minutes belonging to it, then,

Seek in the table, among the proper columns, for the nearest logarithm to the given one; and the corresponding degrees and minutes will be found; observing to reckon them from the top or bottom, according as the column is titled, wherein the nearest logarithm to the given one is found.

43. It may sometimes happen that a log. sine or log. tang. may be wanted to degrees, minutes, and parts of minutes; which may be thus found.

Take the difference between the logs. of the degrees and minutes next less, and those next greater, than the given number.

Then for $\frac{1}{2}$, take a quarter of this difference; for $\frac{1}{3}$, take a third; for $\frac{1}{4}$, take a half; for $\frac{2}{3}$, take two thirds; for $\frac{3}{4}$, take three quarters, &c.

Add the parts taken of this difference to the right hand figures of the log. belonging to the deg. and min. next less, and the sum will be the log. to the deg. min. and parts proposed.

EXAMPLE I. *Required the log. tang. to $60^{\circ} 56\frac{1}{2}'$.*

Log. tan. $60^{\circ} 57'$ is
Log. tan. $60^{\circ} 56'$ is

The diff. is

Its half is

Add it to tan. $60^{\circ} 56'$

Gives tan. $60^{\circ} 56\frac{1}{2}'$

EXAMPLE II. *Required the log. sine to $32^{\circ} 15\frac{1}{4}'$.*

Log sine $32^{\circ} 16'$ is
Log. sine $32^{\circ} 15'$ is

The diff. is

Its three fourths is

Add it to sine $32^{\circ} 15'$

Gives sine $32^{\circ} 15\frac{1}{4}'$

In most cases the work may be done by inspection.

44. And if a given log. sine or log. tangent falls between those in the tables; then the degrees and minutes answering may be reckoned $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{4}$, &c. minutes more than those belonging to the nearest less log. in the tables, according as its difference from the given one is $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{4}$, &c. of the difference between the logarithm next greater and next less than the given log.

SECTION III.

Of the Solution of Plane Triangles.

45.

PROBLEM I.

In any plane triangle (ABC), if among the things given there be a side and its opposite angle, to find the rest.

Then say, *As a given side,*

To the sine of its opposite angle;

So is another given side,

To the sine of its opposite angle.

Therefore, to find an angle, begin with a side opposite to a known angle.

Also, *As the sine of a given angle,*

To its opposite side;

So is the sine of another given angle,

To its opposite side.

Therefore, to find a side, begin with an angle opposite to a known side.

DEM. Take $BD = CF =$ radius of the tables.

Draw DE, AH, FG , each perpendicular to BC . (II. 59.)

Then DE and FG are the sines of the angles B and C .

(3)

Now the triangles BDE, BAH , are similar, and so are the triangles CFG, CAH .

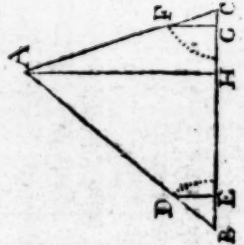
Therefore $BD : DE :: BA : AH$. (II. 167)

And $(CF =) BD : FG :: CA : AH$. (II. 167)

But $(BD \times AH =) DE \times BA = FG \times CA$.

Therefore $DE : CA :: FG : BA$. Or, $s, \angle B : AC :: s, \angle C : BA$. (II. 162)

SCHOL. Or, by circumscribing the triangle with a circle, it will readily appear, that the half of each side is the sine of its opposite angle. And halves have the same proportion as the wholes.



(H. 162)

(II. 163)

46.

PROBLEM II.

In a right angled plane triangle (ABC), if the two sides containing the right angle (B) are known, to find the rest.

Then, *As one of the known sides,*

To the radius of the tables (or tangent of 45°);

So is the other known side,

To the tangent of its opposite angle.

(AB)

(AD)

(BC)

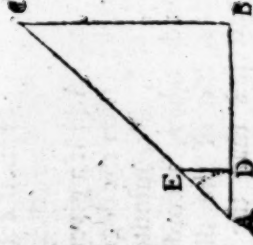
(DE)

DEM. Take $AD =$ radius of the tables.

Then DE perpendicular to AD , is the tangent of the angle A . (4)

And the triangles ADE, ABC , are similar.

Therefore $AB : AD :: BC : DE$, (II. 167)



47. PRO.

47. PROBLEM III.

In any two quantities, their half difference added to their half sum, gives the greater,

The half diff. subtracted from the half sum, gives the less.

And the half sum taken from the greater, the remainder will be the half difference of those quantities.

DEM. Let AB be the greater, and BC the less, of two quantities. Take $AD = BC$; then BD is their difference.

Bisect DB in E; then $DE = EB$, is the half diff.

And $AD + DE = BC + BE$ (II. 47); therefore AE is the half sum.

Now $AE + EB = AB$, is the greater.

And $AE - ED = (AD =) BC$, is the less.

Also $AB - AE = BE$, is the half diff.



48. PROBLEM IV.

In any plane triangle (ABC); if the three things known, be two sides (AC, CB) and their contained angle (C), to find the rest.

Find the sum and difference of the given sides.

Take half the given angle from 90 degrees, and there remains half the sum of the unknown angles. Then say,

As the sum of the given sides;

To the difference of those sides;

So is the tangent of half the sum of the unknown angles,

To the tangent of half the difference of those angles.

Add the half difference of the angles to half the sum, and it will give the greater angle = B.

Subtract the half difference of the angles from the half sum, and it will give the lesser angle = A.

$$\begin{array}{l} AC + CB \\ AC - CB \\ t. \frac{1}{2} \frac{B+A}{B-A} \\ t. \frac{1}{2} \frac{B-A}{B+A} \end{array}$$

DEM. On C, with the radius CB, describe a circle, cutting AC produced, in E and D; draw EB, and BD; also draw DF parallel to EB.

Then $AE = AC + CB$, is the sum of the sides.

And $AD = AC - CB$, is the difference of the sides.

Now $\angle CDB = \angle CBD$.

And $(\angle CDB + \angle CBD =) 2\angle CDB = \angle CBA + \angle A$. (II. 104)

Therefore $\frac{1}{2} \angle CBA + \angle A = \angle CDB$, is half the sum of the unknown angles.

And BE (II. 123) is the tangent of CDB, to the radius DB.

Also $(\angle CBA - \angle CBD =) \angle DBA = \frac{1}{2} \angle CBA - \frac{1}{2} \angle A$.

Therefore $\frac{1}{2} \angle CBA - \angle A = \angle DBA$, is half the difference of the unknown angles.

And DF is the tangent of DBA, to the radius DB.

Now the triangles AEB, ADF, are similar, DF being parallel to EB.

Therefore $AE : AD :: BE : DF$. (II. 167)

Or $AC + CB : AC - CE :: t. \frac{1}{2} \angle CBA + \angle A : t. \frac{1}{2} \angle CBA - \angle A$.



49.

PROBLEM V.

In a plane triangle (ABC), if the three sides are known, and the angles required.

From the greatest angle (B) Suppose a line (BD) drawn perpendicular to its opposite side, or base, dividing it into two segments (AD, DC), and the given triangle into two right angled triangles (ADB, CDB): Then say,

As the base, or sum of the segments,
Is to the sum of the other two sides;
So is the difference of those sides,
To the difference of the segments of the base.

$$\begin{array}{l} AC \\ AB + BC \\ AB - BC \\ AD - DC \end{array}$$

Add half the difference of the segments to half the base, gives the greater segment (AD). (47)

Subtract half the difference of the segments from half the base, gives the lesser segment (DC). (47)

Then, in each of the triangles (ADB, CDB), there will be known two sides, and a right angle opposite to one of them; therefore the angles will be found by Problem I. (45)

When two of the given sides are equal; then a line drawn from the included angle, perpendicular to the other side, bisects the side. (II. 103)

And the angles being found in one of the right angled triangles, will also give the angles of the other.

DEM. of the foregoing proportion.

In the triangle ABC, the line BD, perpendicular to AC, divides AC into the segments AD, DC.

On B with the radius BC, describe a circle GCE. cutting AB, continued, in G, E; and AC in F; draw BF.



Then $DC = DF$.

Now $AC (=AD + DC)$ is the sum of the segments.

And $AF (=AD - FD)$ is the difference of the segments.

Also $AE (=AB + BC)$ is the sum of the other sides.

And $AG (=AB - BC)$ is the difference of those sides.

But $AC \times AF = AE \times AG$.

Therefore $AC : AE :: AG : AF$.

Of $AC : AB + BC :: AB - BC : AD - DC$; or $AD - DF$.

(II. 103)

(II. 172)

(II. 163)

50. PROBLEM VI.

In any plane triangle (ABC), the three sides being known, to find either of the angles.

Put E and F for the sides including the angle sought.
G for the side opposite to that angle.

D for the difference between the sides E and F.

Find half the sum of G and D.

And half the difference of G and D.

Then write these four logarithms under one another, namely,

The Arithmetical complement of the logarithm of E;

The Arithmetical complement of the logarithm of F;

The logarithm of the aforesaid half sum of G and D;

The logarithm of the aforesaid half difference of G and D.

Add them together, take half their sum; which sek among the log. sines.

And the degrees and minutes answering, being doubled, will give the measure of the angle sought.

DEM. In the triangle ABC, let A be $\angle B$ the angle sought.

Take AH = AB, draw BH; and through K, the middle of BH, draw AP, which bisects the angle A, and is perpendicular to BH. (II. 103)

Through K draw KL, KQ, parallel to BC, AC; which will bisect HC, BC, in L and I; then KL = IC, KI = LC.

And the difference between AC and AB is HC = D; then KI = $\frac{1}{2}$ D.

From I with the radius IK describe a circle cutting AP, BH, KQ, BC, in P, O, Q, M, N, and join CQ; now IQ = IK = LC = LH; therefore KQ = HC, and CQ = KH, as the triangles CQI, KHL, are congruous. (II. 99)

Therefore CQ parallel to KH (II. 28) being produced, will meet AP at right angles (II. 53), in the point P, by the reverse of (II. 130).

Then PQ = KO, as the triangles KQP, QOK, are congruous. (II. 95. 100)

Now BM = (BF + IM = $\frac{1}{2}$ BC + $\frac{1}{2}$ HC = $\frac{1}{2}$ G + D: And BN = $\frac{1}{2}$ G - D.

Also BO = CP: For BK = (KH =) CQ, and KO = PQ.

Let Ar = radius of the tables; then $r \sin \frac{1}{2} \angle A$ (parallel to BH) = sine of $\frac{1}{2} \angle A$. (3)

Then the triangles ANP, AKB, APC, are similar.

And Ar : $r \sin \frac{1}{2} \angle A$:: AB : BK; also Ar : $r \sin \frac{1}{2} \angle A$:: AC : (CP =) BO. (II. 167)

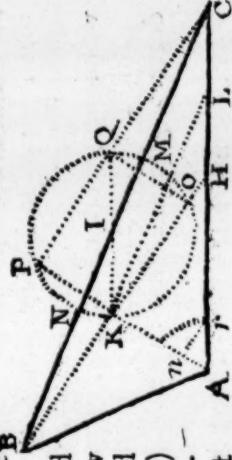
Therefore $\overline{Ar}^2 : \overline{r \sin \frac{1}{2} \angle A}^2 :: AC \times AB : (BK \times BO =) BM \times BN$. (II. 161, 172)

Or (sq. rad. =) $R^2 : \text{sq. sine } \frac{1}{2} \angle A :: AC \times AB : \frac{1}{2} G + D \times \frac{1}{2} G - D$.

Therefore the square of the sine $\frac{1}{2} \angle A = \frac{\frac{1}{2} G + D \times \frac{1}{2} G - D}{AC \times AB} \times R^2$. (II. 164)

Therefore sine $\frac{1}{2} \angle A = \sqrt{\frac{\frac{1}{2} G + D \times \frac{1}{2} G - D}{E \times F}}$; (II. 113)

And R, the radius of the tables, being 1.



$$\text{Or Log. } s, \frac{1}{2} \angle A = \frac{\text{Log. } \frac{1}{2} G + D + \text{Log. } \frac{1}{2} G - D - \text{Log. } F - \text{Log. } F}{2} \left. \begin{array}{l} 85 \\ 86 \\ 91 \end{array} \right\} \text{I.}$$

$$\text{Or Log. } s, \frac{1}{2} \angle A = \frac{L. E + L. F + L. \frac{1}{2} G + D + L. \frac{1}{2} G - D}{2}$$

Where L. stands for logarithm, and L. for Arith. comp. of the logarithm.

51. Every possible case in plane Trigonometry may be readily solved by the preceding Problems, observing the following Precepts.

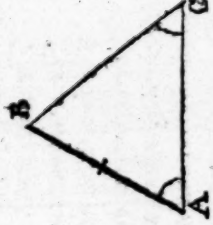
I. Make a rough draught of the triangle, and put the letters A, B, C, at the angles.

II. Let such parts of this triangle be marked, as represent the things which are given in the question. Thus, mark a given side with a scratch across it; and a given angle by a little crooked line; as in the figure; where the side AB, and the angles A and C, are marked as given.

III. If two angles are known, the third is always known.

For if one angle is 90 degrees, the other given angle (which (II. 97) will be acute) taken from 90 degrees, leaves the third angle.

And if both the given angles are oblique; their sum taken from 180 degrees, gives the other angle.



IV. Compare the given things together, and thereby determine to which Problem the question proposed belongs.

V. Then according as the Problem directs, perform the preparatory work; and write down, under one another, in four lines, (or more if necessary), the *literal stating*; expressing each angle by a letter, or by three; each line by two letters; and the sums, or differences, of lines, by proper marks.

VI. Against such terms that are known, write their numeral value as given in the question, or as found in the preparatory work; and against these numbers write their logarithms; those for the lines being found (by I. 81) in the table of the logarithms of numbers; and those for the angles, found (by 41) in the table of logarithm fines and tangents: Observing that an Arithmetical complement (see I. 88) is always used in the first term: And that when an angle is greater than 90 degrees, its supplement is used.

VII. Add these logarithms together, and seek the sum (I. 81) in the log. numbers, when a line is wanted; or (42) in the log. fines or tangents, when an angle is wanted. Then the number or degree, answering to a logarithm the nearest to the said sum, will be the thing required.

A SYNOPSIS

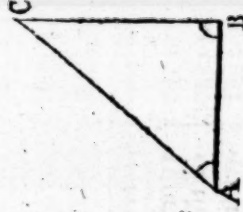
Of the Rules in Plane Trigonometry.

Given	Required	SOLUTION.
All the angles and one side	either of the other sides	Since two angles are known, the third is known. And, As sin. of $\angle$ opp. to side given, is to that opp. side; So sin. of another angle, to its opp. side.
Two sides and an $\angle$ oppof. to one side	The angle oppof. to the other given side	As one given side, is to the fine of its opp. angle; So is the other given side, to the fine of its opp. angle. Then two angles being known, the third is known. And the other side is found as before.
Two sides and the included right $\angle$	either of the other angles	As one of the given sides, is to the Radius; So is the other given side, to the tangent of its opp. $\angle$ Then two $\angle$ s being known, the third is known. The other side is found by opp. sides and $\angle$ s.
Two sides and the included oblique angle	The other angles	Find the sum and diff. of the given sides. Take $\frac{1}{2}$ given $\angle$ from 90° leaves $\frac{1}{2}$ sum of the other $\angle$ s. Then, As sum sides, is to different sides; So tan. $\frac{1}{2}$ sum other $\angle$ s, to tan. $\frac{1}{2}$ diff. those $\angle$ s. The $\frac{1}{2}$ sum $\angle$ s $\left\{ \begin{array}{l} + \\ - \end{array} \right\} \frac{1}{2}$ diff. $\angle$ s, gives $\left\{ \begin{array}{l} \text{greater} \\ \text{lesser} \end{array} \right\} \angle$. Find the other side by opp. sides and angles.
The three sides	All the angles	Draw a line perpend. to the greatest side, from the opp. $\angle$, dividing that side into two parts. Then, As the longest side, is to sum other two sides; So is the diff. those sides, to the diff. parts of longest. Then $\frac{1}{2}$ long. side $\left\{ \begin{array}{l} + \\ - \end{array} \right\} \frac{1}{2}$ diff. pts. gives the $\left\{ \begin{array}{l} \text{great} \\ \text{lesser} \end{array} \right\}$ part. Now the said perp. cuts the triangle into 2 right $\angle$ d ones. In both, are known the Hyp. a Leg. and the right $\angle$. The angles are found by Problem I.
The three sides	Either Angle	Having chose which angle to find; call the sides including that angle E and F. The side opp. that $\angle$, call G. Put D for the difference between E and F. Find the half sum, and half diff. of G and D. Then write these four Logs. under one another; viz. $\left\{ \begin{array}{l} \text{The Ar. Co. Log. of E, The Ar. Co. Log. of F,} \\ \text{The Log. of } \frac{1}{2} \text{ sum, And the Log. of } \frac{1}{2} \text{ difference.} \end{array} \right.$ Add the four logs. together, take half their sum. Seek it among the log. fines; and the corresponding deg. and min. doubled, is the angle sought.

53.

EXAMPLE I. *In the plane Triangle ABC,**Given* $AB = 195$ Poles. $\angle B = 90^\circ 00'$ $\angle A = 47^\circ 55'$ *Required the other parts.*

FOR THE LINEAR SOLUTION.

1st. Draw AB equal to 195 poles, taken from a scale of equal parts.2d. From B , draw BC making with AB an angle of 90° . (Il. 84)3d. From A , draw AC , making with AB an angle of $47^\circ 55'$; and meeting BC in the point C .Then is the triangle ABC such, whose parts correspond with the things given; and the sides CA , CB , being applied to the scale that AB was taken from, their measures will be found, *wiz.* $AC = 291$; and $BC = 216$.

FOR THE NUMERAL SOLUTION, OR COMPUTATION.

Since two angles are known; Therefore,

From $90^\circ 00' = \angle B$ Take $47^\circ 55' = \angle A$ Remains $42^\circ 05' = \angle C$

Now in this triangle, there are known all the angles and one side; therefore among the known things, there is a side and its opposite angle; which belongs to the first problem.

Then to find the side AC , begin with the angle C opposite AB .*As* the sine of the $\angle C$,*To* the opposite side AB ;*So* the sine of the $\angle B$,*To* the opposite side AC .Or thus, *As* $s, \angle C = 42^\circ 05'$ 0,17379 A. C.*To* $AB = 195$ po. 2,29003*So* $s, \angle B = 90^\circ 00'$ 10,00000*To* $AC = 291$ po. 2,46382And to find the side BC , begin with the angle C opposite AB .*As* the sine of the $\angle C$,*To* the opposite side AB ;*So* the sine of the $\angle A$,*To* the opposite side BC .Or thus, *As* $s, \angle C = 42^\circ 05'$ 0,17379 A. C.*To* $AB = 195$ po. 2,29003*So* $s, \angle A = 47^\circ 55'$ 9,87050*To* $BC = 216$ po. 2,33432So that AC is 291 poles, and BC is 216 poles.The letters $A. C.$ standing on the right of the first line, signify the arithmetical complement of the log. sine of $42^\circ 05'$. (See I. 88)

54.

EXAMPLE II. *In the plane Triangle ABC,**Given* $AB = 117$ miles. $\angle B = 134^\circ 46'$ $\angle A = 22^\circ 37'$ *Required the other parts.*

FOR THE LINEAR SOLUTION OR CONSTRUCTION.

Make $AB = 117$ equal parts; at A make an angle $= 22^\circ 37'$ (II. 84); and at B make an angle of $134^\circ 46'$; then the lines making with AB those angles will meet in C , and form the triangle proposed. And the measure of BC will be 117, and of AC 216.

By COMPUTATION. See art. 45.

Since two angles are known,

Namely, $\angle B = 134^\circ 46'$ Now from $180^\circ 00'$ And from $180^\circ 00'$
 $\angle A = 22^\circ 37'$ Take $157^\circ 23'$ Take $134^\circ 46'$

Their sum $= 157^\circ 23'$ Leaves $\angle C = 22^\circ 37'$ The sup. $\angle B = 45^\circ 14'$

Since the angle $C = \angle A$,
 Therefore $BC = AB$.

(II. 104)

To find the side AC.
 As $s, \angle C = 22^\circ 37'$ $0,41503$ A. C.
 To $AB = 117$ M. $2,06819$
 So $s, \angle B = 134^\circ 46'$ $9,85125$ sup.
 To $AC = 216$ M. $2,33447$

55.

EXAMPLE III. *In the plane Triangle ABC,**Given* $AB = 408$ yards. $\angle B = 22^\circ 37'$ $\angle A = 58^\circ 07'$ *Required the other parts.*

CONSTRUCTION.

Make $AB = 408$ yards, or equal parts; make the angle $A = 58^\circ 07'$; and the $\angle B = 22^\circ 37'$; then the lines forming these angles will meet in C ; and the measure of AC is 159 yards, and of BC is 351.

COMPUTATION. See art. 45.

Two angles being known, viz. $\angle A = 58^\circ 07'$ From $180^\circ 00'$
 $\angle B = 22^\circ 37'$ Take $80^\circ 44'$

Their sum $= 80^\circ 44'$ Leaves $99^\circ 16' = \angle C$.

To find the side AC.

As $s, \angle C = 99^\circ 16'$
 To $AB = 408$ Y. $2,61066$
 So $s, \angle B = 22^\circ 37'$ $9,58497$

To $AC = 159$ Y. $2,20133$ *To find the side BC.*

As $s, \angle C = 99^\circ 16'$ $0,00570$ A. C.
 To $AB = 408$ Y. $2,61066$
 So $s, \angle A = 58^\circ 07'$ $9,92897$

To $BC = 351$ Y. $2,54533$ In these operations the supplement of the angle C is used. (12)

56. Ex.

56.

EXAMPLE IV. In the plane Triangle ABC.

Given $AB=195$ } Furlongs. $AC=291$ $\angle B=90^\circ 00'$.

Required the rest.

CONSTRUCTION.

Make $AB=195$ equal parts; draw BC , making an angle at $B=90^\circ 00'$: From A with 291 equal parts cut BC in C , and draw AC .

Then the $\angle A$ measured on the scale of chords will be about 48 degrees, and $\angle C$ about 42° : Also BC , on the equal parts, measures about 216.

COMPUTATION.

Here being two sides and an angle opposite to one of them, the solution falls under problem the first. See art. 45.

To find the angle C .As $AC=291$ F. 7,53611 A. C.To $s, \angle B=90^\circ 00'$ 10,00000So $AB=195$ F. 2,29003To $s, \angle C=42^\circ 05'$ 9,82614

From 908 00'

Take 42 05= $\angle C$,Leaves 47 55= $\angle A$.

57.

EXAMPLE V. In the plane Triangle ABC.

Given $AC=216$ } Yards, $AB=117$ $\angle C=22^\circ 37'$.

Required the rest.

CONSTRUCTION.

Make $AC=216$ yards; the $\angle C=22^\circ 37'$; and draw CB : Then form A with 117 yards, cut CB in b or in B ; and either of the triangles ACb or ACB will answer the conditions proposed: But the triangle to be used is, generally determined by some circumstances in the question it belongs to. Thus if the angle opposite to AC is to be obtuse, the triangle is ABC .

COMPUTATION.

The solution belongs to problem the first. See art. 45.

To find the angle B .As $AB=117$ Y.To $s, \angle C=22^\circ 37'$ So $AC=216$ Y.To $s, \angle B=134^\circ 46'$ $\angle C + \angle B=157 23$

From 180° 00'

Take 157 23= $\angle C + \angle B$,Leaves 22 37= $\angle A$.

9,85123

And as the $\angle A = \angle C$,Therefore $BC=AB$. (II. 104.)

If the angle required be obtuse, subtract the deg. and min. corresponding to the fourth log. from 180; the remainder is the $\angle B$. For the fourth log. gives the $\angle b$, which is the supplement to the angle B . (II. 104, 96) 58. Ex.



58. EXAMPLE VI. In the plane Triangle ABC.

Given $AC=408$ } Fathoms.
 $AB=159$
 $\angle C=22^\circ 37'$.
 Required the rest.



CONSTRUCTION.

Make $AC=408$ fathoms; the $\angle C=22^\circ 37'$; and draw cb ; from a with 159 fathoms cut cb in b or in B , and draw Ab or AB : Then if the angle opposite to AC is to be acute, the triangle AcB is that which is required; but if the angle is to be obtuse, ACB is the triangle sought.

COMPUTATION. See art. 45.

Here being a side and its opposite angle known, the solution falls under problem the first; the $\angle B$ is to be obtuse.

To find the angle B obtuse.

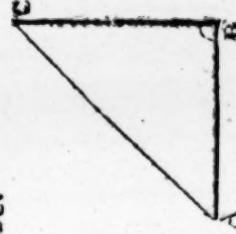
As $AB=159$ F.	7,79860	To find BC .	0,41503
To $s, \angle C=22^\circ 37'$	9,58497	As $s, \angle C=22^\circ 37'$	2,20140
So $AC=408$ F.	2,61066	To $AB=159$ F.	9,92874
To $s, \angle B=99^\circ 19'$	9,99423	So $s, \angle A=58^\circ 04'$	2,54517

$\angle C + \angle B = 121^\circ 56'$ taken
 From 180 00

Leaves $\angle A = 58^\circ 04'$

59. EXAMPLE VII. In the plane Triangle ABC.

Given $AB=195$ } Furlongs.
 $BC=216$
 $\angle B=90^\circ 00'$.
 Required the rest.



CONSTRUCTION.

Make the angle $ABC=90^\circ$; take $BA=195$ equal parts, and $BC=216$; and draw AC ; then ABC is the triangle proposed; wherein the parts required may be measured by the proper scales.

COMPUTATION. See art. 46.

As two sides and the contained right angle are known, the solution belongs to problem the second.

To find the angle A .

As $AB=195$ F.	7,70996	To find AC .	0,12950
To Radius $=45^\circ 00'$	10,00000	As $s, \angle A=47^\circ 55'$	2,33445
So $BC=216$ F.	2,33445	To $BC=216$ F.	10,00000
To $\angle A=47^\circ 55'$	10,04441	So $s, \angle B=90^\circ 00'$	2,46395

90 00

$\angle C=42^\circ 05'$

60 Ex-

60. EXAMPLE VIII. In the plane Triangle ABC.

Given $AB=117$ } Yards.
 $BC=117$ }
 $\angle B=134^\circ 46'$
Required the rest.



CONSTRUCTION.

Make the $\angle ABC=134^\circ 46'$; take BA and BC each equal to 117 equal parts, from the same scale, and draw AC; then is the triangle ABC equal to that proposed; and the parts required measured on their proper scales will give their values.

COMPUTATION. See art. 48.

Now as AB and AC are equal; therefore the angles A and C are also equal.

To find AC.

From $180^\circ 00'$

As $s, \angle A=22^\circ 37'$
 To $BC=117$ Y.

Take $134^\circ 46'=\angle B$.

So $s, \angle B=134^\circ 46'$

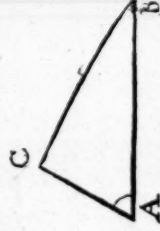
Leaves $45^\circ 14'=\angle A+\angle C$.

To $AC=216$ Y.

The half $22^\circ 37'=\angle A=\angle C$.

61. EXAMPLE IX. In the plane Triangle ABC.

Given $AB=408$ } Yards.
 $AC=159$ }
 $\angle A=58^\circ 07'$
Required the rest.



CONSTRUCTION.

Make an angle $CAB=58^\circ 07'$; take $AC=159$, $AB=408$, from the same scale of equal parts; and draw CB; then will the triangle ACB be equal to that which was proposed.

COMPUTATION.

Here being two sides and their contained angle known, the solution belongs to art. 48.

$AB=408$
 $AC=159$

The half of $58^\circ 07'$
 Is $29^\circ 03\frac{1}{2}'$, which

$AB+AC=567$ =sum of sides,

Taken from $90^\circ 00'$

$AB-AC=249$ =diff. of sides.

Leaves

$60^\circ 56\frac{1}{2}'=\frac{1}{2}\angle C+\frac{1}{2}\angle B$.

To find the angles.

As $AB+AC=567$

To $AB-AC=249$

So $t, \frac{1}{2}\angle C+\angle B=60^\circ 56\frac{1}{2}'$ (See 43)

To $t, \frac{1}{2}\angle C-\angle B=38^\circ 19\frac{1}{2}'$

Then (47)

$99^\circ 16'=\angle C$

And $22^\circ 37'=\angle B$.

To find AC.

As $s, \angle C=99^\circ 16'$

To $AB=408$ Y.

So $s, \angle A=58^\circ 07'$

To $BC=351$ Y.

(11. 105)

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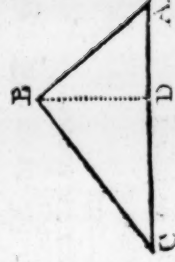
62. EXAMPLE X. In the plane Triangle ABC.

Given $AB=195$

$BC=216$

$AC=291$.

Required the angles.



CONSTRUCTION.

Make $CA=291$ equal parts; from C, with 216, describe an arc B; from A with 195 cut the arc B in B; draw BC, BA, and the triangle is constructed; then the angles may be measured by the help of a scale of chords.

COMPUTATION.

The three sides being given, the solution falls under either Problem V, or Problem VI. But that the use of these Problems may be sufficiently illustrated, the solutions according to both of them are here annexed.

Solution by Problem V. (49)

From the angle B, draw BD perpendicular to CA, which will be divided into the segments CD, DA, whose sum AC is known.

Now $BC=216$

And $AB=195$

$BC+AB=411$ =sum of sides.

$BC-AB=21$ =diff. of the sides.

Now the half of 291 is 145,5

And the half of 29,66 is 14,83

Therefore (47) the sum

the difference

Then in the triangle CDB.

As $BC=216$

To s, $\angle CDB=90^\circ 00'$

So $CD=160,3$

To s, $\angle CBD=47^\circ 55'$

Wh. taken from $90^\circ 00'$

Leaves $\angle C=42^\circ 05'$

To find the diff. of the segments.

As $AC=291$

To $BC+AB=411$

So $BC-AB=21$

To $CD-AD=29,66$

$160,33=CD$; or $CD=160,3$;

$130,67=AD$; or $AD=130,7$.

And in the triangle ADB.

As $AB=195$

To s, $\angle ADB=90^\circ 00'$

So $AD=130,7$

To s, $\angle ABD=42^\circ 05'$

Wh. taken from $90^\circ 00'$

Leaves $\angle A=47^\circ 55'$, & $\angle B=90^\circ 00'$.

Solution by Problem VI. (50)

To find the angle C.

Put $E=291=AC$

$F=216=BC$

$G=195=AB$

$D=75=E-F$

$2)270(135=\frac{1}{2}G+D$

$2)120(60=\frac{1}{2}G-D$

The angle C being found, the other angles may be found by Prob. I.

63 Ex-

63. EXAMPLE XI. In the plane Triangle ABG.

Given $AC=408$ $BC=351$ $AB=159$

Required the angles.



CONSTRUCTION.

The construction and mensuration is performed as in the last EXAMPLE.

COMPUTATION BY PROB. V. art. 49.

From the $\angle B$ draw the perpendicular BD ; and find the segments AD , DC ; which may be done without logarithms.

Thus $BC=351$ Then (I. 46), As $408 : 510 : : 192 : 240 \equiv DC - DA$.

$AB=159$ For $192 \times 510 = 97920$; which divided by 408 gives 240 .

$BC + AB = 510$ Now half of $408 = 204$; and half of 240 is 120 .

$BC - AB = 192$ Then $204 + 120 = 324 = DC$; and $204 - 120 = 84 = DA$.

In the triangle ADB.

As $AB = 159$

To $s, \angle D = 90^\circ 00'$

So $AD = 84$

To $s, \angle ABD = 31^\circ 53'$

And $\angle A = 58^\circ 07'$

In the triangle BDC.

As $BC = 351$

To $s, \angle D = 90^\circ 00'$

So $DC = 324$

To $s, \angle CBD = 67^\circ 23'$

And $\angle C = 22^\circ 37'$

Then $\angle ABD + \angle CBD = \angle ABC = 99^\circ 16'$.

COMPUTATION BY PROB. VI. art. 50.

To find the angle C.

Put $E = 408 = AC$

$F = 351 = BC$

$G = 159 = AB$

$D = 57 = E - F$.

$$2) 216 (108 = \frac{1}{2}G + D$$

$$2) 102 (51 = \frac{1}{2}G - D$$

Now the angle C being known, the other angles may be found by Prob. I. But for a farther illustration of Prob. VI. the work for another angle is here repeated.

To find the angle B.

Put $E = 351 = BC$

$F = 159 = AB$

$G = 408 = AC$

$D = 192 = E - F$

$$2) 600 (300 = \frac{1}{2}G + D$$

$$2) 216 (108 = \frac{1}{2}G - D$$

Then, To Ar. Co. log. $E = 408$

Add. Ar. Co. log. $F = 351$

And the log. $\frac{1}{2}G + D = 108$

Also the log. $\frac{1}{2}G - D = 51$

The half of this sum

Is the log. sine of $11^\circ 18' \frac{1}{2}$

Which doubled, is $22^\circ 37' = \angle C$.

9,29251

2) 18,58502

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64. EXAMPLE XII. In the plane Triangle ABC.



CONSTRUCTION.

The construction of this triangle, and the measuring of the angles, is performed as in the Xth and XIth Examples.

COMPUTATION.

In the triangle ABC, as $AB=BC$; therefore the angles A and C are equal (II. 104); and the perpendicular BD bisects the side AC; so that the right angled triangles ADB, CDB, are congruous; consequently, the angles being found in one triangle, will give those of the other.

Now in the triangle ADB, the side $AB=117$; the side $AD=\frac{1}{2}AC$, is 108; and the $\angle D$ is $90^\circ 00'$: Here being a side and its opposite angle, the solution belongs to Prob. I.

To find the angle ABD.	And $67^\circ 23'$ doubled
As $AD=117$	<u> </u>
To s, $\angle D=90^\circ 00'$	Gives $134.46=\angle ABC$.
So $AD=108$	
<u> </u>	
To s, $\angle ABD=67^\circ 23'$	A like process is to be used in every
<u> </u>	triangle, wherein are two equal sides.

Wh. taken from 90 00
 Leaves $22\ 37=\angle A=\angle C$.

In the foregoing examples are contained all the variety that can happen in the solutions of plane triangles, considered only with regard to their sides and angles; but besides the methods shewn of resolving such triangles by construction and computation, there is another way to find these solutions, called Instrumental; and this is of two kinds, viz. either by a ruler called a Sector, or by one called the Gunter's scale: The method by the sector, the curious reader may see in many books, particularly in a treatise on Mathematical Instruments published in the year 1757, 2d edition *: But the other method by the Gunter's scale being in great use at sea, it will be proper in this place to treat of it.

* By the author of these Elements.

65.

SECTION IV.

Description and Construction of the Gunter's Scale.

Mr. *Edmund Gunter*, Professor of Astronomy at *Gresham College*, did, some time about the year 1624, apply to a flat Ruler the Logarithms of numbers; this he effected, by taking from a scale of equal parts the lengths expressed by the figures in those logarithms, and transferring them to a line or scale drawn on such a ruler; and this is the line, which from his name is called the *Gunter's line*: He also, in like manner, constructed lines containing the logarithms of the fines and tangents; and since his time there have been contrived other logarithmic scales adapted to various purposes.

The Gunter's scale is a ruler, commonly two feet long; having on one of its flat sides several lines, or logarithmic scales; and on the other side various other scales; which, to distinguish them from the former, may be called natural scales.

While the reader is perusing what follows, it is proper he should have a Gunter's scale before him.

66.

Of the Natural Scales.

The half of one side is filled with different scales of equal parts, for the convenience of constructing a larger, or smaller figure: The other half contains scales of Rhumbs, marked Ru; Chords, marked Ch; Sines, marked Sin; Tangents, marked Tan; Secants, marked Sec; Semitangents, denoted by S. T. and Longitude distinguished by M. L. The descriptions and uses of these scales will be considered hereafter, in the places where they will be wanted.

67.

Of the Logarithmic Scales.

On the other side of the scale are the following lines.

- I. A line marked s. r. (sine rhumbs) which contains the logarithm fines of the degrees to each point and quarter point of the compass.
- II. A line signed t. r. (tan. rhumbs), whose divisions correspond to the logarithm tangents of the said points and quarters.
- III. A line marked Num. (numbers), wherein the logarithms of numbers are laid down.
- IV. A line marked Sin. containing the log. fines.
- V. A line of log. versed fines, marked v. s.
- VI. A line of log. tangents, marked Tan.
- VII. A meridional line signed Mer.
- VIII. A line of equal parts, marked E. P.

68. I. Of

68. I. *Of the Line of Numbers.*

The whole length of this line, or scale, is divided into two equal spaces, or intervals; the beginning, or left hand end of the first, is marked 1; the end of the first interval, and beginning of the second, is also marked 1; and the end of the second interval, or end of the scale, is marked with 10: Both these distances are alike divided, beginning at the left hand ends, by laying down in each the lengths of the logarithms of the numbers 20, 30, 40, 50, 60, 70, 80, 90; taken from a scale of equal parts, such that 10 of its primary divisions make the length of one interval: And the intermediate divisions are found, by taking the logarithms of like intermediate numbers.

From this construction it is evident, that when the first 1 stands for 1, the second 1 stands for 10, and the end 10 denotes 100;

And if the first 1 is called
 $\left\{ \begin{array}{l} 10, \\ 100, \\ \text{℥c.} \\ 1000, \\ \text{℥c.} \end{array} \right.$
 the 2d 1 stands for
 $\left\{ \begin{array}{l} 100, \\ 1000, \\ \text{℥c.} \\ 1, \\ 10, \\ \text{℥c.} \end{array} \right.$
 And the 10 at the end stands for
 $\left\{ \begin{array}{l} 1000, \\ 10000, \\ \text{℥c.} \\ 10, \\ 1, \\ \text{℥c.} \end{array} \right.$

And the primary and intermediate divisions in each interval, must be estimated according to the values set on their extremities, viz. at the beginning, middle, and end of the scale.

Now the examples most proper to be worked by this scale, are such wherein the numbers concerned do not exceed 1000, and then the first 1 stands for 10, the middle 1 for 100, and the 10 at the end for 1000: The primary divisions in the first interval, viz. 2, 3, 4, 5, 6, 7, 8, 9, stand for 20, 30, 40, 50, 60, 70, 80, 90, and the intermediate divisions stand for units. In the second interval, the primary divisions signed 2, 3, 4, 5, 6, 7, 8, 9, stand for 200, 300, 400, 500, 600, 700, 800, 900; each of these divisions are also divided into ten parts, which represent the intermediate tens. Between 100 and 200, the divisions for tens are each subdivided into five parts; so that these lesser divisions stand for two units. The tens between 200 and 500, are divided into two parts, each standing for five units: The units between the tens from 500 to 1000 are to be estimated by the eye; which by a little practice is readily done.

From this description it will be easy to find the division representing a given number not exceeding 1000: Thus the number 62, is the second small division from the 6, between the 6 and 7 in the first interval: The number 435 is thus reckoned; from the 4 in the second interval, count towards the 5 on the right, three of the larger divisions, and one of the smaller; and that will be the division expressing 435. And the like of other numbers.

69.

II. *For the Line of Sines.*

This scale terminates at 90 degrees, just against the 10 at the end of the line of numbers; and from this termination the degrees are laid backwards, or from thence towards the left: Now seeking in a table of logarithm sines, for the numbers expressing their arithmetic complements, without the index, take those numbers from the scale of equal parts the logs. of the numbers were taken from, and apply them to the scale of sines from 90°, and they will give the several divisions of this scale.

Thus the arith. comp. of the log. sines (or the co-secants) abating the index, of 10°, 20°, 30°, 40°, &c. are the numbers 76033, 46595, 30103, 19193, &c. then the equal parts to those numbers, laid from 90°, will give the divisions for 10°, 20°, 30°, 40°, &c. and the like for the intermediate degrees.

Proceeding in this manner, the arith. comp. of the sine of 5° 45' will be about equal to 10 of the primary divisions of the scale of equal parts, or to one interval in the log. scale; so that a decrease of the index by unity, answers to one interval; then a decrease of the index by 2 answers to 2 intervals, or the whole length of the log. scale; and this happens about the sine of 35 min. and the divisions answering to the sine of a little above 3 min. viz. 3' 26", will be equal to 3 intervals; and the sine of about 26" will be 4 intervals, &c. so that the sine of 90° being fixed, the beginning of the scale is vastly distant from it.

It is usual to insert the divisions to every 5 minutes, as far as 10 degrees; from 10° to 30°, the small divisions are of 15 minutes each; from 30° to 50°, contains every half degree; from 50° to 70°, are only whole degrees; the rest are easily reckoned.

70.

III. *For the Line of Tangents.*

As the tangent of 45 degrees is equal to the radius, or sine of 90°; therefore 45° on this scale, is terminated directly opposite to 90° on the sines: and the several divisions of this scale of log. tangents are constructed in the same manner as those of the sines, by applying their arith. comp. backwards from 45°, or towards the left hand.

The degrees above 45, are to be counted backwards on the scale: Thus the division at 40, represents both 40° and 50°; the division 30, serves for 30° and 60°; and the like of the other divisions, and their intermediates.

71.

IV. *For the Line of Versed Sines.*

This line begins at the termination of the numbers, sines, and tangents: But as the numbers on those lines descend from the right to the left, so these ascend in the same direction: Now having a table of logarithmic versed sines to 180 degrees, let each log. versed sine be subtracted from that of 180 degrees; then the remainders being successively taken from the said scale of equal parts, and laid on the ruler backwards from the common termination, the several divisions of this scale will be obtained.

The

The numbers for each 10 deg. are in the following table.

D. Numb.	D. Numb.	D. Numb.	Deg.	Numb.	Deg.	Numb.	Deg.	Numb.
100,0033	40,0,0540	70,0,1733	100	0,3839	130	0,7481	160	1,5207
200,0133	50,0,0854	80,0,2315	110	0,4828	140	0,9319	170	2,2194
300,0301	60,0,1219	90,0,3010	120	0,6021	150	1,1740	180	10,3010

The other scales will be described in their proper places.

72. Demonstration of the foregoing constructions.

That of the log. numbers, is evident from the nature of logarithms.

For the Sines and Tangents.

Now cosine : rad. :: rad. secant (34). Then $\text{co-f.} \times \text{secant} = 1$
 sine : rad. :: rad. co-sec. (35). $\text{fine} \times \text{co-fec.} = 1$
 tang. : rad. :: rad. co-tan. (36). $\text{tan.} \times \text{co-tan.} = 1$

The radius of the tables being supposed equal to 1.
 Hence it is evident, that in either case, one of the quantities will be equal to the quotient of unity divided by the other.

But division is performed, by subtraction with logarithms.

And to subtract a log. is the same as to add its arith. comp.

Consequently, the logarithmic co-sine and secant of the same degrees are the arithmetical complements of one another.

And so are the logarithmic sines and co-secants: Also the logarithmic tangents and co-tangents are the arith. comp. of one another.

73. Now as the arith. comp. of any number, is what that number wants of unity in the next superior place:

Therefore every natural sine and its arith. comp. together make the radius.

And the sines begin at one end of a radius, and end in 90° at the other end.

Therefore in a scale of sines, the arith. comp. of any sine, or its co-secant, laid backwards from 90° , gives the division for that sine. And the like must happen in a scale of log. sines.

74. Also, as the logarithmic tangents and co-tangents are the arith. comp. of one another. Therefore in a scale of log. tangents, the divisions to the degrees both under and above 45, are equally distant from the division of 45° .

Consequently the divisions serving to the degrees under 45, will serve, by reckoning backwards, for those above 45.

75. For the Versed Sines.

Although the numbers in the line of versed sines ascend from right to left, yet they are only the supplements of the real versed sines, whose order in numbering is the same as the sines, that is, from left to right: But as the beginning of the versed sines falls without the ruler, therefore it is most convenient to lay down the divisions from the point where the versed sines terminate at 180° degrees, that is, against 90° on the sines.

Now it is evident that the divisions laid off from this termination must be the differences between the log. versed sines of the several degrees, &c. and that of 180° degrees.

SECTION V.

76. *The use of the Gunter's Scale in Plane Trigonometry.*

When a Trigonometrical Question is to be solved by the Gunter's scale, it must first be stated by the precepts to that problem under which the question falls, whether it be by opposite sides and angles, or by two sides and their included angle, or by the three sides.

77. In all proportions wrought by the Gunter's scale, when the first and second terms are of the same kind, then

The extent from the first term to the second, will reach from the third term to the fourth.

Or, when the first and third terms are of the same kind,

The extent from the first term to the third, will reach from the second term to the fourth.

That is, set one point of the compasses on the division expressing the first term, and extend the other point to the division expressing the second (or third) term; then, without altering the opening of the compasses, set one point on the division representing the third term (or second term), and the other point will fall on the division shewing the fourth term or answer.

In working by these directions, it is proper to observe,

78. First The extent from one side to another side, is to be taken from the scale of numbers; and the extent from one angle to another is to be taken from the scale of sines, in working by opposite sides and angles; or from the scale of tangents, in working by two sides and the included angle.

Secondly. When the extent from the first term to the second (or third) is decreasing, or is from the right to the left, then the extent from the third term (or second) must be also decreasing; that is, applied from the right towards the left: And the like caution is necessary when the extent is from the left towards the right.

These precepts being carefully attended to, what follows will be readily understood.

79.

In EXAMPLE I. See article 53.

As $s, \angle C : AB :: s, \angle B : AC$. Or $s, 42^\circ 05' : 195 :: s, 90^\circ 00' : Q$, where Q stands for the number sought.

Now the extent from $42^\circ 05'$ to $90^\circ 00'$, taken on the scale of fines, and applied to the scale of numbers, will reach from 195 to 291. See art. 69.

Also. As $s, \angle C : AB :: s, \angle A : BC$. Or $s, 42^\circ 05' : 195 :: s, 47^\circ 55' : Q$.

Then the extent from $42^\circ 05'$ to $47^\circ 55'$ on the fines, being applied to the numbers, will reach from 195 to 216. See art. 68.

In each of these operations, the first extent was from the left to the right, or increasing; therefore the second extent must be from left to right.

80.

In EXAMPLE IV. See art. 56.

As $AC : s, \angle B :: AB : s, \angle C$. Or $291 : s, 90^\circ 00' :: 195 : Q$.

Here the extent from 291 to 195, taken on the numbers, and applied to the fines, will reach from $90^\circ 00'$ to $42^\circ 05'$.

The first extent being from the right towards the left, or decreasing; therefore the second extent must be also from the right to the left.

In EXAMPLE VII. See art. 59.

As $AB : Rad. :: BC : t, \angle A$. Or $195 : t, 45^\circ 00' :: 216 : Q$.

Then the extent from 195 to 216 on the numbers, will reach from $45^\circ 00'$ to $47^\circ 55'$ on the tangents.

Here the first extent being from left to right, or increasing; therefore the second extent must also be increasing: Now on the tangents, this increase above 45° does not proceed from left to right, but from right to left, the same way that the decrease proceeds (70); consequently the division, which the point falls on for the fourth term, must be estimated according as the first extent is increasing or decreasing.

Thus had the proportion been,

As $BC : Rad. :: AB : t, \angle C$. Or $216 : t, 45^\circ 00' :: 195 : Q$.

Then the extent from 216 to 195 on the numbers, will reach from $45^\circ 00'$ to $42^\circ 05'$, estimated as decreasing,

81. When two sides and the included angle are given, and the tangent of half the difference of the unknown angles is required.

Then, on the line of numbers take the extent from the sum of the given sides to their difference; and on the line of tangents apply this extent from 45° downwards, or to the left; let the point of the compasses rest where it falls, and bring the other point (from 45°) to the division answering to the half sum of the unknown angles; then this extent applied from 45° downwards, will give the half difference of the unknown angles: Whence the angles may be found. (47)

In EXAMPLE IX. See the art. 61.

$$\begin{array}{l} \text{As } AB + AC : AB - AC :: t, \frac{1}{2} \angle C + \angle B : t, \frac{1}{2} \angle C - \angle B. \\ \text{Or } 567 : 249 :: t, 60^\circ 56' : Q. \end{array}$$

Now the extent from 567 to 249 on the numbers, being applied to the tangents, will reach from 45° to about $23^\circ 40'$: Let one point of the compasses rest on this division, and bring the other to $60^\circ 56'$; then this extent will reach from 45° to $38^\circ 19'$, the half difference sought.

And this method will always give the half difference, whether the half sum of the angles is greater or less than 45° .

82. But when the half sum and half difference are greater than 45° ; then the extent from the sum of the sides to their difference on the scale of numbers, will (on the tangents) reach from the half sum of the angles to their half difference, reckoning from left to right.

And when the half sum and half difference are both less than 45° ; then the extent from the sum of the sides to their difference, taken from the numbers and applied to the tangents, will reach from the half sum of the angles downwards to their half difference.

83. When the three sides are given to find an angle, and a perpendicular is drawn from an angle to its opposite side. See Ex. X. art. 62.

$$\begin{array}{l} \text{As } AC : BC + AB :: BC - AB : CD - AD. \\ \text{Or } 291 : 411 :: 21 : Q. \end{array}$$

Now the extent from 291 to 411 on the scale of numbers, will reach from 21 to 29,6 also on the numbers.

Then the extent for the angles is performed in the same manner as shewn in Ex. I. (78)

84. Or an angle may be found by Problem VI. as follows.

In the scale of numbers, take the extent from the half sum (of G and D) to either of the containing sides (as E); apply this extent from the other containing side (as F), to a fourth term: Let one point of the compasses rest on this fourth term, and extend the other to the half difference (of G and D); then this extent applied to the versed sines from the beginning, will give the supplement of the angle sought.

In EXAMPLE X. See art. 62.

$$E = 291; F = 216; \text{ half sum} = 135; \text{ half diff.} = 60$$

Then on the numbers, the extent from 135 to 291, will reach from 216 to 465; let the point rest there, and extend the other to 60; then this extent applied to the versed sines, will reach from the beginning to $137^\circ 56'$; which taken from 180° , leaves $42^\circ 04'$ for the angle sought.

85. In

85.

In EXAMPLE XI. See art. 63.

Here $E=408$; $F=351$; half sum of G and $D=108$; half diff. $=51$. Then on the numbers, the extent from 108 to 408, will reach from 351 in the second interval, to a 4th number: But as the point of the compasses falls beyond the end of the scale, therefore let the extent from 108 to 408 be applied in the first interval, which will reach from 351 to 132,6; let one point rest on 132,6 and extend the other point of the compasses to 51. Now as this extent of the compasses is less than it ought to be, by one interval, or half the length of the scale of numbers; therefore the last extent, when applied to the versed sines, must be from that division, on the versed sines, opposite to the middle of the scale of numbers, which is nearly at 143° ; and it will reach from thence to the versed sine of 157° , $23'$; which taken from 180° , leaves $22^\circ 37'$ for the angle sought.

86 Most of the writers on Plane Trigonometry treat of right angled, and of oblique angled triangles separately; making seven cases in the former and six cases in the latter: But as every one of these thirteen cases fall under one or other of the foregoing Problems, therefore such distinctions are here avoided, it being conceived that they rather tend to perplex than instruct a learner: Also in the generality of the treatises on this subject, it is usually shewn how the solutions of right angled triangles are performed, by making (as it is called) each side radius; that is, by comparing each side of the triangle with the radius of the tables: And although these considerations are here omitted, yet the inquisitive reader will find them in Book VII. near the beginning.

87. Beside the demonstration of Problem IV. at art. 48 it has been thought proper to give another demonstration; because there arises from it a Theorem useful on some occasions: Moreover, there is also added methods of deriving other rules for the solution of the case where the three sides are given to find an angle; which, if they should be found on any other use, will perhaps be agreeable exercises of Geometry to those who are delighted with these studies.

88. In

88.

In any plane triangle ABC,

Given CA, CB, and $\angle C$

Required the angles B and A.

SOLUTION. Take $CD=CB$, and draw DB.

Bisect DB in F, DA in E, draw CFG, and EF,

which is parallel to AB

(II. 165.)

Now DE or AE is equal to half the difference of CA and CB.

And $CE(=CA-AE)$ is equal to the half sum A

of CA and CB.

The sum of the equal angles CBD, CDB (II. 104) is equal to the sum of the unknown angles CBA, CAB. (II. 98.)

Then the angle CBD is the half sum, and the angle ABD is the half difference of the unknown angles CBA, CAB. (47)

And as CFG is at right angles to DB (II. 103); CF is the tangent of $\angle CBD$, and GF is the tangent of $\angle ABF$. (47)

Then $CE:EA::CF:GF$ (II. 165): Or $2CE:2EA::CF:FG$. (II. 151)

That is, $CA+CB:CA-CB::\tan.\frac{1}{2}\angle CBA+CAB:\tan.\frac{1}{2}\angle CBA-CAB$.

89. AGAIN. From H, the middle of CD, draw HI at right angles, and equal to CH; draw DI, and EK parallel to DI, meeting CI produced in K; and join IE.

Now $HE=(\frac{1}{2}CD+\frac{1}{2}DA)=\frac{1}{2}CA$; and $CH=\frac{1}{2}CB$.

And HE=tangent of the angle HIE to the radius HI=HC.

Then $CB:CA::(2HC:2HE::HC:HE::) \text{Radius}:\tan.\angle HIE$.

And $\angle HIE-(\angle HID=)45^\circ=\angle DIE=\angle KEI$ (II. 95) is known.

Then rad. : $\tan.\angle KEI::EK:KI::CK:KI$; because $\angle KCE=\angle KEC$.

But

$$::CK:KI::CE:ED. \quad (\text{II. 166})$$

$$::2CE:2ED. \quad (\text{I. 151})$$

$$::CA+CB:CA-CB::\frac{1}{2}\text{sum}.\angle s:\frac{1}{2}\text{diff}.\angle s. \quad (93)$$

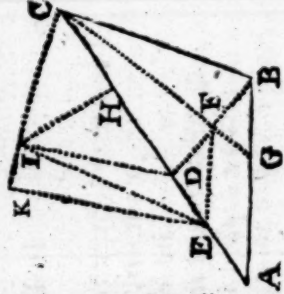
Consequently rad. : $\tan.\angle KEI::\tan.\frac{1}{2}\angle CBA+CAB:\tan.\frac{1}{2}\angle CBA-CAB$.

This rule is often useful in Astronomical calculations, when the logarithms of the sides AB, BC are only known, and the angles BAC, BCA, are required, without finding, from those logarithms, the sides themselves.

For the difference between the logarithms of the sides, increased by radius, gives the tangent of an arc; which arc lessened by 45° leaves a second arc.

Then as rad. : $\tan$. second arc :: $\tan.\frac{1}{2}\text{sum}.\angle s:\tan.\frac{1}{2}\text{diff}.\angle s$.

90. In



90. In any plane triangle ABC, where the three sides are known, the measure of either angle (as the angle A, included between the sides AB, AC,) may be found several ways, as shewn in the following articles.

The letters s, t, v , stand for sine, tangent, verfed sine.

s', t' , stand for co-sine, co-tangent.

v' , stands for the verfed sine of the supplement.

Also ss, tt , stand for the squares of the sine and tangent.

And $s's', t't'$, stand for the squares of the co-sine and co-tangent.

Radius $= R = AB = AE = AD$; and $H = \frac{1}{2}AB + \frac{1}{2}AC + \frac{1}{2}BC$.



$$91. s, \frac{1}{2} \angle A = R \times \sqrt{\frac{H - AC \times H - BC}{AC \times AB}}. \text{ Here } \angle E = \frac{1}{2} \angle A.$$

For $R : s, \frac{1}{2} \angle A :: (DE =) 2AB : BD$.

And $RR : ss, \frac{1}{2} \angle A :: 4AB^2 : BD^2$.

Now $ss, \frac{1}{2} \angle A = RR \times \frac{BD^2}{4AB^2}$.

(45)
(II. 161)

(II. 164)

$$= \frac{2AB}{4AB^2} \times \frac{2H - 2AC \times 2H - 2BC}{2AC} \times RR.$$

(II. 179)

Then $s, \frac{1}{2} \angle A = R \times \sqrt{\frac{H - AC \times H - AB}{AC \times AB}}$.

$$92. s, \angle A = \frac{2R}{AB \times AC} \times \sqrt{H \times H - CB \times H - AC \times H - AB}.$$

For $AB : BG :: R : s, \angle A$.

And $\frac{AB}{AB^2} : \frac{BG}{BG^2} :: RR : ss, \angle A$.

(45)
(II. 161)

(II. 164)

Now $ss, \angle A = \frac{RR}{AB^2} \times BG^2$.

$$= \frac{RR}{AB^2} \times \frac{4}{AC^2} \times H \times H - CB \times H - AC \times H - AB.$$

(II. 180)

$$93. s', \frac{1}{2} \angle A = R \times \sqrt{\frac{H \times H - CB}{AC \times AB}}. \text{ Here } \angle E = \frac{1}{2} \angle A; \text{ and } \angle D = \text{comp. } \angle E.$$

For $R : s', \frac{1}{2} \angle A :: (DE =) 2AB : BE$.

And $RR : s's', \frac{1}{2} \angle A :: 4AB^2 : BE^2$.

(45)
(II. 161)

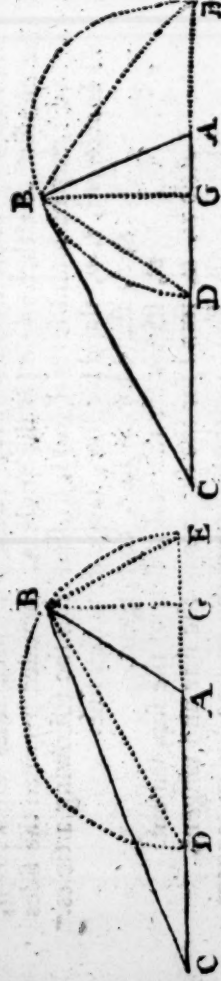
(II. 164)

Now $s's', \frac{1}{2} \angle A = RR \times \frac{BE^2}{4AB^2}$.

$$= RR \times \frac{2AB \times 4 \times H \times H - CB}{4AB^2 \times 2AC}.$$

(II. 178)

94. $t, \frac{1}{2}$



$$94. t, \frac{1}{2}\angle A = R \times \sqrt{\frac{H-AC \times H-AB}{H \times H-CB}}.$$

For $R:t, \frac{1}{2}\angle A :: BE:BD.$

And $RR:t, \frac{1}{2}\angle A :: BE^2:BD^2.$

Now $t, \frac{1}{2}\angle A = RR \times \frac{BD^2}{BE^2}.$

$$= RR \times \frac{H-AC \times H-AB}{H \times H-CB}.$$

(46)
(II. 161)

(II. 164)

(II. 149)

$$95. t', \frac{1}{2}\angle A = R \times \sqrt{\frac{H-H-CB}{H-AC \times H-AB}}.$$

For $\overline{BD}^2 : \overline{BE}^2 :: (tt, \frac{1}{2}\angle A : RR ::) RR : t', \frac{1}{2}\angle A.$

Then $t', \frac{1}{2}\angle A = RR \times \frac{\overline{BE}^2}{\overline{BD}^2}.$

(36)

(II. 178. 179)

$$96. v, \angle A = 2R \times \frac{H-AC \times H-AB}{AC \times AB}$$

For $R:v, \angle A :: AB:GD.$

Then $v, \angle A = R \times \frac{GB}{AB} = R \times \frac{4 \times H-AC \times H-AB}{2 \times AC \times AB}.$

(45)

(II. 177)

$$97. v', \angle A = 2R \times \frac{H \times H-CB}{AC \times AB}.$$

For $R:v', A :: AB:GE.$

Then $v', \angle A = R \times \frac{GE}{AB} = R \times \frac{4 \times H \times H-CB}{2 \times AC \times AB}.$

(45)

(II. 176)

$$98. s', \angle A = \frac{1}{2}R \times \frac{\overline{CB}^2 - \overline{AC}^2 - \overline{AB}^2}{AC \times AB}.$$

For $R:s', \angle A :: AB:AG.$

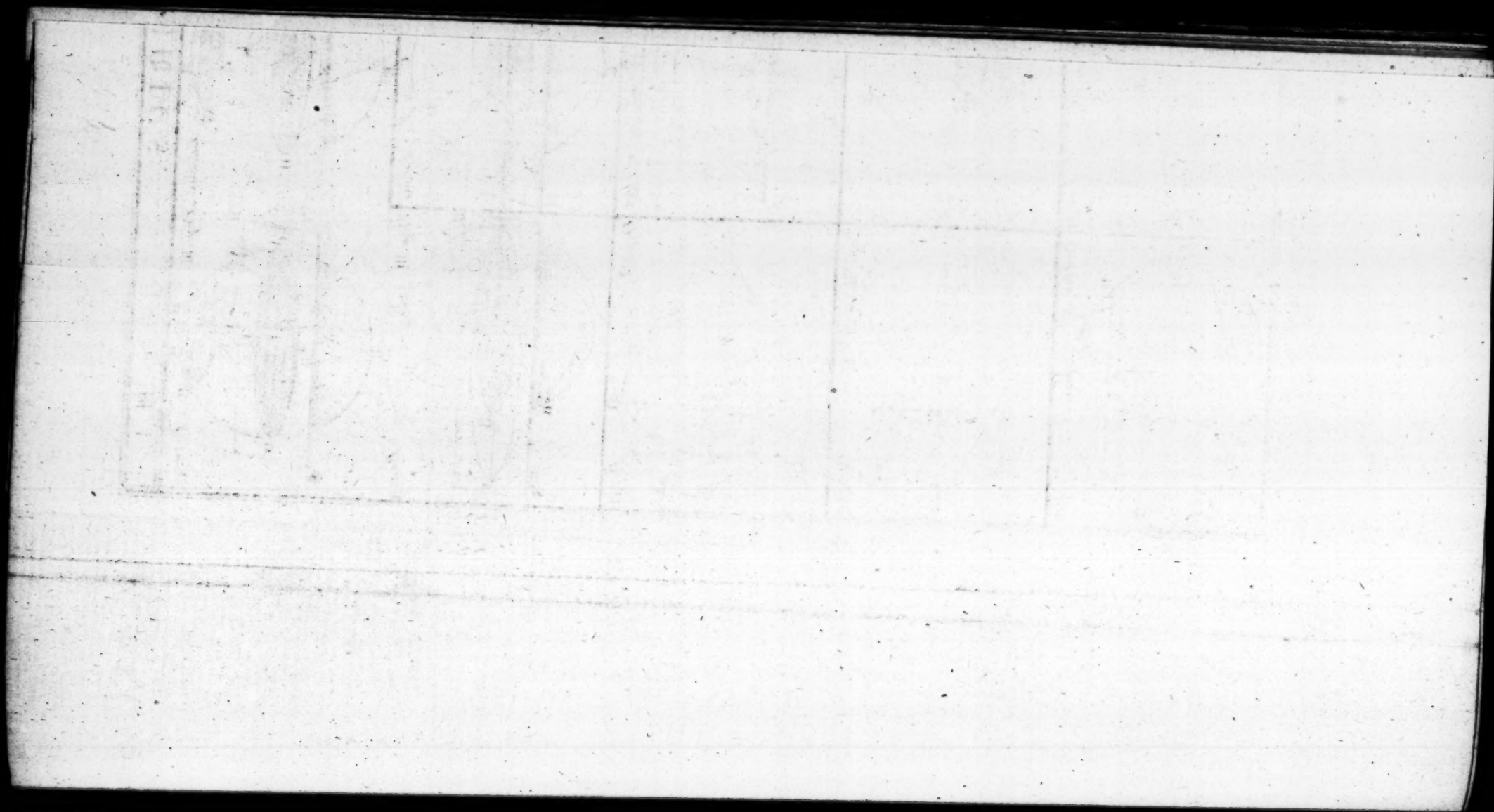
Then $s', \angle A = R \times \frac{AG}{AB} = R \times \frac{\overline{BC}^2 - \overline{AC}^2 - \overline{AB}^2}{2 \times AC \times AB}.$

(45)

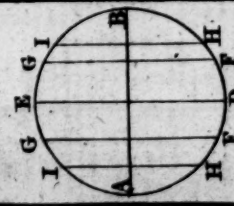
(II. 174)

END OF BOOK III.

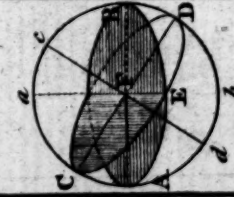
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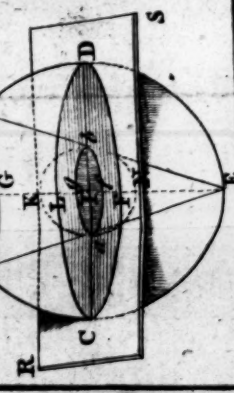
Definitions. Pl. 125



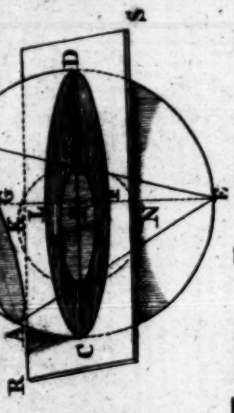
Prop. I. Th. 128



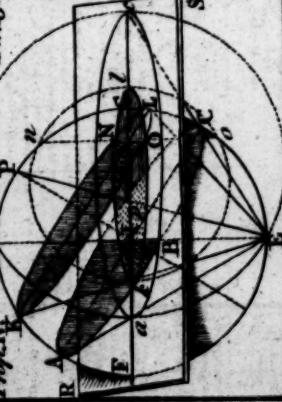
Prop. III. Pl. 129



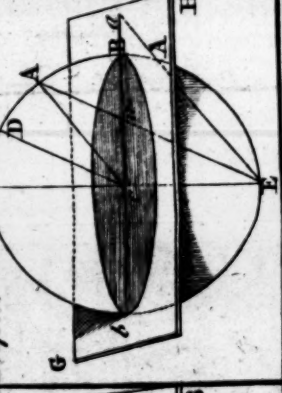
Prop. III. Pl. 129



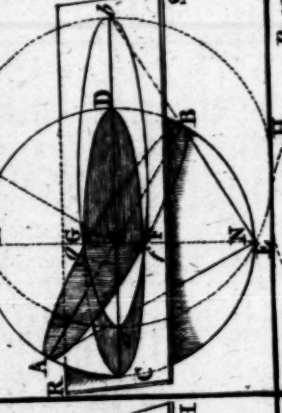
Prop. IV. Pl. 130



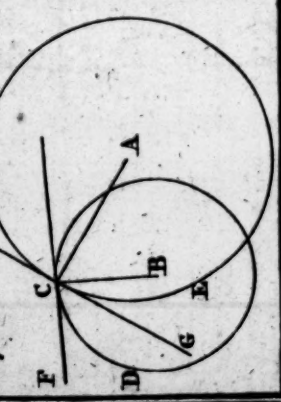
Prop. V. Pl. 131



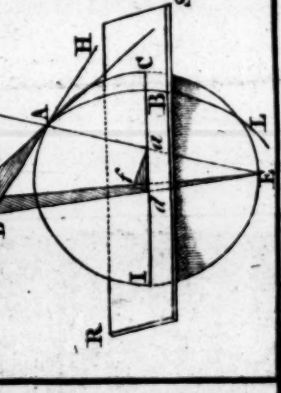
Prop. III. Pl. 129



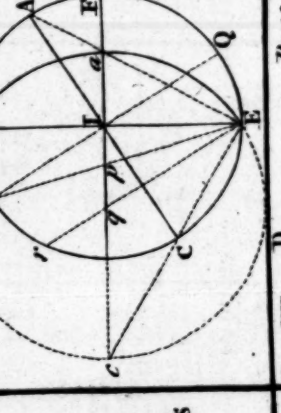
Prop. VI. Pl. 131



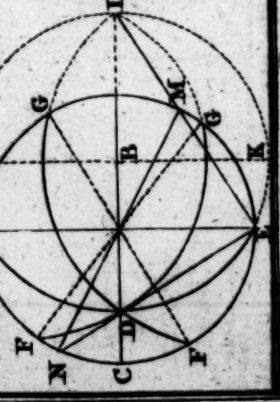
Prop. VII. Pl. 132



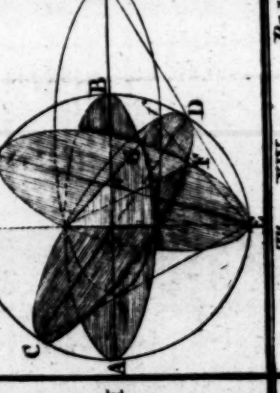
Prop. VIII. Pl. 132



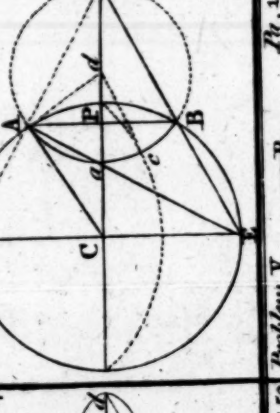
Prop. IX. Pl. 133



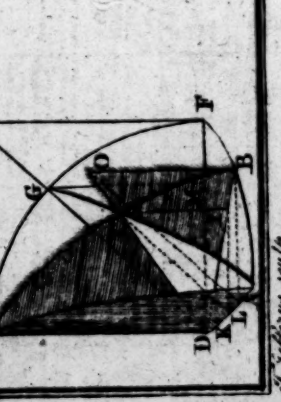
Prop. X. Pl. 134



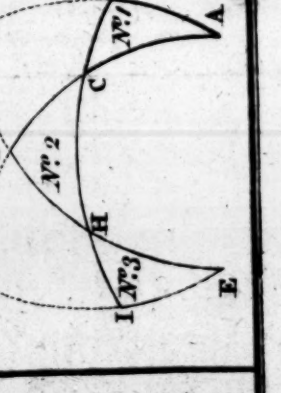
Prop. XI. Pl. 134



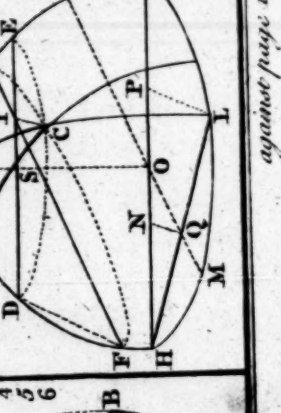
Theor. XIII. Pl. 153



Theor. XIV. Pl. 152



Problem. V. Pl. 160



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THE ELEMENTS OF NAVIGATION.

BOOK IV. OF SPHERICS.

SECTION I.

Definitions and Principles.

1. **SPHERICS** is that part of the Mathematics which treats of the position and magnitude of arcs of circles described on the surface of a sphere.

2. A **SPHERE** is a solid contained under one uniform round surface, such as would be formed by the revolution of a circle about a diameter thereof, that diameter being immovable during the motion of the circle, Thus the circle $AEBD$ revolving about the diameter AE , will generate a sphere, whose surface will be formed by the circumference $AEBD$. See Plate I.

3. The **CENTER** and **AXIS** of a sphere are the same as the center and diameter of a generating circle: And as a circle has an indefinite number of diameters, so a sphere may be considered as having also an indefinite number of axes, round any one of which the sphere may be conceived to be generated.

4. **CIRCLES OF THE SPHERE**, are those circles described on its surface by the motion of the extremities of such chords in the generating circle as are at right angles to the diameter thereof, or to the axis of the sphere.

Thus by the motion of the circle $AEBD$ about the diameter AB , the extremities of the chords ED , GF , IH , at right angles to AB , will describe circles whereof the diameters are equal to those chords respectively. Plate I.

5. The

5. The POLES of a circle on the sphere, are those points on its surface equally distant from the circumference of that circle.

Thus A and B are the poles of the circles described on the sphere by the ends of the chords ED, GF, IH. Plate I.

6. A GREAT CIRCLE of the sphere, is that circle which is equally distant from both its poles.

Thus the circle described by the extremities E, D, of the diameter ED, at right angles to AB, being equally distant from its poles A and B, is called a great circle.

7. LESSER CIRCLES of the sphere, or small circles, are those circles which are unequally distant from both their poles.

Thus the circles of which FG, HI are diameters, having their poles A and B unequally distant from them, are called lesser circles.

8. PARALLEL CIRCLES of the sphere, are those circles whereof the planes are considered as parallel to the plane of some great circle.

Thus the circles having the diameters FG, HI, are called parallel circles in respect to the great circle whereof ED is the diameter.

9. A SPHERIC ANGLE is the inclination of two great circles of the sphere meeting one another.

10. A SPHERIC TRIANGLE is a figure formed on the surface of a sphere by the mutual intersections of three great circles.

11. The STEREOGRAPHIC PROJECTION of the sphere, is such a representation of its circles, upon the plane of one of them passing through the center, and called the PLANE OF PROJECTION, as would appear to an eye placed in one of the poles of that great circle, and thence viewing the circles on the sphere.

12. The place of the Eye is called the PROJECTING POINT, or lower pole: and the point diametrically opposite is called the remotest, or opposite, or upper pole.

Also, the projection of any point on the sphere, is that point in the plane of projection, through which the visual ray passes to the eye.

13. The PRIMITIVE CIRCLE is that great circle which limits or bounds the representation, or projection.

14. An OBLIQUE CIRCLE is the projection of such a great circle whereof the plane lies obliquely to the eye.

15. A RIGHT CIRCLE is represented by a diameter of the primitive circle, and is the projection of a great circle when its plane passes through the eye, or is seen edgewise.

AXIOMS.

16. The diameter of every great circle passes through the center of the sphere; but the diameters of small circles do not pass through the same center: Also the center of the sphere is the common center of all its great circles.

17. Every section of a sphere, by a plane passing through its circumference, is a circle.

18. A sphere is divided into two equal parts by the plane of every great circle; and into two unequal parts by the plane of every small circle.

19. The pole of every great circle is at 90 degrees distance from it on the surface of the sphere: And no two great circles can have a common pole.

20. The poles of a great circle are at the extremities of that diameter, or axis, of the sphere, which is perpendicular to the plane of that circle.

21. Lines flowing to the projecting point, or place of the eye, from every point in the circumference of a circle which it views, form the convex surface of a Cone.

22. A plane passing through three points on the surface of a sphere, equally distant from the pole of a great circle, will be parallel to the plane of that circle.

23. The shortest distance between two points on the surface of a sphere, is the arc of a great circle passing through those points.

24. If one great circle meets another, the angles on either side are supplements to one another; and every spheric angle is less than 180 deg.

25. A spheric angle is measured by an arc of a great circle intercepted between the legs of that angle, at 90 degrees distant from the angular point.

26. If two circles intersect one another, the opposite angles are equal.

27. Two spheric triangles are congruous, if two sides and their contained angle in one, are equal to two sides and their contained angle in the other, each to each: Or if two angles and the contained side in the one, are equal to two angles and their contained side in the other, each to each: Or if the three sides in the one, are respectively equal to the three sides in the other.

28. All parallel circles having the same pole, may be conceived to be concentric to the great circle to which they are parallel.

29. All parallel circles on the sphere, having the same pole, are cut into similar arcs by two great circles passing through that pole.

SECTION

SECTION II.

Stereographic Propositions.

30.

PROPOSITION I.

Great circles of a sphere mutually cut one another into two equal parts.

DEMONSTR. Any two great circles have the same common center. (16)
 And their planes intersect in a right line.
 Now the center must lie in the line of their intersection.
 Therefore this right line is a diameter common to both.
 But every circle is bisected by its diameter.
 Therefore the circles mutually bisect one another.

31. COROL. I. The circumferences of any two circles intersecting one another twice, make the angles at both sections equal.

For the planes of those circles have the same inclination at both ends of their intersection, or where the circumferences intersect.

32. COROL. II. Two great circles of the sphere will cut each other twice at the distance of 180 degrees, or in opposite points of the sphere.

PROP. II.

33. *The distance of the poles of two great circles, is equal to the angle formed by the inclination of those circles.* Plate I.

DEM. Let AEB, CED, be two great circles of the sphere, their planes passing through its center F; and let a, b , be the poles of the circle AEB, and c, d , the poles of the circle CED.

Then is the arc $Aa = \text{arc } Cc = 90^\circ$.

And the arc ca is common to both the arcs Aa and Cc .

Therefore the arc AC measuring the inclination CPA of the circles, is equal to the arc ac measuring the distance of the poles. (II. 48)

34. COROL. I. Two great circles are at right angles to one another, when they pass through each other's poles.

35. COROL. II The pole of a great circle is 90 degrees distant from it, taken in another great circle, or in an arc thereof, drawn perpendicular to the former circle.

36. COROL. III. Two or more great circles at right angles to another great circle, intersect one another at 90° distant from it, or in the pole of the latter circle. And the like of arcs of great circles.

37. COROL. IV. If several great circles intersect one another in the pole of another great circle; then are the former circles perpendicular to the latter.

38. PROP.

38.

PROP. III.

In the Stereographic projection of the sphere, the representations of all circles, not passing through the projecting point, will be circles. Plate I. three figures.

Let $ACEDB$ represent a sphere, cut by a plane RS , passing through the center I , at right angles to the diameter EH , drawn from E , the place of the eye.

And let the section of the sphere (17) by the plane RS , be the circle $CFDL$, its poles being H , and E .

Suppose AGB is a circle on the sphere to be projected, its pole, most remote from the eye, being F : And the visual rays from the circle AGB meeting in E , form the cone $ACBE$ (21), whereof the triangle AEB is a section through the vertex E , and diameter of the base AB . (II. 204)

Then will the figure $agbf$, which is the projection of the circle BGA , be a circle.

DEMONSTRATION. Since the $\angle EAB$ is measured by $\frac{1}{2}$ arc $AC + (\frac{1}{2}$ arc $ED =) \frac{1}{2}$ arc CE .

And the $\angle EBA$ is measured by $\frac{1}{2}$ arc $AC + \frac{1}{2}$ arc CE . (II. 137)

Therefore the angle $EBA =$ angle Eab . (II. 128)

And so the triangles EAB , Eba , are similar, the $\angle E$ being common. (II. 50)

Therefore ab cuts the sides EA , EB , of the cone in a subcontrary position to AB ; and consequently the section $agbf$ is a circle. (II. 213)

Now suppose the plane RS to revolve on the line CD , till it coincides with the plane of the circle $ACEB$;

Then it is evident, that the point L will fall in H , the point F in E , and the circle $CFDL$ will coincide with the circle $CEDH$, which now becomes the primitive circle, where the point F , or E , is the projecting point: Also the projected circle $agbf$ will become the circle $anbk$.

39. COROL. I. Hence the middle of the projected diameter is the center of the projected circle, whether it be a great circle or a small one.

40. COROL. II. Hence in all circles parallel to the plane of projection, their centers and poles will fall in the center of the projection.

41. COROL. III. The centers and poles of circles inclined to the plane of projection, fall in that diameter of the primitive circle which is at right angles to the diameter drawn through the projecting point; but at different distances from its center.

42. COROL. IV. All oblique great circles cut the primitive circle in two points diametrically opposite.

K

43. PROP.

43.

PROP. IV.

The projected diameter of any circle subtends an angle at the eye whose measure is equal to the distance of that circle from its nearest pole, taken on the sphere. And that angle is bisected by a right line joining the eye and that pole.
Plate I.

Let the plane RS cut the sphere $HPEG$, as in the last.

And let ABC be any oblique great circle, whose diameter AC is projected in ac ; and KOL any small circle parallel to ABC , whose diameter KL is projected in kl .

The distances of those circles from their pole P , being the arcs AHP , KHP , and the angles aEC , kEl , are angles at the eye subtended by their projected diameters ac , kl .

Then is the angle aEC measured by the arc AHP , the angle kEl is measured by the arc KHP ; and those angles are bisected by EP .

DEM. For arc $PHA = \text{arc } PC$; and arc $PHK = \text{arc } PL$.

And the $\angle AEC$ is measured by $\frac{1}{2}$ arc $APC = \text{arc } PHA$. (II. 128)

Also the $\angle KEL$ is measured by $\frac{1}{2}$ arc $KPL = \text{arc } PHK$. (II. 128)

Therefore the angles AEC , KEL , are respectively measured by the arcs PHA , PHK .

And it is evident those angles are bisected by the line EP .

44. COROL. I. Hence as the line EP projects the pole P in p ; so the same line refers a projected pole to its place on the sphere, in the circumference of the primitive circle.

45. COROL. II. Hence on the primitive circle may be described the representation of any circle, whose distance from its pole, and the projected place of that pole, are given.

For PA and PC are projected into pa and pc ; and the bisection of ac gives the center of the circle sought.

46. COROL. III. Hence every projected oblique great circle cuts the primitive circle in an angle equal to the inclination of the plane of that oblique circle to the plane of projection.

For Pa is equivalent to PA the inclination.

And Pa measures the angle FHa , since FH , Ha are each 90° . (9)

47. COROL. IV. The distance between the projections of a great circle and any of its parallels is equivalent to their distance on the sphere.

Thus the projection ak is equivalent to AK .

48. PROP.

PROP. V.

Any point of a sphere stereographically projected, is distant from the center of projection, by the tangent of half the arc intercepted between that point and the pole opposite to the eye: The semidiameter of the sphere being made radius. Plate I.

Let $CDEB$ be a great circle of the sphere whose center is c , GH the plane of projection cutting the diameter of the sphere in b , B ; E , C , the poles of the section by that plane; and a the projection of A . Then is ca equal to the tangent of half the arc AC .

DEM. Draw CF , a tangent to the arc $CD = \frac{1}{2}$ arc CA , and join cF . Now the triangles cFe , cae , are congruous: For $ce = ce$, $\angle c = \angle eca = \text{right } \angle$, $\angle cFe = \angle cea$ (II. 128): Therefore $ca = cF$. Consequently ca is equal to the tangent of half the arc CA .

" PROP. VI.

The angle made by the intersection of the circumferences of two circles in the same plane, is equal to the angle made by tangents to those circles in the point of section; and also is equal to the angle made by their radii drawn to that point. Plate I.

Let CE , CD , be two arcs of circles in the same plane cutting in the point c ; AC , BC , their radii; GC , FC , tangents at the point C . Then is the curve lined angle $ECD = \angle GCF = \angle ACB$.

DEM. Since the radii AC , BC , are at right angles to their tangents GC , FC (II. 126); and are also at right angles to the arcs CE , CD . (II. 136) Therefore the position of the tangents and arcs at the point c are the same; and consequently the $\angle ECD = \angle GCF$.

Also the $\angle ACB + \angle BCG = (\text{right } \angle =) \angle FCG + \angle BCG$.

Therefore the angle ACB is equal to the angle FCG , by taking away the common angle BCG .

Consequently $\angle ECD = \angle GCF = \angle ACB$. (II. 48)

50. SCHOLIUM. If the arcs CE , CD , were in different planes, the same would hold true with regard to their tangents.

For suppose the circle CD to revolve on the fixed radius BC , still cutting the circle CE in c : Then the tangent cF revolving with it, has still the same inclination to BC : And as the inclination of the planes of the circles vary, so much will the inclination of the tangents vary.

Therefore the angle made by the tangents, in all positions of the circular planes, is the same as the angle made by their circumferences.

51. COROL. Hence if a plane touches a sphere, at the point where two circles thereof intersect one another, the tangents to both circles will lie in that plane; and consequently, in all oblique positions, a right line perpendicular to one tangent will cut the other tangent.

52. PROP. VII.

The angle which any two circles make, when stereographically projected, is equal to the angle which those circles make on the sphere. Plate I.

Let $IACE$ and ABL be two circles on a sphere intersecting in A ; E the projecting point; and RS the plane of projection whereon the point A is projected in a , in the line IC , the diameter of the circle ACE .

Also let DH , FA , be tangents to the circles ACE , ABL .

Then if ad , af , are the projections of the tangents AD , AF ; the projected $\angle daf$ will be equal to the spheric angle BAC .

DEM. As the tangents AD , AF , are obliquely posited in a plane touching the sphere in A ;

From any point D in the tangent AD , draw DF perpendicular to AD meeting the tangent AF in F ; draw DG parallel to IC meeting EA produced in G ; and draw FE cutting the plane of projection in f .

Now as the tangent DH is in the same plane with the circle ACE passing through the eye E , the line AD will be projected in the line ad .

And as $\angle DGE = \angle dae$ (II. 95) $\therefore \angle EAH$. (II. 132, 137)

And $\angle EAH = \angle DAG$. (II. 93)

Therefore $\angle DGE = \angle DAG$. And $DG = DA$. (II. 104)

Also DF being perpendicular to AD , is at right angles to the plane ACE ; and consequently is in a parallel position to the plane RS .

Therefore df , the projection of DF , is at right angles to da , the projection of DA .

Therefore the triangles dfa , DFG , are parallel sections of the pyramid $DFGE$: So that $ad : df :: (DG =) DA : DF$. (II. 211)

Now the triangles ADF , adf , having an equal angle in each, and the sides about that angle proportional, are similar.

Consequently the angle $daf =$ angle DAF . (II. 168)

But $\angle DAF = \angle BAC$ (49): Therefore $\angle daf = \angle BAC$.

53. PROP. VIII.

The distance between the poles of the primitive circle and an oblique circle, in stereographic projections, is equal to the tangent of half the inclination of those circles; and the distance of their centers is equal to the tangent of their inclination: The semidiameter of the primitive circle being made radius. Plate I.

Let AC be the diameter of a circle whose poles are P and Q , and inclined to the plane of projection in the angle AIF .

And let a , c , p , be the projections of the points A , C , P .

Also let HAE be the projected oblique circle, whose center is q .

Now when the plane of projection becomes the primitive circle, whose pole is I .

Then is $ip =$ tangent of half $\angle AIF$, or of half the arc AF .

And $iq =$ tangent of AF , or of the $\angle FHA = AIF$.

DEM. For $AH + HP = AH + AF$. Therefore $HP = AF$.

But $ip =$ tangent of half HP , or of half AF . (48)

Again. As AC is projected in ac , then q , the middle of ac , is the center of the projected circle, and of its representative HAE . (45)

Draw eq produced to r : Then as $qa = qe$; the $\angle qEA = \angle qae$. (II. 104)

But

But the $\angle qae$ is measured by half the arc EFA .
 Therefore the arc $AHr = \text{arc } AFE$ (II. 50): And as the arc $AHP = FQF$;
 Therefore $Pr = AF = HP$; and $HPr = \text{twice the arc } AF$.
 Therefore (II. 127) the $\angle IEq = \angle AIF$, the inclination of the circles.
 But Iq is the tangent of the $\angle IEq$, EI being the radius.

54. COROL. Hence the radius of an oblique circle is equal to the secant of the obliquity of that circle to the primitive.
 For Eq is the secant of the angle IEq , to the radius EI .

PROP. IX.

55. *If through any given point in the primitive circle an oblique circle be described; then the centers of all other oblique circles passing through that point, will be in a right line drawn through the center of the first oblique circle at right angles to a line passing through that center, the given point, and the center of the primitive.*

Let $GACE$ be the primitive circle, $ADEI$ a great circle described through D , its center being B .

HK is a right line drawn through B , perpendicular to a right line CI passing through D , B , and the center of the primitive circle.

Then the centers of all other great circles FDG passing through D will fall in the line HK .

DEM. For if E be the projecting point, the circle $EDAI$ will be the projection of a circle whose diameter is NM . (38)

Therefore D and I are the projections of N , M , which are opposite points on the sphere; or of points at a semicircle's distance.

Therefore all circles passing through D and I must be the projections of great circles on the sphere.

But DI is a chord in every circle passing through the points D , I .

Consequently the centers of all those circles will be found in HK drawn perpendicularly through B , the middle of DI . (II. 125)

PROP. X.

56. *Equal arcs of any two great circles of the sphere, will be intercepted between two other circles drawn on the sphere through the remotest poles of those great circles. Plate I.*

Let $PBEA$ be a sphere, whereon AGB , CFD , are two great circles, whose remotest poles are E , P ; and through these poles let the great circle $PBEC$, and small circle PGE , be drawn, intersecting the great circles AGB , CFD , in the points B , G , and D , F .
 Then are the intercepted arcs BG and DF equal to one another.

DEM. For the arcs $ED + DB = \text{arcs } PE + DB$; therefore $ED = PE$.

And the arcs $EF + FG = \text{arcs } PG + FG$ (19); therefore $EF = FG$.

For the points F and G are equally distant from their poles P , E .

Also the $\angle DEF = BPG$; for intersecting circles make equal angles at the sections. (31)

Therefore the triangles EFD and PGB are congruous. (27)

Therefore the arc $BG = \text{arc } DF$.

57.

PROP. XI.

If lines be drawn from the projected pole of any great circle, cutting the peripheries of the projected circle and plane of projection, the intercepted arcs of those circumferences are equal. Plate I.

On the plane of projection AGB , let the great circle CFD be projected in fd , and its pole P in p . Then the lines pd , pf , being drawn, cutting the circumference of the plane of projection in s , G , and that of the projected circle in d , f , the arc $GB = \text{arc } fd$.

DEM. For as the points D , F , are projected in d , f . (38)

The arc fd is equivalent to the arc FD .

But the arc FD is equal to the arc GB . (56)

Therefore the arc GB is equal to the arc fd . (II. 46)

58.

PROP. XII.

The radius of any small circle, whose plane is perpendicular to that of the primitive circle, is equal to the tangent of that lesser circle's distance from its pole; and the secant of that distance, is equal to the distance of the centers of the primitive and lesser circle. Plate I.

Let P be the pole, and AB the diameter of a lesser circle, the plane being perpendicular to the plane of the primitive circle, whose center is C : Then d being the center of the projected lesser circle, dA is equal to the tangent of the arc PA , and $dc = \text{secant of } PA$.

DEM. Draw the diameter ED parallel to AB , and through P draw cb . Now E being the projecting point, the diameter AB is projected in ab . (22)

And d , the middle of ab , is the center of a circle on ab . (39)

Then a right-line drawn from D through A , will meet b : (II. 130)

And draw CA' , DA .

Now the right angled triangles Dcb , DAE , having the angle D common: the $\angle Dbc = \angle DEA$. (II. 98)

But $\angle DEA = \frac{1}{2} \angle DCA$; and $\angle Dbc = \frac{1}{2} \angle Adc$: (II. 127)

Therefore $\angle DCA = \angle Adc$.

Now $\angle DCA + \angle Acd = \text{right angle}$;

Then $\angle Adc + \angle ACD = \text{right angle}$: Therefore $\angle cad$ is right. (II. 96)

Consequently $dA = \text{radius of the circle } AaB$, is the tangent of the arc PA , to the radius CA . (II. 126)

And dc , the distance of the centers, is the secant of the arc AP . (III. 5)

59. COROL. Hence the tangent and secant of any arc of the primitive circle, belongs also to an equal arc of any oblique circle; those arcs being reckoned from their intersection.

For the arc Pc of every oblique circle intercepted between P and the arc of the small circle AaB , is equivalent to the arc PA of the primitive circle: Because the arc AaB is equally distant from its pole P . (5)

SECTION

SECTION II.

Spherical Geometry

Spheric Geometry, or Spheric projection, is the art of describing, or representing, on the plane of a great circle, such circles or arcs of circles as are usually drawn upon a sphere; and of measuring such arcs and their positions to one another when projected.

60.

PROBLEM I.

To describe a great circle that shall pass through two given points in the primitive circle, or plane of projection.

Let the given points be A, B; and c the center of the prim. circle.

CASE I. *When one point A is the center of the primitive circle.*

CONST. A diameter drawn through the given points A, B, will be the great circle required. (15)



CASE 2. *When one point A is in the circumference of the primitive circle.*

CONST. Through A draw a diameter AD.

Then an oblique circle described through the three points A, B, D, (II. 72) will be the great circle required. (42)



61. CASE 3. *When neither point is at the center, or circumference of the primitive circle.*

CONST. Through one point A, and the center G, draw AG, and draw CE at right angles to AG.

A ruler by E and A gives D; by D and c gives F; and by E and F gives G, in AC continued.



Through the three points G, B, A, describe a circumference (II. 72) cutting the primitive circle in H and I.

Then the oblique circle HBAI will be the great circle required.

For AG may be taken as the projection of the great circle FD. Therefore A and G are the projections of opposite points on the sphere. (12)

Consequently, all circles passing through G and A will be the representatives of great circles on the sphere. (32)

62.

PROBLEM II.

About any given point as a pole, to describe a great circle in a given primitive circle.

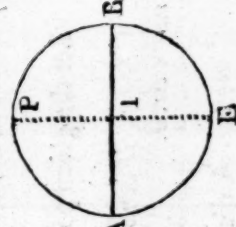
Let P be the given point, and I the center of the primitive circle.

CASE 1. When the given pole P is in the center of the primitive circle.



CONST. The primitive circle will be the great circle required. (13)

CASE 2. When the given pole P is in the circumference of the primitive circle.



CONST. Through the given pole P , draw PE a diameter to the primitive circle.

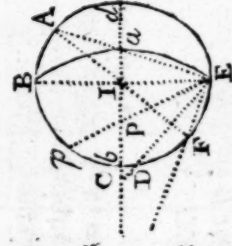
Then another diam. AB , drawn at right angles to PE , will be the great circle required. (20, 15)

63. CASE 3. When the given pole P is neither in the center or circumference of the primitive circle.

CONST. Through P draw a diameter bd , and another BE at right angles to bd ; then a ruler by E and P gives p .

Make the arc $pA = 90^\circ$; a ruler by E and A gives a in the diameter bd .

Then a circle described through the three points B , a , E , is the great circle required.



Or thus. Make the arc $pD = \text{arc } pB$; a ruler on E and D gives c in db produced.

Then on c with the radius ca describe BaE .

For, As E is the projecting point, and P the projected pole;

Therefore p is the pole of the circle aE to be projected. (44)

And BaE is the projection of the circle aE . (38)

Now $\angle caE$ is measured by half the arc aE . (II. 137)

But arc $ABD = \text{arc } aE$: For $Ap = (bd =) dE$; and $pD = Ad$ by construction.

Therefore $\angle AEC = \angle caE$; and $CE = ca$.

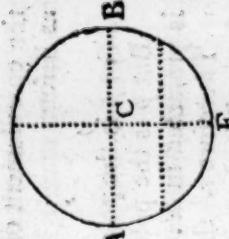
Consequently c is the center required. (II. 104)

64.

PROBLEM III.

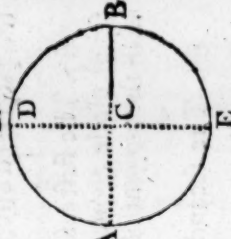
A projected circle being given; to find its poles.

CASE I. *When the given circle AEB is the primitive.*



CONST. Find the center c , (II. 70) and it is the pole sought.

CASE 2. *When the given circle ACB is a right circle.*



CONST. Draw a diameter ED at right angles to AB, and the ends or points D, E, of that diameter are the poles required.

65. CASE 3. *When the given circle ABE is oblique.*

CONST. Through the interfections of the primitive and oblique circles draw a diameter AE, and another at right angles to AE, cutting the given oblique circle in B.

A ruler by E and B gives b ; make bp , bq , each an arc of 90° .

A ruler by E and p gives, in the diameter through B, the point P, which is the pole required.

And a ruler by E and q gives, in CB continued, the point Q for the other, or opposite or exterior pole.

Make $pd = pa$; then a ruler by E and D gives, in BC continued, the point F, which is the center of the oblique circle ABE.

THE reason of this operation is evident from that of the last Problem.



66.

PROBLEM IV.

About any given projected pole, to describe a circle at a given distance from that pole.

Or, *At a proposed distance from a given great circle, to describe a parallel circle.*

Let P be the given pole, belonging to the given great circle DFE.

GENERAL SOLUTION. Through the given pole P, and C the center of the primitive circle, draw a diameter, and another DE at right angles to it.

A ruler on E and P gives p in the primitive circle.

Make pa and pB , each equal to the proposed distance from the pole.

A ruler on E and A, and then on E and B, will cut the diameter CP in a and b .

Bisect ab in c ; and on c as a center describe a circle passing through a and b , which will be the circle required.

But

But when the parallel circle is to be at a proposed distance from the given great circle DPE ,

Find p as before; and make $PA = pb$ equal to the complement of the proposed distance; the rest R as before.

For p is the pole whose projection is P . (44)

But p is the pole of a circle whose diameter AB is projected in ab . (12)

Therefore c , the middle of ab , is the center of the projected circle. (39)

67. The first case is readily done, by describing the small circle about the center of the primitive circle with the tangent of half its distance from the pole P .

68. The second case is soonest performed thus.

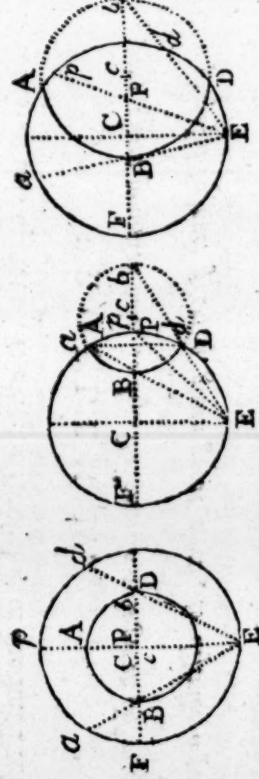
From the points A, B , (found as above) with the tangent of their distance from P , the pole of the right circle, describe arcs cutting in c , which is the center of a small circle parallel to the right circle DPE .

For Ap is the tangent of the arc AP . (58)

69.

PROBLEM V.

The primitive circle, and the projection of a small circle, being given; to find the pole of that small circle.



Let c be the center of the primitive circle, and ABD a projected small circle, whose center is c , and radius CB .

GENERAL SOLUTION. Through c the center of the small circle, and c the center of the primitive, draw a diameter CF , and another CE at right angles to it.

Find the projected diameter $Bb = 2CB$.

Lines drawn from c through B and b , cut the primitive circle in d , then bisect the arc ad in p .

A ruler by c and p cuts the diameter Bb in P , the pole sought.

THE truth of this construction is evident by that of the last Prob.

70. PROBLEM VI.

To measure any arc of a projected great circle: Or, in a given projected great circle, to take an arc of a given number of degrees.

GENERAL SOLUTION. Find the pole of the given circle. (64)
From that pole draw lines through the ends of the proposed arc, cutting the primitive circle.

Then the intercepted arc of the primitive circle applied to the scale of chords will give the measure sought.

Thus, if AB be the arc to be measured, and P the pole of the given circle DAF .

Then lines drawn from P through A and B , gives the arc ab in the primitive circle, corresponding to AB in the projected circle.

Now if an arc of a given number of degrees was to be taken from a given point A , in the given projected circle DAF .

Draw from the pole P , through A , the line Pa to the primitive circle,

Apply the given number of degrees from a to b .

Draw Pb , and the intercepted arc AB will contain the degrees proposed.

71. Any number of degrees are readily applied to a right circle by the scale of half tangents. Thus

When the distance of the point A from the center c is known, and the given quantity of the arc is to be laid from A towards F ;

To the known distance CA add the proposed arc AB , the degrees in the sum taken from the scale of half tangents, and laid from c to B , will make the arc AB equal to the degrees proposed.

But when the arc AB is to be laid from A towards D ;

Then the difference between the arcs AB and AC , taken from the scale of half tangents and laid towards D from c to B , will make the arc AB equal to the degrees proposed.

The reason of all these operations is evident from art. 57.

Note, The half, or semi-tangents, are only the tangents, of half the arcs the scale of tangents is made to; their construction depends on art. 48. On the plane scale they are put under the Tangents, and marked s. t.

72.

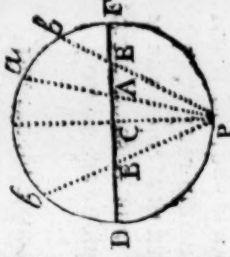
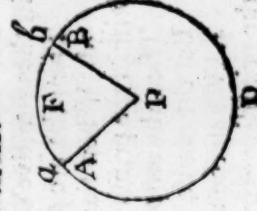
PROBLEM VII.

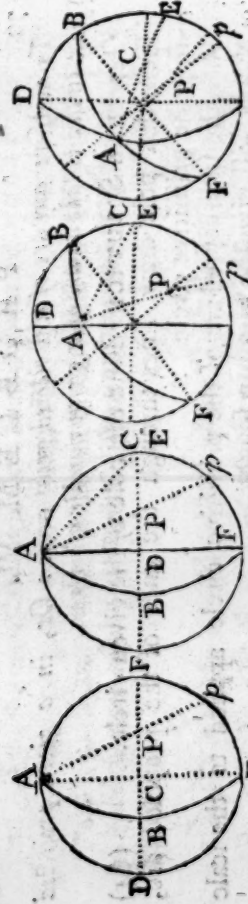
To measure any projected spherical angle.

GENERAL SOLUTION. Find the poles of the two circles which form the angle (64); and from the angular point draw lines through those poles to cut the primitive circle.

Then the measure of that angle, if acute, will be the intercepted arc of the primitive circle; or the supplement of that arc will be the measure of the angle when obtuse.

Let the proposed angle DAB , formed by the great circles AD , AB , whose poles are c and P ; and lines drawn from the angular point A , through the poles c and P , cut the primitive circle in E and F . 1st.





1st. When the angle is formed by the primitive and oblique circles.

Then the arc ρE measures the acute angle DAB .

But the obtuse angle BAF is measured by the supplement of ρE .

2d. When the angle is formed by right and oblique circles meeting in the primitive's circumference.

Then the arc ρE measures the angle DAB .

3d. When the angle is formed by right and oblique circles meeting within the primitive circle.

Then the arc ρE measures the acute angle DAB .

But the obtuse angle DAF is measured by the supplement of ρE .

4. When the angle is formed by two oblique circles meeting within the primitive circle.

Then the acute angle DAB is measured by the arc ρE .

But the supplement of ρE measures the obtuse angle DAF .

For, as the angular point A is in both circles, and 90° distant from their poles c and P (19). Therefore a great circle described about A as a pole, will pass through the poles c and P .

And lines drawn from A through c and P , cut off, in the circumference of the plane of projection, an arc equal to the distance of the poles c and P . (57)

But the measure of the distance of the poles c , P , is equal to the inclination of the planes of the circles AD , AE ;

And consequently measures the angle DAB . (33)

73. PROBLEM VIII.

Through a given point in any projected great circle, to describe another great circle at right angles to the given one.

GENERAL SOLUTION. Find the pole of the given circle. (64)

Then a great circle described through that pole and the given point will be at right angles to the given circle.

Let the given projected great circle be BAD ; and A the given point.

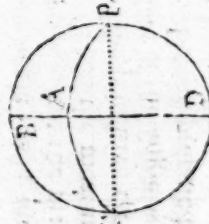
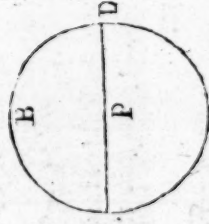
1st. When BAD is the primitive circle, whose pole is P .

A diameter through A will be perpendicular to BAD . (II. 136)

2d. When BAD is a right circle, whose poles are P and C .

An oblique circle described through the points C , A , P , (II. 72) will be at right angles to BAD .

3d. When



3d. When BAD is an oblique circle, whose pole is P .

Through the points P and A , a great circle PAC being described (61), will be at right angles to BAD .

The truth of these operations are evident from art. 34.



PROBLEM IX.

74

Through any assigned point in a given projected great circle, to describe another great circle cutting the former in an angle of a given number of degrees.

Let P be a given point in any great circle APB .

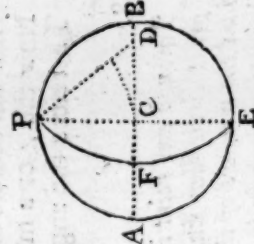
1st. When APB is the primitive circle.

Through the given point P draw a diameter PE , and draw the diameter AB at right angles to PE .

Draw PD cutting AB in D , so that the angle CPD be equal to the angle proposed.

On D with the radius DP describe the great circle APF .

Then will the angle APF contain the given degrees.



For the $\angle FPA = \text{angle made by the radii } PC, PD$.

And D being equally distant from P and E , is the center sought. (49)

75. Or thus. Make CD equal to the tangent of the given angle to the radius CP .

Or, Make PD equal to the secant of that angle.

76. 2d. When APB is a right circle.

Draw a diameter GH at right angles to APB .

Then a ruler by G and P gives a in the primitive circle.

Make $Hb = 2Aa$; a ruler by G and b gives c in AB .

Draw CD at right angles to AB .

Draw PD cutting CD in D , so that the $\angle CPD = \text{complement of the degrees given}$. (II. 84)

On D with the radius DP describe a circle FPE ; which will be a great circle making with APB the angle APF as required.



For, c is the center of a great circle GPH , by the demonstration art. 63.

And the centers of all great circles through P , will be in CD . (55)

Now $\angle DPE = 90^\circ$. (II. 136)

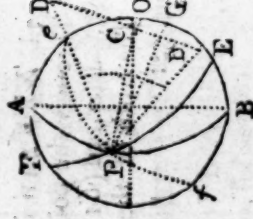
Therefore $\angle APF$ or $\angle BPE$ (26), the compl. of CPD , is the angle sought.

77. 3d. When APB is an oblique circle.

From the given point P , draw the lines PG , PC , through the centers of the primitive and given oblique circles, and through G the center of APB draw CD at right angles to PG . (II. 59)

Draw PD , making the $\angle CPD$ = given degrees, and cutting CD in D . (II. 84)

From D with the radius DP , a circle FPE being described, will be a great circle cutting APB in the angle proposed.



For C , the center of APD , is in a line perpendicular to PG drawn through P and the center of the primitive, by construction.

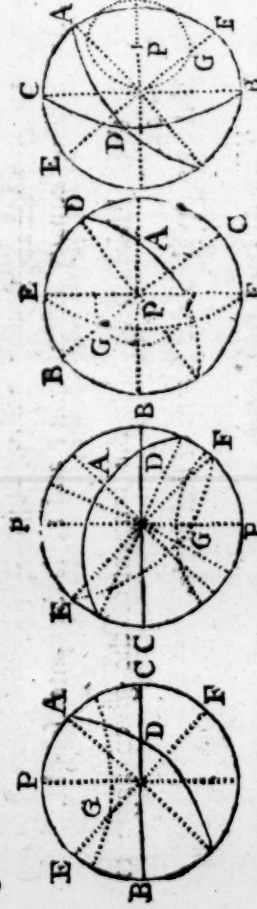
And the centers of all great circles through P will be in cn . (55)

Now the $\angle CPD$ made by the radii PC , PD , contains the given degrees. Therefore the angle APF is equal to the angle required. (49)

78.

PROBLEM X.

Through any point in the plane of projection, or primitive circle, to describe a great circle that shall cut a given great circle in any angle proposed: Provided the measure of that proposed angle is not less than the distance between the given point and circle.

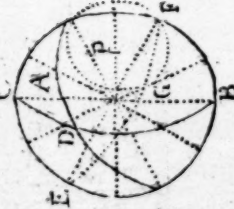


Let the given point be A , through which a circle is to be described to cut a given great circle BDC , whose pole is P , in an angle equal to a proposed E number of degrees.

GENERAL SOLUTION. About the given point (A) as a pole, describe a great circle EGF . (62)

About (P) the pole of the given circle BDC , describe a small circle at a distance equal to the given angle, cutting the great circle EGF in G . (66)

About the point G as a pole (62), describe a great circle cutting the given circle BDC in D . Then will ADG be the angle required.



Note, When the given angle is equal to the distance between the given point and circle, the problem is limited: When the measure of the angle is greater, the problem has two solutions by the circle described cutting the given one in two points: But when the measure of the angle is less, the problem is impossible. This construction is thus proved.

For P and G are the poles of BC and AD .

And

And the distance of p and g is equal to the degrees in the proposed angle, by construction. But $\angle ADC =$ distance of p and g . (33)
Therefore the $\angle ADC$ is the angle required.

79. *When the required circle is to make a given angle with the primitive.*

Then, from the center of the primitive, with the tang. of the given angle, describe an arc; and from the given point A , with the secant of the given angle, cut the former arc.

On this intersection, a circle being described through the given point A , will cut the primitive circle in the angle proposed.

This depends on art. 75.

80.

PROBLEM XI.

Any great circle, cutting the primitive, being given, to describe another great circle, which shall cut the given one in a proposed angle, and have a given arc intercepted between the primitive and given circles.

Let ABC be the primitive circle, whose center is p ; and the given great circle be ADC , whose center is E .

SOLUTION. Draw a diameter EBD at right angles to ADC ; and make the angle BDF equal to the complement of the given angle; suppose $=$ complement of 35° .

Make DF equal to the tangent of the given arc ED (suppose 58°); and from p , with the secant of that arc, describe an arc gg .

Now when ADC is an oblique circle; from E the center of ADC , with the radius EF , cut the arc gg in G .

But when ADC is a right circle; through F draw FG parallel to ADC , cutting the arc gg in G .

From G , with the tangent DF , describe an arc BD , cutting ADC in I ; and draw GI .

Through G and the center p draw OK , cutting the primitive circle in H , K ; draw PL perpendicular to OK ; and IL at right angles to IG , cutting PL in L .

And L will be the center of a circle passing through H , I , K , which will be the great circle required.

Then the $\angle AIH = 35^\circ$; and arc $IH = 58^\circ$, as was proposed.

For GP is the secant, and GI is the tang. of the arc HI . (59)

And as the triangles EGI , EFD , are congruous; the $\angle EIG = \angle EDF$. (11. 101)

But the $\angle EIG$ made by the tangent of the arc HI and the radius of arc AI , is the complement of the angle made by those arcs. (49)

Consequently the $\angle AIH$ is the complement of $\angle EDF$.

The center of the right circle AC being supposed at an infinite distance, therefore any circle FG described from that center, will be parallel to AC .

When

When the given arc is more than 90° , the tangent and secant of its supplement is to be applied on the line DF the contrary way, or towards the right; the former construction being reckoned to the left.

81.

PROBLEM XII.

Any great circle in the plane of projection being given; to describe another great circle, which shall make given angles with the primitive and given circles.

Let the given great circle be ADC , whose pole is Q .

SOLUTION. About P the pole of the primitive circle, describe an arc mn , at the distance of as many degrees, as are in the angle which the required circle is to make with the primitive. Suppose 62° . (67) C

About Q the pole of the other given circle, and at a distance equal to the measure of the angle which the required circle is to make with the given circle ADC (suppose 48°), describe an arc mn , cutting mn in n . (66)

About n , as a pole, describe the great circle EDF , cutting the given circles in E and D . (62)

Then is the angle $AED = 62^\circ$; and $ADE = 48^\circ$.

For the distance of the poles of any two great circles, is equal to the angle which those circles make with one another. (33).

REMARK. The 11th Problem, which is particularly useful in constructing a spherical triangle, wherein are given two angles and a side opposite to one of them, includes only two cases of a more general Problem, viz.

Any two great circles being given in position; to describe a third, which shall cut one of those given in an angle proposed, and have a given arc intercepted between the given circles.

Also the 12th Problem, used when the three angles are given, contains only two cases of another Problem; viz.

Any two great circles being given in position, to describe a third which shall cut the given circles in given angles.

The solution of these two general Problems not being wanted in any part of this work, it was not thought necessary here to annex them; more having been already delivered in the preceding pages than it is usual to meet with. However, their solution is recommended as exercises to speculative learners.

SECTION IV.

Spherical Trigonometry.

DEFINITIONS.

82. SPHERIC TRIGONOMETRY is the art of computing the measures of the sides and angles of such triangles as are formed on the surface of a sphere, by the mutual interfections of three great circles described thereon.

83. A SPHERIC TRIANGLE consists of three sides and three angles.

The measures of unknown sides or angles of spheric triangles, are estimated by the relations between the sines, or the tangents, or the secants, of the sides or angles known, and of those that are unknown.

84. A RIGHT ANGLED SPHERIC TRIANGLE has one right angle: The sides about the right angle are called Legs; and the side opposite to the right angle is called the Hypothenufe.

85. A QUADRANTAL SPHERIC TRIANGLE has one side equal to ninety degrees.

86. An OBLIQUE SPHERIC TRIANGLE has all its angles oblique.

87. The CIRCULAR PARTS of a triangle, are the arcs which measure its sides and angles.

88. Two spheric triangles are said to be supplements to one another, when the sides and angles of the one are respective supplements of the angles and sides of the other: And one, in regard to the other, is called the supplemental triangle.

89. Two arcs or angles, when compared together, are said to be alike, or of the same kind, when both are acute, or less than 90° , or when both are obtuse, or greater than 90° : But when one is greater and the other less than 90° , they are said to be unlike.

The lesser circles of the sphere do not enter into Trigonometrical computations, because of the diversity of their radii.

SECTION V.

Spherical Theorems.

90.

THEOREM I.

In every spheric triangle (ABC), equal angles (B, C) are opposite to equal sides (AC, AB): And equal sides (AB, AC) are opposite to equal angles (C, B).
 DEM. Since $AB = AC$, make $AE = AD$; and draw BD , CE .

Then is $BD = CE$; and $\angle AEC = \angle ADB$. (27)

For the triangles AEC , ADB are congruous,

Since $AB = AC$; $AD = AE$; $\angle A$ common.

Also, the triangles BEC , CDB are congruous, (27)

Therefore $\angle EBC = \angle DCB$.

For $EC = BD$ $EB = (AB - AE =) DC (= AC - AD)$.

And $\angle BEC = \angle CDB$, they being the suppl of equal angles AEC , ADB . (II. 48)

Again, if $\angle ABC = \angle ACB$: Then is $AB = AC$.

For take $BE = CD$; and describe the arcs CE , BD .

Then is $EC = DB$, $\angle BEC = \angle CDB$; $\angle BCE = \angle CBD$. (27)

For $\Delta^* BCE = \Delta CBD$; since BC is common, $BE = CD$, $\angle BEC = \angle DCB$.

Also the triangles ABD , ACE are congruous,

Since $EC = DB$, $\angle ACE = (\angle BCA - BCE =) \angle ABD (= \angle CBA - CBD)$. (II. 48)

And $\angle AEC = (\text{sup. } \angle BEC =) \angle ADB (= \text{sup. } \angle CDB)$. (II. 48)

Therefore $AE = AD$; and $AB = (AE + EB =) AC (= AD + DC)$. (II. 47)

91. COROL. A line drawn from the vertex of an isosceles spheric triangle, to the middle of the base, is perpendicular to the base.

This is easily proved from art. 90, 27.

92.

THEOREM II.

Either side of a spheric triangle is less than the sum of the other two sides.

DEM. For on the surface of a sphere, the shortest distance between two points, is an arc of a great circle passing through those points. (23)

But each side of a spheric triangle is an arc of a great circle. (10)

Therefore either side being the shortest distance between its extremities, is less than the sum of the other two sides.

93.

THEOREM III.

Each side of a spheric triangle is less than a semicircle, or 180 degrees.

DEM. Two great circles intersect each other twice at the distance of 180 degrees. (32)

The sides about any spheric angle are arcs of two great circles. (10)

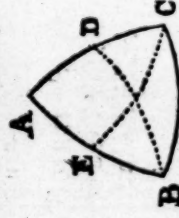
But a spheric triangle has three sides.

Therefore every two sides before their second meeting must be intersected by the third side.

Consequently each side is less than a semicircle.

* The mark Δ stands for the word triangle.

94. THEO-



94. THEOREM IV.

In every spheric triangle (ABC) the greatest side (BC) is opposite to the greatest angle (A).

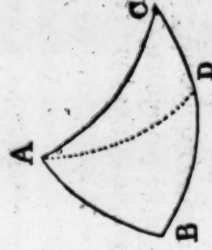
DEM. Make $\angle BAD = \angle ABC$.

Then $AD = BD$ (90); and $BC = AD + DC$.

But $AD + DC$ is greater than AC .

Therefore BC is greater than AC .

(92)



95. THEOREM V.

If from the three angles of a spheric triangle (ABC), as poles, be described three arcs of great circles, forming another spheric triangle (EDF); then will the sides of the latter, and the opposite angles of the former, be the supplements of one another: Also the angles in the latter, and their opposite sides in the former, are the supplements of one another.

That is, FE and $\angle CAB$, FD and $\angle ABC$, DE and $\angle ACB$, are supplements to one another.

Also $\angle E$ and AC , $\angle D$ and CB , $\angle F$ and AB , are the supplements to one another.

DEM. Now E , the intersection of the arcs about the poles A and C , being 90° distant from them, is the pole of the arc AC . (19)

And for the same reason, D is the pole of CB , and F of AB .

Let the sides of the triangle ABC be produced to meet the sides of the triangle DEF , in G and H , I and L , M and N .

Then $FI = DL = 90^\circ$: Therefore $(DL + FI = DL + FL + LI = DF + LI = 180^\circ)$. (11. 47)

Therefore DF and LI are supplements to one another.

But LI measures the angle ABC . (9)

Therefore $\angle ABC$ and DF are the supplements of one another.

And in the same manner it may be demonstrated, that the $\angle BAC$ and FE , $\angle ACB$ and DE , are the supplements of one another.

Again, since $BI = AH = 90$ degrees; (19)

Therefore $(IB + AH = IB + BH + AB =) IH + AB = 180$ degrees.

But IH measures the angle F ; (9)

Therefore AB and $\angle F$ are the supplements of one another.

And the same may be shewn of AC and $\angle E$, CB and $\angle D$.

96. THEOREM VI.

The sum of the three sides of every spheric triangle (ABC) is less than a circumference, or 360 degrees.

DEM. Continue the sides AC , AB , till they meet in D .

Then the arcs ACD , ABD , are each 180° . (32)

But $DC + DB$ is greater than BC . (92)

Therefore $AC + AB + DC + DB$ is greater than $AC + AB + BC$.

Or the femicircles $ACD + ABD$ is greater than $AC + AB + BC$.

That is, 360° is greater than the three sides of the triangle ABC .



97.

THEOREM VII.

The sum of the three angles of a spheric triangle (ABC) is greater than two right angles, and less than six; or will always fall between 180 and 540 degrees.

DEM. Since $\angle A$ and FE , $\angle B$ and FD , $\angle C$ and DE , are supplements to one another. (95)

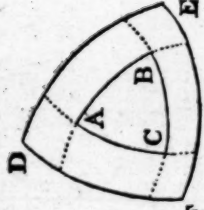
Therefore the three angles A, B, C , together with the three sides FE, FD, DE , makethrice 180° , or 540° .

Now the sum of the three sides $FE + FD + DE$, is less than twice 180° . (96)

Therefore the sum of the three angles $A + B + C$ is greater than 180° .

Again, As a spheric angle is ever less than 180° ;

Therefore the sum of any three spheric angles is ever less than thrice 180° , or 540 degrees. (24)



98.

THEOREM VIII.

If one side (AB) of a spheric triangle (ABC) be produced, then the outward angle (CBD) is either equal to, less or greater than, the inward opposite angle (A) adjacent to that side; according as the sum of the other two sides ($CA + CB$) is equal to, greater or less than, 180 degrees.

DEM. Produce AC, AB , to meet in D .

Then arc $ACD = \text{arc } AED = 180^\circ$. (32)

And $\angle D = \angle A$. (31)

Now if $AC + CB$ is equal to 180° ; then $CB = CD$.

And $\angle CBD = (\angle D =) \angle A$. (90)

If $AC + CB$ is greater than 180° ; then CB is greater than CD .

And $\angle CBD$ is less than $(\angle D =) \angle A$. (94)

If $AC + CB$ is less than 180° ; then CB is less than CD . (94)

And $\angle CBD$ is greater than $(\angle D =) \angle A$.



99.

THEOREM IX.

In right angled spheric triangles, the oblique angles and their opposite sides are of the same kind: That is, if a leg is less or greater than 90° , its opposite angle is also less or greater than 90° .

In the right angled spheric triangle ABC , right angled at A ,

If AC is greater than 90° ; then $\angle ABC$ is greater than 90° .

If AC is less than 90° ; then $\angle ABC$ is less than 90° .

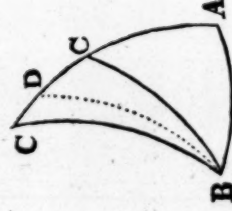
DEM. Let the leg AC be less, AD equal, AC greater, than 90° , and describe the arc DB .

Now D being the pole of AB (37). Therefore $\angle DBA$ is right.

Consequently if AC is less than AD , the $\angle CBA$ is less than $\angle DBA$.

But if AC is greater than AD , the $\angle CBA$ is greater than $\angle DBA$.

And the same may be proved of the leg AB and its opposite angle.



THEOREM. X.

100.

In right angled spheric triangles (BAC), the hypothenuse (BC) is less than 90° , when the legs (AB, AC) are of a like kind: But the hypothenuse is greater than 90° , when the legs are of different kinds.

1st. When the legs AB, AC, are both less than 90° .

DEM. In BA, AC produced, take BD, AF, equal to quadrants; through F and D describe an arc FD meeting BC produced in E.

Now F being the pole of BD (19). Therefore B is the pole of ED.

Consequently BC is less than $(BE =) 90^\circ$.

2^d. When the legs AB, AC, are both greater than 90° .

Produce AC, AB, till they meet in D.

Now the hypothenuse CB is common to both the right angled triangles BAC and BDC.

And the legs DC, DB, being both less than 90° .

Therefore the hypothenuse BC is less than 90° , by the 1st case of this Theorem.

3^d. When the legs AB, AC, are one greater, the other less than 90° .

In AB, and AC produced, take BD, AF each of 90° , and describe the arc FED.

Then B is the pole of FD; and since F is the pole of BA, and FD is at right angles to BD. Therefore $BE = 90^\circ$.

Consequently BC is greater than 90° degrees. (37)

101. COROL. I. The hypothenuse is less or greater than 90° , according to the oblique angles are of a like, or of different kinds.

For if legs are like, or unlike, the angles are like or unlike. (99)

And if legs are like, or unlike, the hypothenuse is acute, or obtuse. (100)

Therefore if the angles are like, the hypothenuse is acute, or less than 90° ; but if unlike, the hypothenuse is obtuse, or greater than 90° .

102. COROL. II. The legs and their adjacent angles are like, or unlike, as the hypothenuse is less, or greater than 90° degrees.

For like legs, or like angles, make the hypothenuse acute (by 1st and 2^d of 100).

And unlike legs, or unlike angles, make the hypothenuse obtuse (by 3^d of 100 and 101).

103. COROL. III. A leg and its opposite angle are both acute, or both obtuse, according as the hypothenuse and other leg are like, or unlike.

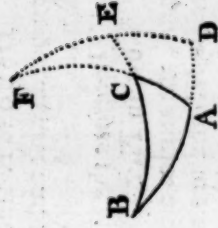
This is evident from the three cases of this Theorem.

104. COROL. IV. Either angle is acute, or obtuse, as the hypothenuse and the other angle are like, or unlike.

This follows from case 1st and 2^d of this Theorem.

L 3

105. THEO-



105.

THEOREM. XI.

In every spheric triangle (ABC), if the angles adjacent to either side (AB) be alike, then a perpendicular (CD) drawn to that side from the other angle, will fall within the triangle: But the perpendicular (CD) falls without the triangle, when the angles adjacent to the side (AB) it falls on are unlike.

DEMONST. Since in all right angled triangles the perpendicular and its opposite angle are of the same kind. (99)

Therefore the $\angle CAD$ and CBD , are each like CD .

Now in Fig. 1. the angles CAD , CBD , or CAB , CBA , are angles adjacent to the base AB within the triangle, and are therefore alike.

Therefore the perpendicular falling between A and B , falls within the triangle.

In Fig. 2. the angles CAD and CAB are the supplements of each other, and are therefore unlike, as CA falls obliquely on AB .

Therefore $\angle CAB$ is unlike to $\angle CBA$.

Consequently the perpendicular CD cannot fall between A and B : Therefore it must fall without.

106.

THEOREM XII.

If the two lesser sides (CA, CB) of a spheric triangle (ABC) are of the same kind, then an arc (CD) drawn from their included angle (ACB) perpendicular to the opposite side (AB), will fall within the triangle.

DEMONST. In AB take $AF = AC$; draw CF , and AH at right angles to CF .

Then $CH = HF$ (91) are each less than 90° . (93)

Also take $BE = BC$; draw CE and BG at right angles to CE .

Then $CG = GE$ (91) are each less than 90° . (93)

Now in the right angled triangles FHA , EGB ; if the hypothenuses $AF (= AC)$, and $BE (= BC)$, are acute, or like FH and EG ;

Then the angles $A FH$ and BEG are acute, and like AC and BC ; (103)

Therefore the perpendicular CD falls on EF , within the triangle. (105)

Also if the hypothenuses AF and BE are obtuse, or unlike to FH and GE ;

Then the angles $A FH$ and BEG are obtuse, and also like CA and CB . (103)

Consequently the perpendicular will fall on EF . (105)

Therefore in either case the perpendicular falls within the triangle.

107.

THEOREM XIII.

In all right angled spheric triangles,

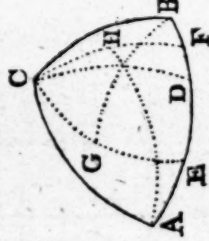
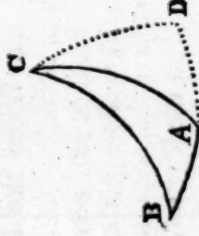
As sine hypoth. : Rad. :: sine of a leg : sine of its opposite angle.

108. *And sine a leg : Rad. :: tan. other leg : tan. of its opposite angle.* Pl. I.

DEMONST. Let $EDAFG$ represent the eighth part of a sphere, where the quadrantal planes $EDFG$, $EDBC$, are both perpendicular to the quadrantal plane $ADFB$; and the quadrantal plane $ADGC$ is perpendicular to the quadrantal plane $EDFG$; and the spheric triangle ABC is right angled at B , where CA is the hypothenuse, and BA , BC , are the legs.

To the arcs GF , CB , draw the tangents HF , OB , and the sines GM , CI , on the radii DF , DB ; also draw BL the sine of the arc AB , and CK the sine of AC ; then join IK and OL .

Now



Now HF, OB, GM, CI, are all perpendicular to the plane ADFB. And HD, CK, OL, lie all in the same plane ADGC.

Also FD, IK, BL, lie all in the same plane ADFB.

Therefore the right angled triangles HFD, CIK, OBL, having the equal angles HDF, CKI, OLB, (II. 199) are similar. (II. 167)

Therefore $CK : DG :: CI : GM$.

That is, As fin. hyp. : Rad :: fin. of a leg : fin. opp. angle.

For GM is the sine of the arc GF, which measures the angle CAB. (9)

Also, As LB : DF :: BO : FH.

That is, As sin. of a leg : Rad :: tan. of other leg : tan. opp. angle.

109.

THEOREM XIV.

In right angled spheric triangles (ABC) if about the oblique angles (A, C) as poles, at 90° distance, there be described arcs (DE, FE) cutting one another (in E); and the sides (AB, AC, BC) of the triangle be produced to cut those arcs (in D; G, F; H, I), there will be constituted two other triangles (CGH, HIE), whose parts are either equal to, or are the complements of the parts of the given triangle (ABC). PI. I.

DEMONST. Now since A is the pole of ED (19). Therefore AD, AG are at right angles to ED; and so is ED to AD. (37)

And since BI and DE are at right angles to AD, their intersection H is the pole of AD (36). Therefore HB, HD are quadrants. (35)

Then in the triangle CGH, right angled at G.

$CG =$ complement of AC.

$HG =$ comp. $\angle A$; For HG is the comp. of GD, which (9) mea. $\angle A$.

HC the hypoth. is the comp. of CB.

The $\angle HCG = \angle ACB$. (26)

The $\angle CHG =$ comp. $\angle C$; For BD, the comp. of AB, measures $\angle CHG$.

Also in the triangle EIH, right angled at I: Because CF, CI are at right angles to EF; and EF, EG being also at right angles to AF; therefore E is the pole of AF; (36) consequently EF and EG are quadrants. (35)

Then the hypoth. $EH = \angle A$; For $GH =$ comp. of EH and GD; and GD measures the angle A.

$HI = CB$; For $HC =$ comp. of HI and CB.

$EI =$ comp. $\angle C$; For $EI =$ comp. of IF, which measures $\angle C$.

The $\angle H =$ comp. AB; For BD, the comp. of AB, measures $\angle H$.

The $\angle E = A$; For GF, the measure of $\angle E$, is equal to AC.

110.

THEOREM XV.

In every spheric triangle, it will be,

As the sine of either angle, is to the sine of its opposite side;

So is the sine of another angle, to the sine of its opposite side.

Let ABC be a spheric triangle, wherein BD is perpendicular to AC produced; forming the two right angled triangles ADB, CDB.

DEM. Now sin. AB : rad :: sin. BD : sin. $\angle A$. (107)

And sin. BC : rad :: sin. BD : sin. $\angle C$.

Therefore sin. AB $\times$ sin. $\angle A =$ rad $\times$ sin. BD.

And sin. BC $\times$ sin. $\angle C =$ rad $\times$ sin. BD.

Therefore sin. AB $\times$ sin. $\angle A =$ sin BC $\times$ sin. $\angle C$.

Therefore sin. $\angle A : \sin. BC :: \sin. \angle C : \sin. AB$.

L 4



(II. 162)

(II. 162)

(II. 46)

(II. 162)

SECTION

SECTION VI.

Of the Solution of right angled Spheric Triangles.

In every case of right angled spheric triangles, three things beside the radius enter the proportion, whereof two are given, and the third is sought.

Now the solution of every case will be obtained by the application of the two following rules to Theorem XIII and XIV. (107, 108, 109)

III. RULE I. If of the three things concerned, or their complements, two are opposite to one another, and the third is opposite to the right angle, in one of the triangles marked 1, 2, 3, in the fig. to Theo. XIV. Pl. I. Then the thing sought will be found by the first proportion (107) either directly, or by inversion.

III. RULE II. If of the three things concerned, or their complements, two are sides and the third is an oblique angle, in either of the three triangles marked 1, 2, 3, in fig. to Theo. XIV. Pl. I. Then the thing sought will be found by the second proportion (108, either directly, or by inversion.

III.

PROBLEM I.

In the right angled spheric triangle ABC. Plate I. Theorem XIV.

Given the hypothenuse AC } required the rest.
and one of the legs AB }

1st. To find the angle ACB opposite the given leg AB.

Here the things concerned are AC, $\angle B$, AB, $\angle C$; which are found in the triangle, N^o 1, to be opposite; and so fall under Rule I. (111)

Then $\sin. AC : \text{rad} :: \sin. AB : \sin. \angle ACB$. (107)

Or $\sin. \text{hyp.} : \text{rad} :: \sin. g. \text{leg} : \sin. \text{op. } \angle$. Like the g. leg. (99)

2d. To find the angle CAB adjacent to the given leg AB.

Here the things concerned are AC, $\angle B$, AB, $\angle A$.

Now trying in the triangle, N^o 1, I find the things concerned will fall under neither of the Rules.

But trying in the triangle, N^o 2, the things concerned, or their complements, fall under Rule II. (112)

Then $\sin. HG : \text{rad} :: \tan. GC : \tan. \angle CHG$. (108)

Or $\cos. \angle CAB : \text{rad} :: \cot. AC : \cot. AB$.

Or $\cos. \angle CAB : \cot. AC :: (\text{rad} : \cot. AB) : \tan. AB : \text{rad}$. (III. 36)

Therefore $\text{rad} : \cot. \text{hyp.} :: \tan. g. \text{leg} : \cos. \text{adj. angle}$. (II. 145)

Like, or unlike the given leg; as the hyp. is acute, or obtuse. (102)

3d. To find the other leg BC.

Here the things concerned are AC, $\angle B$, AB, BC; which in the triangle, N^o 1, do not fall under either Rule: But in N^o 2 they will be found to fall under the first Rule. (111)

Then $\sin. HC : \text{rad} :: \sin. CG : \sin. \angle CHG$. (107)

Or $\cos. CB : \text{rad} :: \cos. AC : \cos. AB$.

Therefore $\cos. g. \text{leg} (AB) : \text{rad} :: \cos. \text{hyp.} (AC) : \cos. \text{req. leg} (CB)$.

And is acute, if hyp. and given leg are like; but obtuse, if unlike. (103)

114. PRO.

114.

PROBLEM II.

In the right angled Spheric triangle ABC. Pl. I. Theorem XIV.

Given the hypothenuse AC

And one of the oblique angles A } Required the rest.

1st. *To find the leg CB opposite to the given angle A.*In the triangle, N^o 1, the things concerned fall under Rule I. (111)Then $\text{rad} : \sin. AC :: \sin. \angle CAB : \sin. CB.$ (107)Or $\text{rad} : \sin. \text{hyp.} :: \sin. \text{given angle} : \sin. \text{opp. side.}$

And is like the given angle. (99)

2d. *To find the leg AB adjacent to the given angle A.*In the triangles, N^o 2, the things concerned fall under Rule II. (112)Then $\sin. HG : \text{rad} :: \tan. EG : \tan. \angle CHG.$ (108)Or $\text{cof.} \angle BAC : \text{rad} :: (\text{cof.} AC : \text{cot. } AB ::) \tan. AB : \tan. AC.$ (III. 37)Therefore $\text{rad} : \tan. AC :: \text{cof.} \angle BAC : \tan. AB.$ Or $\text{rad} : \tan. \text{hyp.} :: \text{cof. given angle} : \tan. \text{adjacent leg.}$

And is acute, if hyp. and given angle are alike; but obtuse, if unlike. (104)

3d. *To find the other angle ACB.*In the triangle, N^o 2, the things concerned fall under Rule II. (112)Then $\sin. CG : \text{rad} :: \tan. GH : \tan. \angle HCG.$ (108)Or $\text{cof.} AC : \text{rad} :: (\text{cot.} \angle BAC : \tan. \angle BCA ::) \cot. \angle BCA : \tan. \angle BAC.$ (III. 37)Therefore $\text{rad} : \tan. \angle BAC :: \text{cof.} AC : \cot. \angle BCA.$ (II. 145)Or $\text{rad} : \text{cof. hyp.} :: \tan. \text{given angle} : \cot. \text{req. angle.}$

And is acute, if hyp. and given angle are alike; but obtuse, if unlike. (104)

115.

PROBLEM III.

In the right angled Spheric triangle ABC. Plate I. Theorem XIV.

Given one of the legs AB

And its opposite angle ACB } Required the rest.

1st. *To find the hypothenuse AC.*In the triangle, N^o 1, the things concerned fall under Rule I. (111)Then $\sin. \angle ACB : \sin. AB :: \text{rad} : \sin. AC.$ (107)Or $\sin. \text{given angle} : \sin. \text{given leg} :: \text{rad} : \sin. \text{hyp.}$

And is either acute or obtuse.

2d. *To find the other leg CB.*In the triangle, N^o 1, the things concerned fall under Rule II. (112)Then $\sin. CB : (\text{rad} ::) \tan. AB (: \tan. \angle ACB) :: \cot. \angle ACB : \text{rad.}$ (III. 36)Or $\text{rad} : \cot. \text{given angle} :: \tan. \text{given leg} : \sin. \text{req. leg.}$

And is either acute or obtuse.

3d. *To find the other angle CAB.*In the triangle, N^o 3, the things concerned fall under Rule I. (111)Then $\sin. EH : \text{rad} :: \sin. EF : \sin. \angle IHE.$ (107)Or $\sin. \angle BAC : \text{rad} :: \text{cof.} \angle ACB : \text{cof. } AB.$ Or $\text{cof. given leg} : \text{cof. given angle} :: \text{rad} : \sin. \text{required angle.}$

And is either acute or obtuse.

116.

PROBLEM IV.

In the right angled spheric triangle ABC. Plate I. Theorem XIV.

Given one of the legs AB } Required the rest.
And its adjacent angle BAC }

1st. To find the other angle BCA.

In the triangle N° 3, the things concerned fall under Rule I. (111)

Then rad : sin. EH : : sin. $\angle$ EHI : sin. EI. (107)

Or rad : sin. $\angle$ BAC : : cof. AB : cof. $\angle$ ACB.

Therefore rad : cof. given leg : : sin. given angle : cof. req. angle.

And is like the given leg. (99)

2d. To find the other leg BC.

In the triangle, N° 1, the things concerned fall under Rule II. (112)

Then sin. AB : rad : : tan. BC : tan. $\angle$ CAB. (108)

Or rad : sin. AB : : tan. $\angle$ CAB : tan. BC.

Therefore rad : sin. given leg : : tan. given angle : tan. req. leg.

And is like the given angle. (99)

3d. To find the hypotenuse AC.

In the triangle, N° 2, the things concerned fall under Rule II. (112)

Then sin. GH : rad : : tan. CG : tan. $\angle$ CHG. (108)

Or cof. $\angle$ CAB : rad : : cot. AC : cot. AB.

Therefore rad : cof. given angle : : cot. given leg : cot. hypotenuse.

And is acute, if the given leg and angle are alike ; but obtuse, if unlike.

(102)

117.

PROBLEM V.

In the right angled spheric triangle ABC. Plate I. Theorem XIV.

Given both the legs AB, BC.

Required the rest.

1st. To find either of the oblique angles, as BAC.

In the triangle, N° 1, the things concerned fall under Rule II. (112)

Then, As sin. AB : rad : : tan. BC : tan. $\angle$ BAC. (108)

Or rad : sin. AB : : (tan. $\angle$ BAC : tan. BC :) cot. BC : cot. $\angle$ BAC. (III 37)

Therefore rad : sin. one leg : : cot. oth. leg : cot. opp. angle.

And is like its opposite leg. (99)

2d. To find the hypotenuse AC.

In the triangle, N° 2, the things concerned fall under Rule I. (111)

Then, As rad : sin. HC : : sin. $\angle$ CHG : sin. CG. (107)

Or rad : cof. BC : : cof. AB : cof. AC.

Therefore rad : cof. one leg : : cof. oth. leg : cof. hypotenuse.

And is acute, if the legs are alike, but obtuse, if unlike. (100)

118.

PROBLEM VI.

In the right angled spheric triangle ABC. Plate I. Theorem XIV.

Given both the angles BAC, BCA.

Required the rest.

1st. To find either of the legs, as BC.

In the triangle, N^o 2 or 3, the things concerned fall under Rule I. (111)Then, $\text{rad} : \sin. HC :: \sin. \angle HCG : \sin. HG.$ (107)Or $\text{rad} : \cos. BC :: \sin. \angle ACB : \cos. \angle BAC.$ Therefore sine of one angle : $\text{rad} :: \cos. \text{oth. angle} : \cos. \text{opposite side.}$

And is like its opposite angle. (99)

2d. To find the hypotenuse AC.

In the triangle, N^o 2, the things concerned fall under Rule II. (112)Then, As $\sin. CG : \text{rad} :: \tan. GH : \tan. \angle HCG.$ (108)Or $\cos. AC : (\text{rad} ::) \cot. \angle EAC (: \tan \angle BCA) :: \cot. \angle BCA : \text{rad.}$ (111. 36)Therefore $\text{rad} : \cot. \text{one angle} :: \cot. \text{oth. angle} : \cos. \text{hypotenuse.}$

And is acute, if the angles are like. (101)

But obtuse, if unlike.

In these six Problems are contained sixteen proportions, which are applicable to the like number of cases usually given to right angled spheric triangles; and these proportions being collected and disposed in a Table, will readily shew, by inspection, how any of the cases are to be solved.

The celebrated Lord NEPIER, the inventor of logarithms, contrived a general rule, easy to be remembered, whereby the solution of every case in right angled spheric triangles is readily obtained, where the table of proportions is wanting; which rule is as follows.

GENERAL RULE.

119. Radius multiplied by the sine of the middle part, is either equal to the product of the tangents of extremes conjunct.

Or to the product of the cosines of extremes disjunct.

Observing ever to use the complements of the hypoth. and angles.

Lord Nepier called the five parts of every right angled spheric triangle, omitting the right angle, circular parts; which he thus distinguished; the two legs, the complements of the two angles, and the complement of the hypotenuse; and any two of these circular parts being given, the others are to be found by this rule, as is shewn in what follows.

Now, In all the proportions about right angled spheric triangles, there are, besides the radius, three things concerned; one whereof may be called the middle term in respect of the other two; and these two, in respect to the middle term, may be called extremes.

When the two extremes are joined to the middle, they are called extremes conjunct: But when each of them are disjoined from the middle by an intermediate term (not concerned), they are then called extremes disjunct; taking notice that the right angle does not disjoin the legs.

If the three parts under consideration do all join, the middle one of those three is readily seen, and the other two are extremes conjunct.

But if only two of the three parts are joined, those two are extremes disjunct, and the other term is the middle part.

These

These things duly observed, the practice of the Rule will appear in the following examples.

EXAMPLE I. *When the hypothenuse and the angles are concerned.*

The hypoth. is the middle term, and the two angles are extremes conjunct; then by the rule,

$\text{Rad} \times \sin. \text{hyp.} = \tan. \text{one angle} \times \tan. \text{other angle.}$

But the comp. of the hypoth. and angles are always to be used.

Therefore $\text{rad} \times \text{cof. hyp.} = \cot. \text{one angle} \times \cot. \text{other angle.}$

Hence $\text{rad} : \cot. \text{one angle} :: \cot. \text{other angle} : \text{cof. hypoth. (II. 163)}$

From whence are deduced the 6th and 15th cases.

EXAM. II. *When the hypothenuse and legs are under consideration.*

The hypothenuse is the middle term, and the two legs are extremes disjunct, having the angles between them and the hypothenuse.

Then by the rule. $\text{Rad} \times \sin. \text{hyp.} = \text{cof. one leg} \times \text{cof. other leg.}$

But the complement of the hypothenuse is to be used.

Therefore $\text{rad} \times \text{cof. hypoth.} = \text{cof. one leg} \times \text{cof. other leg.}$

Hence $\text{rad} : \text{cof. one leg} :: \text{cof. other leg} : \text{cof. hypoth. (II. 163)}$

From whence are deduced the 3d and 13th cases.

EXAM. III. *The legs and an angle under consideration.*

Here the angle and its opposite leg are extremes conjunct; and the other leg is the middle part.

And these being resolved into a proportion by the rule, will produce the 8th, 11th, and 14th cases.

EXAM. IV. *The angles and a leg under consideration.*

Here one angle is the middle, and the other angle and leg are extremes disjunct. the hypothenuse and other leg intervening.

Now these being resolved into a proportion, gives the 9th, 12th, and 16th cases.

EXAM. V. *The hypothenuse, a leg, and the angle between them, being under consideration.*

Here the angle is the middle term, and the hypothenuse and leg are extremes conjunct.

And these being resolved into a proportion, will give the 2d, 4th, and 10th cases.

EXAM. VI. *The hypothenuse, a leg, and its opposite angle, being under consideration.*

Here the leg is the middle term, and the hypothenuse and angle are extremes disjunct, the other leg and other angle falling between them and the middle.

And these being converted into a proportion, from thence the 1st, 5th, and 7th cases are deduced.

SECTION

SECTION VII.

Of the Solution of oblique angled Spheric Triangles.

120. All the cases in oblique angled spheric triangles, except where the three sides, or the three angles are given, are most conveniently resolved by drawing a perpendicular from one of the angles to its opposite side, continued if necessary; which perpendicular will either divide the given triangle into two right angled triangles, or make two that are right angled, by joining a right angled one to the given triangle.

In drawing of this perpendicular, observe,

1st. It must be drawn from the end of a given side, and opposite to a given angle.

2^d. It must be so drawn, that two of the given things in the oblique triangle may remain known in one of the right angled triangles.

3^d. This perpendicular is to be used as a known quantity; and being drawn as here directed, will either fall within or without the triangle, as the angles, next the side on which it falls, are of the same or of different kinds. (105)

121.

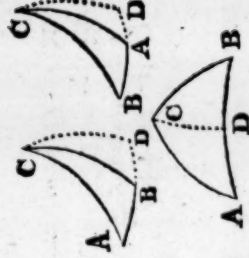
PROBLEM I.

In the oblique angled spheric triangle ABC.

Given two sides

CA, CB

And the angle opp. to one, $\angle CAB$ } *Requir. the rest.* A



1st. To find the angle opposite to the other given side ($\angle CBA$).

As $\sin. BC : \sin \angle CAB :: \sin. AC : \sin \angle CBA$. (110) A

Or, As $\sin.$ one side : $\sin.$ opposite angle :: $\sin.$ other side : $\sin.$ opposite angle. Which may be either acute or obtuse.

2^d. To find the angle between the given sides ($\angle ACB$).

Now $\text{rad} : \tan. \angle CAB :: \cos. AC : \cot. (ACD, \text{ call it } m).$ (3d 114)

Or $\text{rad} : \tan.$ given $\angle :: \cos. \text{adj. side} : \cot. (\text{of a fourth} =) m.$

And is acute, if AC and $\angle CAB$ are like; but obtuse, if unlike.

But $\text{rad} : \tan. CD :: \cot. AC : \cot. (ACD = m).$ (2d 113)

$\text{rad} : \tan. CD :: \cot. CB : \cot. (BCD, \text{ call it } n)$

Therefore $\cot. AC : \cot. CB :: \cot. m. : \cot. n.$ (II. 155)

Or $\cot. \text{side adj. given } \angle : \cot. \text{other side} :: \cos. m. : \cos. n.$

And is like the side opposite the given angle, if that angle is acute;

But unlike that side, if the given angle is obtuse.

Then the angle sought, viz. $\angle ACB = \begin{cases} \text{sum of } m \text{ and } n, \text{ if } \angle^* \text{ falls within.} \\ \text{diff. of } m \text{ and } n, \text{ if } \angle \text{ falls without.} \end{cases}$

* The mark $\angle$ signifies the perpendicular.

3d. To find the other side AB.

Now Rad : cof. $\angle CAB :: \tan. AC : \tan. (AD, \text{ call it } M). \quad (2d \text{ 114})$

Or rad : cof. given angle :: tan. adj. side : tan. (of a fourth =) M.

Acute, if the angle and its adj. side are alike; but obtuse, if unlike.

But cof. CD : Rad :: cof. AC : cof. (AD =) M.

Therefore cof. AC : cof. CB : cof. (DB call it) N. (3d 113)

(11. 155)

Or cof. side adj. given angle : cof. other side :: cof. M : cof. N.

Like the side opposite the given angle, if that angle be acute;

But unlike that side, if the angle be obtuse.

Then the side sought AB = $\begin{cases} \text{sum of } M \text{ and } N, & \text{if the } \angle \text{ falls within.} \\ \text{diff. of } M \text{ and } N, & \text{if the } \angle \text{ falls without.} \end{cases}$

But if CA = CB, or if CA = $180^\circ - CB$, or if CA is between BC and $180^\circ - BC$;

Then $\angle B$ is like BC only.

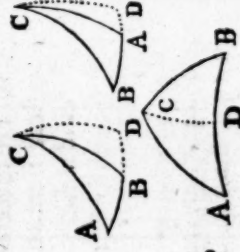
And if BC is $\begin{cases} \text{like} & \text{Then } \angle ACB = m \pm n \text{ only; and } AB = m \\ \text{unlike } \angle A; & \pm n \text{ only.} \end{cases}$

122. PROBLEM II.

In the oblique angled spheric triangle ABC.

Given two angles $\angle CAB, \angle CBA$ } Required

And a side opposite one of them AC } the rest.



1st. To find the side opposite the other given angle, viz. CB. (110)

Then, As sin. $\angle CBA : \sin. AC :: \sin. \angle CAB : \sin. CB.$

Or sin. one angle : sin. opposite side :: sin. other angle : sin. opposite side.

Which may be either acute or obtuse.

2d. To find the side included by the given angles, viz. AB.

Now rad : cof. $\angle CAB :: \tan. AC : \tan. (AD, \text{ call it } M). \quad (11. 114)$

Or Rad : cof. $\angle adj. \text{ given side} :: \tan. \text{ the given side} : \tan. (\text{of a fourth} =) M.$

Like the angle adj. the side given, if that side is acute; but unlike, if obtuse.

But Rad : tan. CD :: cot. $\angle CAB : \sin. (AD =) M.$

Rad : tan. CD :: cot. $\angle CBD : \sin. (DB, \text{ call it } N). \quad (2d \text{ 115})$

Therefore cot. $\angle CAB : \cot. \angle CBD :: \sin. M : \sin. N. \quad (11. 155)$

Or cot. $\angle adj. \text{ given side} : \cot. \text{ other angle} :: \sin. M : \sin. N.$

Which may be either acute or obtuse.

Then side sought AB = $\begin{cases} \text{sum of } M \text{ and } N, & \text{if the given angles are alike.} \\ \text{diff. of } M \text{ and } N, & \text{if the given angles are unlike.} \end{cases}$

3d. To find the other angle, viz. $\angle ACB.$

Now Rad : tan. $\angle CAD :: \cot. AC : \cot. (\angle ACD, \text{ call it } m). \quad (3d \text{ 114})$

Or Rad : tan. $\angle adj. \text{ side given} :: \cot. \text{ of given side} : \cot. (\text{of a fourth} =) m.$

Like $\angle adj. \text{ side given}$, if that side is acute; but unlike, if obtuse.

But cof. CD : Rad :: cof. $\angle CAB : \sin. (\angle ACD =) m.$

cof. CD : Rad :: cof. $\angle ABC : \sin. (\angle BCD, \text{ call it } n). \quad (3d \text{ 115})$

Therefore cof. $\angle CAB : \cot. \angle ABC :: \sin. m : \sin. n. \quad (11. 155)$

Or cof. $\angle adj. \text{ side given} : \cot. \text{ other angle} :: \sin. m : \sin. n.$

Which may be either acute or obtuse.

Then

Then $\angle$ sought $ABC = \begin{cases} \text{sum of } m \text{ and } n, \text{ if the given angles are alike.} \\ \text{diff. of } m \text{ and } n, \text{ if the given angles are unlike.} \end{cases}$

But if $AC = BC$, or to $180^\circ - BC$, or is between BC and $180^\circ - BC$.

Then BC cannot be unlike its opposite angle.

Neither can DB , or the $\angle BCD$ be obtuse.

123.

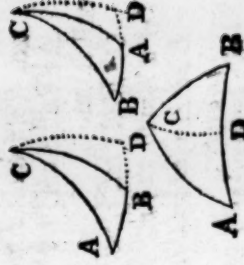
PROBLEM III.

In the oblique angled spheric triangles ABC .

Given two sides

AC, AB

And their contained angle BAC } Required the rest.



1st. To find either of the other angles, as $\angle ABC$.

As Rad : $\cos. \angle CAB :: \tan. AC : \tan. (AD, \text{ call it}) M.$ (2d 114)

Or Rad : $\cos. \text{ given } \angle :: \tan. \text{ side opposite } \angle \text{ sought} : \tan. (\text{of a fourth} =) M.$

Like the side opposite $\angle$ sought, if the given $\angle$ is acute;

But unlike that side, if the given $\angle$ is obtuse.

Take the diff. between (AB) side adj. $\angle$ sought, and $(AD =) M$; call it N .

Now Rad : $\cot. CD :: \sin. (AD =) M : \cot. \angle CAB.$ (1st 117)

Rad : $\cot. CD :: \sin. (DB =) N : \cot. \angle ABC.$

Therefore $\sin. N : \sin. M :: \cot. \angle ABC : \cot. \angle CAB.$ (II. 155)

$:: \tan. \angle CAB : \tan. \angle CBA.$ (III. 37)

Or $\sin. N : \sin. M :: \tan. \text{ given } \angle : \tan. \angle$ sought.

Like the given angle (BAC) , if M is less than (AB) the side adjacent the angle sought; but unlike, if M is greater.

2d. To find the other side CB .

As Rad : $\cos. \angle CAB :: \tan. AC : \tan. (AD, \text{ call it}) M.$ (2d 114)

Or Rad : $\cos. \text{ given } \angle :: \tan. \text{ of either given side} : \tan. (\text{of a fourth} =) M.$

Like the side used in this proportion, if the given angle is acute;

But unlike that side, if the angle is obtuse.

Take the difference between the other side (AB) and $(AD =) M$; call it N .

Now Rad : $\cos. CD :: \cos. (AD =) M : \cos. AC.$ (2d 117)

Rad : $\cos. CD :: \cos. (DB =) N : \cos. CB.$

Therefore $\cos. M : \cos. N :: \cos. AC : \cos. CB.$ (II. 155)

Or $\cos. M : \cos. N :: \cos. \text{ side used in 1st proportion} : \cos. \text{ side required.}$

Like N , if the given $\angle$ is acute; but unlike N , if that $\angle$ is obtuse.

124.

PROBLEM IV.

In the oblique angled spheric triangle ABC .

Given two angles $\angle CAB, \angle ACB$ } Required the rest.

And their included side AC

1st. To find either of the other sides, as CB .

As Rad : $\cos. AC :: \tan. \angle CAB : \cot. (\angle ACD, \text{ call it}) m.$ (3d 114)

Or Rad : $\cos. \text{ given side} :: \tan. \angle \text{ opposite side sought} : \cot. (\text{of a fourth} =) m.$

Like the angle opposite side sought, if the given side is acute;

But unlike that angle, if the given side be obtuse.

Take the diff. between $\angle (ACB)$ adj. side sought, and $(\angle ACD =) m$, call it n .

Then Rad : $\cot. CD :: \cos. (\angle ACD =) m : \cot. AC.$ (3d 116)

Rad : $\cot. CD :: \cos. (\angle BCD =) n : \cot. CB.$

Therefore

3d. To find the other side AB.

Now Rad : $\cos. \angle CAB :: \tan. AC : \tan. (AD, \text{ call it } m). \quad (2d \text{ 114})$

Or rad : $\cos. \text{ given angle } :: \tan. \text{ adj. side } : \tan. (\text{ of a fourth } =) m.$

Acute, if the angle and its adj. side are alike; but obtuse, if unlike.

But $\cos. CD : \text{Rad} :: \cos. AC : \cos. (AD =) m.$

$\cos. CD : \text{Rad} :: \cos. CB : \cos. (DB \text{ call it } n). \quad (3d \text{ 113})$

Therefore $\cos. AC : \cos. CB :: \cos. m : \cos. n.$

Or $\cos. \text{ side adj. given angle } : \cos. \text{ other side } :: \cos. m : \cos. n.$

Like the side opposite the given angle, if that angle be acute;

But unlike that side, if the angle be obtuse.

Then the side sought $AB = \begin{cases} \text{sum of } m \text{ and } n, & \text{if the } \angle \text{ falls within.} \\ \text{diff. of } m \text{ and } n, & \text{if the } \angle \text{ falls without.} \end{cases}$

But if $CA = CB$, or if $CA = 180^\circ - CB$, or if CA is between BC and $180^\circ - BC$;

Then $\angle B$ is like BC only.

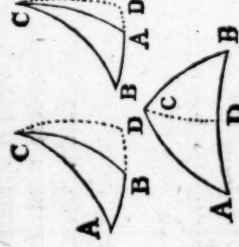
And if BC is $\begin{cases} \text{like} \\ \text{unlike } \angle A; \end{cases} \begin{cases} \text{Then } \angle ACB = m \pm n \text{ only; and } AB = m \\ \pm n \text{ only.} \end{cases}$

122. PROBLEM II.

In the oblique angled spheric triangle ABC .

Given two angles $\angle CAB, \angle CBA$ } Required

And a side opposite one of them AC } the rest.



1st. To find the side opposite the other given angle, AB , viz. CB .

Then, As $\sin. \angle CBA : \sin. AC :: \sin. \angle CAB : \sin. CB.$

Or $\sin. \text{ one angle } : \sin. \text{ opposite side } :: \sin. \text{ other angle } : \sin. \text{ opposite side.} \quad (110)$

Which may be either acute or obtuse.

2d. To find the side included by the given angles, viz. AB .

Now rad : $\cos. \angle CAB :: \tan. AC : \tan. (AD, \text{ call it } m). \quad (11. 114)$

Or Rad : $\cos. \angle \text{ adj. given side } : \tan. \text{ the given side } : \tan. (\text{ of a fourth } =) m.$

Like the angle adj. the side given, if that side is acute; but unlike, if obtuse.

But Rad : $\tan. CD :: \cot. \angle CAB : \sin. (AD =) m.$

Rad : $\tan. CD :: \cot. \angle CBD : \sin. (DB, \text{ call it } n). \quad (2d \text{ 115})$

Therefore $\cot. \angle CAB : \cot. \angle CBD :: \sin. m : \sin. n.$

Or $\cot. \angle \text{ adj. given side } : \cot. \text{ other angle } :: \sin. m : \sin. n.$

Which may be either acute or obtuse.

Then side sought $AB = \begin{cases} \text{sum of } m \text{ and } n, & \text{if the given angles are alike.} \\ \text{diff. of } m \text{ and } n, & \text{if the given angles are unlike.} \end{cases}$

3d. To find the other angle, viz. $\angle ACB$.

Now Rad : $\tan. \angle CAD :: \cos. AC : \cot. (\angle ACD, \text{ call it } m). \quad (3d \text{ 114})$

Or Rad : $\tan. \angle \text{ adj. side given } : \cos. \text{ of given side } : \cot. (\text{ of a fourth } =) m.$

Like $\angle \text{ adj. side given, if that side is acute; but unlike, if obtuse.}$

But $\cos. CD : \text{Rad} :: \cos. \angle CAB : \sin. (\angle ACD =) m.$

$\cos. CD : \text{Rad} :: \cos. \angle ABC : \sin. (\angle BCD, \text{ call it } n). \quad (3d \text{ 115})$

Therefore $\cos. \angle CAB : \cos. \angle ABC :: \sin. m : \sin. n.$

Or $\cos. \angle \text{ adj. side given } : \cos. \text{ other angle } :: \sin. m : \sin. n.$

Which may be either acute or obtuse.

Then

Then $\angle$ sought $ABC = \begin{cases} \text{sum of } m \text{ and } n, & \text{if the given angles are alike.} \\ \text{diff. of } m \text{ and } n, & \text{if the given angles are unlike.} \end{cases}$
 But if $AC = BC$, or to $180^\circ - BC$, or is between BC and $180^\circ - BC$.
 Then BC cannot be unlike its opposite angle.
 Neither can DB , or the $\angle BCD$ be obtuse.

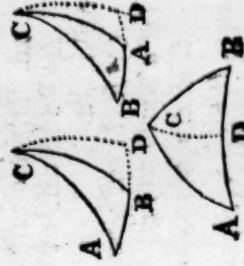
123. PROBLEM III.

In the oblique angled spheric triangles ABC .

Given two sides

AC, AB

And their contained angle BAC } Required the rest.



1st. To find either of the other angles, as $\angle ABC$.

As Rad : $\cos. \angle CAB :: \tan. AC : \tan. (AD, \text{ call it}) M. \quad (2d \text{ 114})$

Or Rad : $\cos. \text{ given } \angle :: \tan. \text{ side opposite } \angle \text{ sought} : \tan. (\text{of a fourth} =) M.$

Like the side opposite $\angle$ sought, if the given $\angle$ is acute;

But unlike that side, if the given $\angle$ is obtuse.

Take the diff. between (AB) side adj. $\angle$ sought, and $(AD =) M$; call it N .

Now Rad : $\cot. CD :: \sin. (AD =) M : \cot. \angle CAB. \quad (1st \text{ 117})$

Rad : $\cot. CD :: \sin. (DB =) N : \cot. \angle ABC.$

Therefore $\sin. N : \sin. M :: \cot. \angle ABC : \cot. \angle CAB.$

$:: \tan. \angle CAB : \tan. \angle CBA. \quad (II. 155)$

$(III. 37)$

Or $\sin. N : \sin. M :: \tan. \text{ given } \angle : \tan. \angle \text{ sought.}$

Like the given angle (BAC) , if M is less than (AB) the side adjacent the angle sought; but unlike, if M is greater.

2d. To find the other side CB .

As Rad : $\cos. \angle CAB :: \tan. AC : \tan. (AD, \text{ call it}) M. \quad (2d \text{ 114})$

Or Rad : $\cos. \text{ given } \angle :: \tan. \text{ of either given side} : \tan. (\text{of a fourth} =) M.$

Like the side used in this proportion, if the given angle is acute;

But unlike that side, if the angle is obtuse.

Take the difference between the other side (AB) and $(AD =) M$; call it N .

Now Rad : $\cos. CD :: \cos. (AD =) M : \cos. AC. \quad (2d \text{ 117})$

Rad : $\cos. CD :: \cos. (DB =) N : \cos. CB.$

Therefore $\cos. M : \cos. N :: \cos. AC : \cos. CB. \quad (II. 155)$

Or $\cos. M : \cos. N :: \cos. \text{ side used in 1st proportion} : \cos. \text{ side required.}$

Like N , if the given $\angle$ is acute; but unlike N , if that $\angle$ is obtuse.

124.

PROBLEM IV.

In the oblique angled spheric triangle ABC .

Given two angles $\angle CAB, \angle ACB$ } Required the rest,
 And their included side AC

1st. To find either of the other sides, as CB .

As Rad : $\cos. AC :: \tan. \angle CAB : \cot. (\angle ACD, \text{ call it}) m. \quad (3d \text{ 114})$

Or Rad : $\cos. \text{ given side} :: \tan. \angle \text{ opposite side sought} : \cot. (\text{of a fourth} =) m.$

Like the angle opposite side sought, if the given side is acute;

But unlike that angle, if the given side be obtuse.

Take the diff. between $\angle (ACB)$ adj. side sought, and $(\angle ACD =) m$, call it n .

Then Rad : $\cot. CD :: \cos. (\angle ACD =) m : \cot. AC. \quad (3d \text{ 116})$

Rad : $\cot. CD :: \cos. (\angle BCD =) n : \cot. CB.$

Therefore

Therefore $\cos. n : \cos. m :: \cot. CB : \cot. AC.$

$n :: \tan. AC : \tan. CB.$

(II. 155)
(III. 37)

Or $\cos. n : \cos. m :: \tan. given\ side : \tan. side\ required.$

Like n , if the angle opposite the side sought be acute;
But unlike n , if the angle is obtuse.

2d. To find the other angle ABC.

As Rad : $\cos. AC :: \tan. \angle CAB : \cot. (\angle ACD,$
call it) $m.$ (3d 114)

Or Rad : $\cos. given\ side :: \tan. either\ given\ \angle : \cot.$
(of a fourth $=$) $m.$

Like $\angle$ used in this proportion, if the given side
(AC) is acute;

But unlike that $\angle$, if the given side is obtuse.

Take the difference between the other $\angle$ (ACB) and
 $\angle$ (ACD $=$) m , call it n .

Now Rad : $\cos. CD : \sin. (\angle ACD =) m : \cos. \angle CAB.$ (1st. 116)

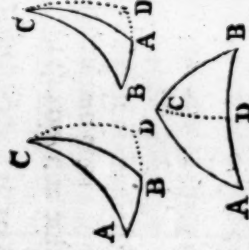
Rad : $\cos. CD : \sin. (\angle BCD =) n : \cos. \angle ABC.$

Therefore $\sin. m : \sin. n :: \cos. \angle CAB : \cos. \angle ABC.$

(II. 155)

Or $\sin. m : \sin. n :: \cos. \angle$ used in 1st prop. : $\cos. \angle$ sought.

Like the used in both proportions, if m is less than the other $\angle$;
But unlike, if m is greater than the other angle.



125.

PROBLEM V.

In the oblique angled spheric triangle ABC. Plate I. Problem V.

Given the three sides AB, BC, AC; Required the angles.

To find the angle ABC.

Let HBKLM represent the quarter of a sphere, whose center is O.

Where the femicircular sections HBK, HLK, are at right angles to one another; and OB is perpendicular to HK.

Then, continuing the side BC to L, the arc HML measures $\angle ABC.$ (9)
And $HQ = \frac{1}{2}$ chord HL, will be the sine of $\frac{1}{2}$ (arc $HMC = \frac{1}{2}$) $\angle ABC.$

Draw the radius OQM; and draw LP, QN at right angles to HK.

Then LP = sine, HP = versed sine, of $\angle ABC$: And $HN = NP.$ (II. 163)

But as HQQ is a right angled triangle; OQ being perp. to HL. (II. 125)
Therefore OH : HQ :: HQ : HN.

(II. 170)

And $OH \times HN = HQ^2$ (II. 162) = square of the sine of $\frac{1}{2} \angle ABC.$

Make $BD = BE = BC$; and $AF = AG = AC.$

Then the femicircular plane DCE, which is parallel to HLK (23), will be cut by the femicircular plane FCG, drawn at right angles to the plane HBK, in the line CI (II. 209) at right angles to DE. (II. 210)

And the arc DC and its versed sine DI, are similar to the arc HL and its versed sine HP.

(29. III. 15)

Then Rad OH : Rad DS :: PH : ID $= \left(\frac{PH}{OH} \times DS = \right) \frac{2HN}{OH} \times DS.$

Draw OR parallel to FG; then arc AR $= (90^\circ =)$ arc BK, and $RK = AB.$

Therefore $\angle DIF = (\angle KOR = \text{arc } RK =) \text{arc } AB.$

Now DS $= (\text{se} = \text{fine arc } BE =) \text{fine arc } BC.$

And AD $= (BD - BA = BC - BA =) \text{diff. sides about } \angle \text{ sought.}$

Also $\angle DFI = \frac{1}{2} \text{ arc } (DG = AG + AD =) AC + AD$, whose fine is $\frac{1}{2}$ ID.
Schol. to art. III. 45.

And arc FD $= (AF - AD =) AC - AD$, whose fine is $\frac{1}{2}$ DF.

Now $\sin. \angle DIF : \sin. \angle DFI :: (FD : ID ::) \frac{1}{2} FD : \frac{1}{2} ID$. (Schol. III. 45)

Or $\sin. \angle DIF : \sin. \angle DFI :: \frac{1}{2} FD : \frac{HN}{OH} \times \frac{1}{2} FD$.

Therefore $\sin. \angle DIF \times DS \times \frac{HN}{OH} = \sin. \angle DFI \times \frac{1}{2} FD$. (II. 163)

Therefore $\sin. \angle DIF \times DS \times \frac{HN}{OH} \times OH = \sin. \angle DFI \times \frac{1}{2} FD \times OH$. (II. 156)

Or $\sin \angle DIF \times DS \times HN = \sin. \angle DFI \times \frac{1}{2} FD \times OH$. (II. 149)

Ther. $\sin. \angle DIF \times DS : \sin. \angle DFI \times \frac{1}{2} FD :: OH : HN$. (II. 163)

Therefore $\sin. \angle DIF \times DS : \sin. \angle DFI \times \frac{1}{2} FD :: OH^2 : HQ^2$. (II. 155)

Or $\sin. AB \times \sin. BC : \sin. \frac{1}{2} AC + AD \times \sin. \frac{1}{2} AC - AD :: Kad^2 : \sin. \frac{1}{2} \angle ABC$

Ther. $\text{sq.} \sin. \frac{1}{2} \angle ABC = \frac{\sin. AB \times \sin. BC}{\sin. \frac{1}{2} AC + AD \times \sin. \frac{1}{2} AC - AD} \times \text{sq. Rad.}$ (II. 164)

Now supposing $\text{Rad} = I$, and L . to stand for logarithm.

Then $2L$, $\sin. \frac{1}{2} \angle ABC = L. \sin. \frac{1}{2} AC + AD + L. \sin. \frac{1}{2} AC - AD - L. \sin. ABC$

And putting l for the arithmetic complement of logarithm.

Then $L. \sin. \frac{1}{2} \angle ABC = \frac{l. \sin. AB + l. \sin. BC + L. \sin. \frac{1}{2} AC + AD + L. \sin. \frac{1}{2} AC - AD}{2}$

That is, having determined which angle to find,

To the arithmetic complement of $\log. \sin.$ of one containing side,

Add the arithmetic complement of $\log. \sin.$ of the other containing side,

And the $\log. \sin.$ of the $\frac{1}{2}$ sum of 3d side and difference of the containing sides,

Also the $\log. \sin.$ of the $\frac{1}{2}$ difference of 3d side and diff. of the containing sides,
Then the degrees answering to half the sum of these four logarithms, found among the sines, being doubled, will give the angle sought.

126.

PROBLEM VI.

In the oblique angled spheric triangle ABC.

Given the three angles A, B, C; Requ. the sides.

To find the side AB.

About the given angles as poles, describe arcs of great circles meeting one another, and forming the triangle FDE.

Then are the sides of FDE, the supplements of the angles A, B, C. (95)

Continue FD, FE, the supplements of the angles

B, A, adjacent to the side AB required, till they meet in G.

Then in the triangle DGE, the sides GD, GE, are the measures of the angles B and A, adjacent to the side sought.

The side DE is the supplement of $\angle C$ opposite the side AB.

Now $\angle G (= \angle F$, by 31) is the supplement of AB.

Therefore the $\angle G$ being found in the triangle DGE by PROB. V. (125) will give the supplement of the side AB required.

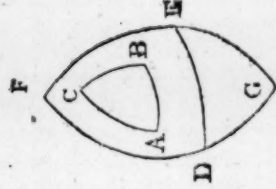
That is, Let the given angles be taken as the sides of another triangle, observing to use the supplement of that angle opposite to the side required.

In this new triangle find (by PROB. V.) the angle opposite to that side where the supplement is used.

Then will the supplement of the angle thus found be the side required.

MI

A TA-



A TABLE containing all the cases of right angled, or Quadrantal, Spheric Triangles, with the Solutions and Determinations.

Pr. Given.	Required.	SOLUTION.		Determination.
I. Hyp. and a leg.	$\angle$ op. gn. leg.	fin. Hyp	Rad : : fin. gn. leg : fin. op. $\angle$	Like given leg.
I. Hyp. and a leg.	$\angle$ ad. gn. leg	Rad	: cor. Hyp. : : tan. gn. leg : col. adj. $\angle$	$\left\{ \begin{array}{l} \text{ac. or ob. as hyp.} \\ \text{lik. or unl. gn. leg} \end{array} \right.$
other leg.	col. gn. leg :	Rad	: col. Hyp. : col. req. leg	$\left\{ \begin{array}{l} \text{ac. or ob. as hyp.} \\ \text{like or unl. gn. leg} \end{array} \right.$
leg. op. gn. $\angle$	Rad	: fin. Hyp. : : fin. gn. $\angle$: fin. op. leg	Like given angle.	
Hyp. and an angle.	leg ad. gn. $\angle$	Rad	: tan. Hyp. : : col. gn. $\angle$: tan. adj. leg	$\left\{ \begin{array}{l} \text{ac. or ob. as hyp.} \\ \text{lik. or unl. gn. \angle} \end{array} \right.$
other angle	Rad	: col Hyp. : : tan. gn. $\angle$: col. req. $\angle$		$\left\{ \begin{array}{l} \text{ac. or ob. as hyp.} \\ \text{lik. or unl. gn. \angle} \end{array} \right.$
Hypoeth.	fin. gn. $\angle$: fin. gn. leg : :	Rad	: fin. Hyp.	either ac. or ob.
A leg and its other leg	Rad	: cor. gn. $\angle$: : tan. gn. leg : fin. req. leg	either ac. or ob.	
op. $\angle$ other angle	col. gn. leg : col. gn. $\angle$: :	Rad	: fin. req. $\angle$	either ac. or ob.
other angle	Rad	: col. gn. leg : : fin. gn. $\angle$: col. req. $\angle$	Like given leg.	
A leg and its other leg	Rad	: fin. gn. leg : : tan. gn. $\angle$: tan. req. leg	Like given angle.	
adj. $\angle$ Hypoeth.	Rad	: col. gn. $\angle$: : cor. gn. leg : col. Hyp.	$\left\{ \begin{array}{l} \text{ac. or ob. as gn. leg} \\ \text{like or unl. gn. \angle} \end{array} \right.$	

ac. or ob. as gn. leg.
like or unl. gn. $\angle$.

Rad : col. gn. $\angle$: : col. gn. leg : col. Hyp.

Hypoth.

IV. and its adj. $\angle$

137.

138.

V.		VI.	
Both legs.	either angle	Hypoth.	Rad
Hypoth.	Rad	gn.eith. $\angle$:	Rad
	Rad	col.either leg :	col.either $\angle$:
Both angles	Rad	col. oth. leg :	col. oth. $\angle$:
	Rad	col. other $\angle$:	col Hyp.
Hypoth.	Rad	col. other $\angle$:	col. oth. $\angle$:
	Rad	col. oth. leg :	col Hyp.
Both legs.	Rad	col. other $\angle$:	col. oth. $\angle$:
	Rad	col. oth. leg :	col Hyp.
Hypoth.	Rad	col. other $\angle$:	col. oth. $\angle$:
	Rad	col. oth. leg :	col Hyp.
Both angles	Rad	col. other $\angle$:	col. oth. $\angle$:
	Rad	col. oth. leg :	col Hyp.

143. In a quadrantal triangle, if the quadrantal side be called radius; the supplement of the angle opposite to that side be called hypothenuse; the other sides be called angles, and their opposite angles be called legs: Then the solution of all the cases will be as in this table; observing, that where the kind of a side or angle is determined by the hypothenuse; or the hypothenuse is to be determined; to use unlike instead of like, and like instead of unlike.

In this table, beside the contractions for sine, tangent, co-sine, co-tangent; *op.* stands for opposite; *adj.* for adjacent; *oth.* for other; *eith.* for either; *ac.* for acute; *ob.* for obtuse; *gn.* for given; *lik. unl.* for like, unlike; *req.* for required; *ang.* or $\angle$, for angle.

A TABLE containing all the cases of Oblique angled Spherick Triangles, with the Solutions and Determinations.

[illegible]

	IV.	Two angles and their included side	either of other sides	Rad : col. given side : : tan. $\angle$ op. req. S. : col. m Take the difference between $\angle$ adj required side and m, call it n. col. n : col. m : : tan. given side : tan. req. S. { like or unlike angle op. req. side, as given side is acute or obtuse. { like or unlike n, as the angle opposite req. side is acute or obtuse { like or unlike angle used, as given side is acute or obtuse. Rad : col. given side : : tan. either gn. $\angle$: col. m Take the difference between the other side and m, call it n. line m : line n : : col. $\angle$ if used : col. req. $\angle$ { like or unlike angle here used, as m is less or greater than other ang.	Call the sides including the angle sought e and f; the opposite side call g. Put d equal to difference between e and f; Find the half sum and half difference of g and d. Then, To the Ar. Co. of Log. sine of e, add the Ar. Co. of Log. sine of f, And the Log. sine of $\frac{1}{2}$ sum of g and d; Also the Log. sine of $\frac{1}{2}$ difference of g and d. Take half the sum of these four Logarithms, which seek among the Log. sines; And the degrees and minutes answering being doubled, will give the angle sought.	Let the given angles be taken as the sides of another triangle, observing to use the supplement of that angle opposite the side required. In this new triangle, find the angle opposite to that side where the supplement is used, by the precepts in Problem V. Then will the supplement of the angle thus found be the side required.
	IV.	included $\angle$ and their	other side	Rad : col. given angle : : tan. either gn. S. : tan. m Take the difference between the other side and m, call it n. col. m : col. n : : col. S. if used : col. req. S. { like or unlike m, as given angle is acute or obtuse. { like or unlike n, as given angle is acute or obtuse.	Call the sides including the angle sought e and f; the opposite side call g. Put d equal to difference between e and f; Find the half sum and half difference of g and d. Then, To the Ar. Co. of Log. sine of e, add the Ar. Co. of Log. sine of f, And the Log. sine of $\frac{1}{2}$ sum of g and d; Also the Log. sine of $\frac{1}{2}$ difference of g and d. Take half the sum of these four Logarithms, which seek among the Log. sines; And the degrees and minutes answering being doubled, will give the angle sought.	Let the given angles be taken as the sides of another triangle, observing to use the supplement of that angle opposite the side required. In this new triangle, find the angle opposite to that side where the supplement is used, by the precepts in Problem V. Then will the supplement of the angle thus found be the side required.
	V.	Three sides	either angle	Rad : col. given side : : tan. either gn. $\angle$: col. m Take the difference between the other angle and m, call it n. col. n : col. m : : tan. given side : tan. req. S. { like or unlike angle op. req. side, as given side is acute or obtuse. { like or unlike n, as the angle opposite req. side is acute or obtuse { like or unlike angle used, as given side is acute or obtuse. Rad : col. given side : : tan. $\angle$ if used : col. req. $\angle$ line m : line n : : col. $\angle$ if used : col. req. $\angle$ { like or unlike angle here used, as m is less or greater than other ang.	Call the sides including the angle sought e and f; the opposite side call g. Put d equal to difference between e and f; Find the half sum and half difference of g and d. Then, To the Ar. Co. of Log. sine of e, add the Ar. Co. of Log. sine of f, And the Log. sine of $\frac{1}{2}$ sum of g and d; Also the Log. sine of $\frac{1}{2}$ difference of g and d. Take half the sum of these four Logarithms, which seek among the Log. sines; And the degrees and minutes answering being doubled, will give the angle sought.	Let the given angles be taken as the sides of another triangle, observing to use the supplement of that angle opposite the side required. In this new triangle, find the angle opposite to that side where the supplement is used, by the precepts in Problem V. Then will the supplement of the angle thus found be the side required.
	V.	Three angles	either side	Rad : col. given angle : : tan. either gn. S. : tan. m Take the difference between the other side and m, call it n. col. m : col. n : : col. S. if used : col. req. S. { like or unlike m, as given angle is acute or obtuse. { like or unlike n, as given angle is acute or obtuse.	Call the sides including the angle sought e and f; the opposite side call g. Put d equal to difference between e and f; Find the half sum and half difference of g and d. Then, To the Ar. Co. of Log. sine of e, add the Ar. Co. of Log. sine of f, And the Log. sine of $\frac{1}{2}$ sum of g and d; Also the Log. sine of $\frac{1}{2}$ difference of g and d. Take half the sum of these four Logarithms, which seek among the Log. sines; And the degrees and minutes answering being doubled, will give the angle sought.	Let the given angles be taken as the sides of another triangle, observing to use the supplement of that angle opposite the side required. In this new triangle, find the angle opposite to that side where the supplement is used, by the precepts in Problem V. Then will the supplement of the angle thus found be the side required.

SECTION VIII.

*The Construction and numeral Solution of the cases
of right angled Spheric Triangles.*

156. EXAM. I. In the right angled spheric triangle ABC.
Given the hypoth. $ac = 64^{\circ} 40'$ } Required the rest.
And one leg $BC = 42^{\circ} 12'$

CONSTRUCTIONS.

1st. To put the given leg on the primitive circle.
Describe the primitive circle, and draw the right circle AB.

Apply the given leg ($42^{\circ} 12'$) to the primitive circle from B to C.

About c as a pole, at a distance equal to the hypotenuse ($64^{\circ} 40'$) describe (68) a small circle *aa*, cutting the right circle AB in A; and draw the right circle CD.

Through C, A, D, describe an oblique circle.
And ABC is the triangle sought.

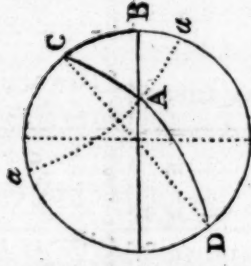
2d. To put the required leg on the primitive circle.
Describe the primitive circle, and draw the right circle CB; on which lay the given leg ($42^{\circ} 12'$) from B to C.

About c as a pole (66), at a distance equal to the hypotenuse ($64^{\circ} 40'$) describe a small circle, cutting the primitive in A; and draw AD.

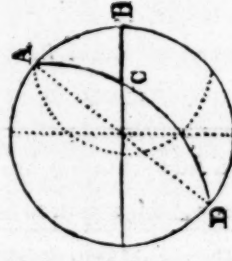
Through A, C, D, describe an oblique circle.

(II. 72)

Then ABC is the triangle required: Whose sides and angles are measured by art. 70, 72.



(II. 72)



COMPUTATION.

To find $\angle$ oppos. the given leg. (127)		To find $\angle$ adj. the given leg. (128)	
As sin. hyp. $= 64^{\circ} 40'$	0,04392	As Rad $= 90^{\circ} 00'$	10,00000
To Rad $= 90^{\circ} 00'$	10,00000	To cot. hyp. $= 64^{\circ} 40'$	9,67524
So sin. gn. leg $= 42^{\circ} 12'$	9,82719	So tan. gn. leg $= 42^{\circ} 12'$	9,95748
To sin. op. $\angle = 48^{\circ} 00'$		To cos. adj. $\angle = 64^{\circ} 35'$	
9,87111		9,63272	

This angle is acute, because it is to be like the given leg, which is acute.

To find the other leg. (129)

As cos. gn. leg $= 42^{\circ} 12'$	0,13030
To Rad $= 90^{\circ} 00'$	10,00000
So cos. hyp. $= 64^{\circ} 40'$	9,63133
To cos. req. leg $= 54^{\circ} 43'$	9,76163

This angle is acute, because the hyp. and given legs are of like kinds.

This leg is acute, because the hyp. and given leg are of like kinds.

NOTE, In these operations, and in all the following ones, although the word co-sine, or co-tangent, are used in the proportions, yet the degrees and minutes set down, are not the complements, but the real sides or angles.

157. EXAMPLE II. In the right angled spheric triangle ABC.
 Given the hypoth. $AC = 64^{\circ} 40'$ } Required the rest.
 And one angle $ACB = 64^{\circ} 35'$

CONSTRUCTIONS.

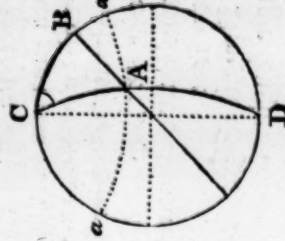
1st. To put the leg adjacent to the given angle on the primitive circle.

Through any point c , in the primitive circle, describe (75) the oblique circle CAD , making with the primitive circle the angle BCA equal to the given angle $64^{\circ} 35'$.

In the oblique circle CAD , take CA , equal to the given hypotenuse $64^{\circ} 40'$. (70)

Through A describe the right circle AB .

And CAB is the triangle required.



2d. To put the leg opposite the given angle on the primitive circle.

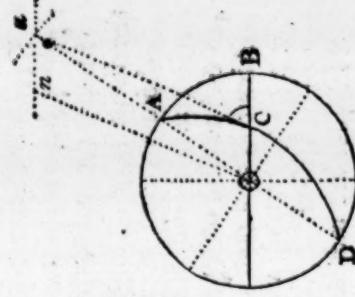
Having described the primitive circle, and drawn the right circle OB ;

Describe (80) an oblique circle ACD , cutting the right circle OB in c , with the given angle $64^{\circ} 35'$, and having the part AC intercepted between the right circle OB and the primitive circle, equal to the given hypotenuse $64^{\circ} 40'$;

Then ABC is the triangle required.

The sides required are measured by art. 70.

And the required angle by art. 72.



COMPUTATION.

To find the leg opp. the giv. $\angle$. (130)
 As Rad $= 90^{\circ} 00'$ 10,00000
 To sin. hyp. $= 64^{\circ} 40'$ 9,95608
 So sin. given $\angle = 64^{\circ} 35'$ 9,95579

To sin. op. leg $= 54^{\circ} 43'$ 9,91187

Like the given angle

To find the leg adj. the giv. $\angle$ (131)
 As Rad $= 90^{\circ} 00'$ 10,00000
 To tan. hyp. $= 64^{\circ} 40'$ 10,32476
 So cof. given $\angle = 64^{\circ} 35'$ 9,63266

To tan. adj. leg $= 42^{\circ} 12'$ 9,95742

Acute, as the hypotenuse and given angle are of like kind.

To find the other angle. (132)

As Rad $= 90^{\circ} 00'$ 10,00000
 To cof. hyp. $= 64^{\circ} 40'$ 9,63133
 So tan. given angle $= 64^{\circ} 35'$ 10,32313
 To cot. required angle $= 48^{\circ} 00'$ 9,95446

And is acute, as the hypotenuse and given angle are of like kind.

M 4

158 Ex-

158. EXAMPLE III. In the right angled spheric triangle ABC.
 Given one leg $CB = 42^\circ 12'$ } Require the rest.
 And its opp. angle $CAB = 48^\circ 00'$

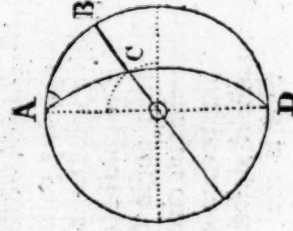
CONSTRUCTIONS.

1st. To put the required leg on the primitive circle.

Describe an oblique circle ACD (75), making with the primitive circle the angle CAB, equal to the given angle $48^\circ 00'$.

About the center O of the primitive circle describe (67) a small circle, at the distance of the complement of the given leg $42^\circ 12'$, cutting ACD in C.

Draw the right circle OCB, and ACB is the triangle sought.



2d. To put the given leg on the primitive circle.

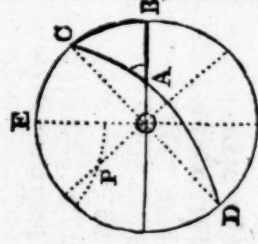
Draw the right circle OAB, and another OE at right angles.

Make BC equal to the given leg $42^\circ 12'$; draw the diameter CD, and another OP at right angles.

About E, the pole of AB, describe a small circle (68), at the distance of the given angle $48^\circ 00'$ cutting OP in P.

About P, as a pole (62), describe the oblique circle CAD, cutting AB in A. Then is CBA the triangle required.

The sides are measured by art. 70, and the angles by art. 72.



COMPUTATION.

To find the hypotenuse. (133)		To find the other leg. (134)	
As fin. $\text{giv. } \angle = 48^\circ 00'$	0,12893	As Rad $= 90^\circ 00'$	10,00000
To fin. $\text{giv. leg} = 42^\circ 12'$	9,82719	To cot. $\text{giv. } \angle = 48^\circ 00'$	9,95444
So Rad $= 90^\circ 00'$	10,00000	So tan. $\text{giv. leg} = 42^\circ 12'$	9,95748
To fin. hyp. $= 64^\circ 40\frac{1}{2}'$		To fin. req. leg $= 54^\circ 44'$	
9,95612		9,91102	

And is either acute or obtuse.

And is either acute or obtuse.

To find the other angle. (135)

As cof. given leg	$= 42^\circ 12'$	0,13030
To cof. given $\angle$	$= 48^\circ 00'$	9,82551
So Rad	$= 90^\circ 00'$	10,00000
To fin. required $\angle$		$= 64^\circ 35'$
9,95581		

And is either acute or obtuse.

19. Ex.

159. EXAMPLE IV. In the right angled spheric triangle ABC.

Given a leg $AB = 54^{\circ} 43'$ } Required the rest.
And its adj. angle $CAB = 48^{\circ} 00'$ }

CONSTRUCTIONS.

1st. To put the given leg on the primitive circle.

Having described the primitive, and right circle

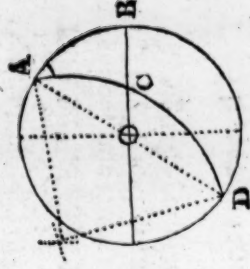
OB;

Make BA equal to the given leg $54^{\circ} 43'$.

Draw the diameter AD.

Through A describe the oblique circle ACD (75) making with the primitive the given angle BAC $48^{\circ} 00'$, cutting OB in C.

Then is ACB the triangle required.



2d. To put the required leg on the primitive.

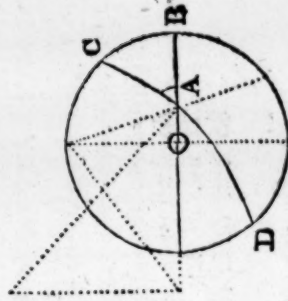
In the right circle OB, take (71) AB, equal to the given leg $54^{\circ} 43'$.

Through the point A, describe (76) the oblique circle CAD, making with AB the angle BAC, equal to the given angle $48^{\circ} 00'$, cutting the primitive circle in C.

Then is ABC the triangle sought.

The sides required are measured by art. 70.

And the required angle by art. 72.



COMPUTATION.

To find the other angle. (136)
As Rad $= 90^{\circ} 00'$ 10,00000
To cof. giv. leg $= 54^{\circ} 43'$ 9,76164
So fin. given $\angle = 48^{\circ} 00'$ 9,87107
To cof. req. $\angle = 64^{\circ} 35'$ 9,63271

And is like the given angle.

To find the other leg. (137)
As Rad $= 90^{\circ} 00'$ 10,00000
To fin. giv. leg $= 54^{\circ} 43'$ 9,91185
So tan. giv. $\angle = 48^{\circ} 00'$ 10,04556
To tan. req. leg $= 42^{\circ} 12'$ 9,95741

And is like the given leg.

To find the hypotenuse. (138)

As Rad $= 90^{\circ} 00'$ [10,00000
To cof. given $\angle = 48^{\circ} 00'$ 9,82551
So cot. given leg $= 54^{\circ} 43'$ 9,84979
To cot. hypoth. $= 64^{\circ} 40'$ 9,67530

And is acute, as the given leg and angle are of a like kind.

160. Example V. In the right angled spheric triangle ABC.

Given one leg $BA = 54^{\circ} 43'$ } Required the rest.
And the other leg $BC = 42^{\circ} 12'$

CONSTRUCTION.

To put either leg on the primitive circle.

Having described the primitive circle, and drawn the right circle OB;

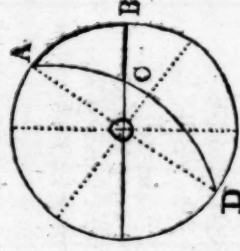
Then let the given legs $54^{\circ} 43'$, and $42^{\circ} 12'$, be applied, one from B to A, and the other from B to C (74); and draw the diameter AD.

Through the points A, C, D, describe an oblique circle. (11. 72)

Then is ABC the triangle required.

The angles A and c may be measured by art. 72.

And the hypotenuse AC by art. 70.



COMPUTATION.

To find the angle A. (139)

As Radius	—	$= 90^{\circ} 00'$	10,00000
To sin. of leg AB		$= 54^{\circ} 43'$	9,91185
So cot. other leg BC		$= 42^{\circ} 12'$	10,04251
To cot. op. angle A		$= 48^{\circ} 00'$	9,95436

And is acute, as the opposite leg CB is acute.

To find the angle C. (139)

As Radius	—	$= 90^{\circ} 00'$	10,00000
To sin. of leg CB		$= 42^{\circ} 12'$	9,82719
So cot. other leg AB		$= 54^{\circ} 43'$	9,84979
To cot. op. angle C		$= 64^{\circ} 35'$	9,67698

And is acute, because the opposite leg AB is acute.

To find the hypotenuse AC. (140)

As Radius	—	$= 90^{\circ} 00'$	10,00000
To cos. either leg AB		$= 54^{\circ} 43'$	9,76164
So cos. other leg CB		$= 42^{\circ} 12'$	9,86970
To cos. hypoth. AC		$= 64^{\circ} 40'$	9,63134

And is acute, as the legs are of the same kind.

161. EXAMPLE VI. In the right angled spheric triangle ABC.

Given one angle $A = 48^\circ 00'$

And the other angle $C = 64^\circ 35'$ } Required the rest.

CONSTRUCTION.

To put either leg, as CB, on the primitive circle.

Having described the primitive circle, and drawn the right circle OB;

Then (81) describe the oblique great circle CAD, cutting the primitive circle in the given angle C, and the right circle OB in the given angle A.

The sides are to be measured by art. 70.



COMPUTATION.

To find the leg CB. (141)

As sin. $\angle$ adj. req. leg. c	$= 64^\circ 35'$	0,04421
To Radius	$= 90^\circ 00'$	10,00000
So cof. other angle A	$= 48^\circ 00'$	9,82551
To cof. of its op. leg CB	$= 42^\circ 12'$	9,86972

And is acute, because the opposite angle is acute.

To find the leg AB. (141)

As sin. $\angle$ adj. req. leg. A	$= 48^\circ 00'$	0,12893
To Radius	$= 90^\circ 00'$	10,00000
So cof. other angle C	$= 64^\circ 35'$	9,63266
To cof. of its op. leg AB	$= 54^\circ 43'$	9,76159

And is acute, because the opposite $\angle$ is acute.

To find the hypotenuse AC. (142)

As Radius	$= 90^\circ 00'$	10,00000
To cot. either angle as A	$= 48^\circ 00'$	9,95444
So cot. other angle as C	$= 64^\circ 35'$	9,67687
To cof. hypoth. AC	$= 64^\circ 40'$	9,63131

And is acute, because the angles are both acute, or alike.

162. EXAMPLE VII. In the quadrantal triangle ABC.

Given the quadrantal side $AC = 90^\circ 00'$
 an adjacent angle $A = 42^\circ 12'$ } Required the rest.
 And the opposite angle $B = 64^\circ 40'$

CONSTRUCTION.

To put the quadrantal side on the primitive circle.

Having described the primitive circle, and drawn the diameters AD, BC, at right angles;

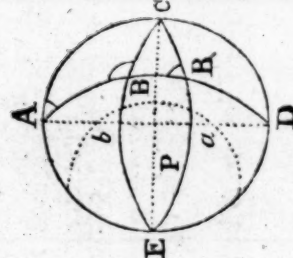
Describe the oblique circle ABD, making with AC an angle of $42^\circ 12'$.

Through c describe a great circle CBE, cutting the circle ABD in an $\angle$ of $64^\circ 40'$. (74)

Then is ABC the triangle sought.

The angle c is to be measured by art 72.

And the sides AB, CB, are measured by art. 70.



COMPUTATION.

Imagine the given triangle ABC to be changed into a right angled triangle, where the supplement of the angle B is to represent the hypotenuse, and the angle A to be one of the legs.

Then will the solution fall under art. 127, 128, 129, in the table; and the numeral computations will be the same as in Example I. Observing that the angles there found are, in this example, the measures of the sides AB, CB; and the side AB in that example stands for the angle c in this.

Now in determining the value of the parts of this triangle, as they arise in the computation, the words like and unlike are to be changed one for the other, where the hypotenuse is concerned in the determination: Thus the leg AB is taken acute, because the supplement of the angle opposite to the quadrantal side, which is here used as the hypotenuse, is unlike to the other given angle: and its opposite angle c is to be acute for the same reason: But the kind of the side BC being known by the kind of its opposite angle A, it must be taken acute, as the opposite angle is acute.

In the construction there arises two triangles, either of which will answer the conditions in the example: For the small circle described about P, the pole of the oblique circle ABD, cuts the diameter AD in the points, a, b; and either of these points may be taken for the pole of the oblique circle wanting to complete the triangle.

Now if a be taken for the pole, then in the triangle ABC, the measure of the things sought, will be equal to those arising from the computation: But the angle B is the supplement of what was given.

And if b is taken for the pole; then the triangle ABC will arise from the construction; wherein the angles A and B are respectively equal to what is propounded: But then the side AB, and the angle c, will both be obtuse.

SECTION

SECTION IX.

The Construction and numeral Solution of the cases of oblique angled Spheric Triangles.

163. EXAMPLE I. In the oblique angled spheric triangle ABC.

Given the side

$$AB = 114^{\circ} 30'$$

the side

$$BC = 56^{\circ} 40'$$

And an angle opposite to one side, $BCA = 125^{\circ} 20'$ } Required the rest.

CONSTRUCTION.

To put the given side, adjacent to the known angle, on the primitive circle.

Describe the primitive circle, and draw the diameter BD.

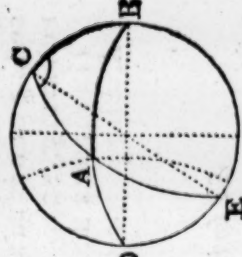
Make BC equal to the side adjacent to the given angle $= 56^{\circ} 40'$. (70)

Describe the great circle CAE, making the angle BCA equal to the given one, $= 125^{\circ} 20'$. (75)

Through B describe a great circle BAD, cutting AE in A, at the distance of AB, the other given side from B, $= 114^{\circ} 30'$. (68)

Then ABC is the triangle sought.

And the parts required are measured by art. 70, 72.



COMPUTATION.

To find the angle A, opposite to the other given side. (144)

$$\text{As fin. one side } AB = 114^{\circ} 30' \quad 0,04098$$

$$\text{To fin. op. } \angle C = 125^{\circ} 20' \quad 9,92194$$

$$\text{Sofin. oth. side } CB = 56^{\circ} 40' \quad 9,91158$$

$$\text{To fin. op. } \angle A = 48^{\circ} 31' \quad 9,87450$$

Which may be either acute or obtuse from the things given : But the construction shews it to be acute.

To find the angle B between the given sides. (145)

$$\text{As Rad} = 90^{\circ} 00' \quad 10,00000$$

$$\text{To tan. giv. } \angle C = 125^{\circ} 20' \quad 10,14941$$

$$\text{Socof. adj. sid. } BC = 56^{\circ} 40' \quad 9,73997$$

$$\text{To cot. m} = 127^{\circ} 46' \quad 9,88,938$$

$$\text{To cof. n} = 64^{\circ} 54,9,62773$$

And is obtuse, as the given angle and its given adjacent side are unlike.

Then as the given sides are unlike, the diff. of m and n, or $62^{\circ} 52' = \angle B$.

To find the other side AC. (146)

$$\text{As Rad} = 90^{\circ} 00' \quad 10,00000$$

$$\text{To cof. giv. } \angle C = 125^{\circ} 20' \quad 9,76218$$

$$\text{Sotan. adj. sid. } BC = 56^{\circ} 40' \quad 10,18196$$

$$\text{To tan. m} = 138^{\circ} 41' \quad 9,94414$$

$$\text{To cof. n} = 55^{\circ} 28,9,75343$$

And is obtuse, as $\angle C$ and CB are unlik.

Then as BC and BA are unlike, the diff. of m and n, or $83^{\circ} 13' = AC$.

164 Ex-

164. EXAMPLE II. In the oblique angled spheric triangle ABC.

Given the angle
the angle

$$\left. \begin{array}{l} BAC = 48^{\circ} 31' \\ BCA = 125^{\circ} 20' \end{array} \right\} \text{Required the rest.}$$

And the side opposite to one angle, $AB = 114^{\circ} 30'$

CONSTRUCTION.

To put the given side AB on the primitive circle.

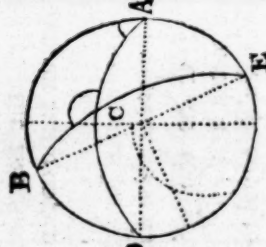
Describe the primitive circle; draw the diameter DA; and through A describe the great circle ACD, making the given angle $BAC = 48^{\circ} 31'$. (75)

Make the arc AB equal to the given side $114^{\circ} 30'$ (70); and draw the diameter BE.

Through B, describe the great circle BCE, cutting ACD in an angle equal to the given angle $BCA = 125^{\circ} 20'$. (78)

Then is ACB the triangle sought.

And the parts required are to be measured by art. 70, 72.



COMPUTATION.

To find the side opposite the other given angle. (147)

As fin. one $\angle C = 125^{\circ} 20'$ 0,08841

To fin. op. side AB = 114 30 9,87457

So fin. other $\angle A = 48^{\circ} 31'$ 9,95902

To fin. op. side BC = 56 40 9,92200

Which may either be acute or obtuse from what is given. But the construction shews it to be acute.

To find the side AC between the given angles. (148)

As Rad = 90° 00' 10,00000

To cof. $\angle$ ad. g. S. A = 48 31 9,82112

So tan. gn. S. AB = 114 30 10,34129

To tan. M = 124 32 10,16241

As cot. $\angle$ ad. g. S. A = 48° 31' 0,05345

To cot. other $\angle C = 125^{\circ} 20'$ 9,85099

So fine M = 124 32 9,91182

To fine N = 41 20 9,81986

And is obtuse, being unlike $\angle A$, as AB is greater than 90° .

Which may be either acute or obtuse; either $41^{\circ} 20'$ or $138^{\circ} 40'$.

Then as the given angles are unlike, the difference of M and N, or $83^{\circ} 12'$, is the side AC. Or the sum of $138^{\circ} 40'$, and $124^{\circ} 32'$, lessened by 180° , leaves $83^{\circ} 12'$.

To find the other angle ABC. (149)

As Rad = 90° 00' 10,00000

To tan. $\angle$ ad. g. S. A = 48 31 10,05319

So cof. gn. S. AB = 114 30 9,61773

To cot. m = 115 07 9,67092

As cof. $\angle$ ad. g. S. A = 48° 31' 0,17888

To cof. other $\angle C = 125^{\circ} 20'$ 9,76218

So fine m = 115 07 9,95886

To fine n = 52 14 9,89792

And is obtuse, being unlike $\angle A$, as its adj. side AB is greater than 90° .

Which may be either acute or obtuse, viz. $52^{\circ} 14'$, or $127^{\circ} 46'$.

Then as the given angles are unlike, the difference of m and n, or $62^{\circ} 53'$, is the angle B required. Or the sum of $115^{\circ} 07'$, and $127^{\circ} 46'$, lessened by 180° , leaves $62^{\circ} 53'$.

165. EXAMPLE III. In the oblique angled spheric triangle ABC.

Given the side	AB = 114° 30'	} Required the rest.
the side	BC = 56 40	
And the contained angle ABC =	62 52	

CONSTRUCTION.

To put either of the given sides, as BC, on the primitive circle.

Describe the primitive circle; draw the diameter BD; and through B describe a great circle BAD, making the given angle ABC = 62° 52'.



On the circles BCD, BAD, take the arcs BC, BA, respectively equal to the given sides, viz. BC = 56° 40', and BA = 114° 30'.

Draw the diameter CE, and through C, A, E, describe the great circle CAE; then ABC is the triangle sought.

The required parts of ABC are measured by art. 70, 72.

COMPUTATION.

To find the angle C. (150)

As Rad	= 90° 00' 10,00000
To cof. given ∠. B =	62 52 9,65902
Sot. S. op. re. ∠. A.B =	114 30 10,34129
To tan. M	= 134 59 10,00031

Obtuse, being like side op. req. ∠, the given angle being acute.

Take the difference between M and BC, and it is 78° 19', call it N.

As sine N	= 78° 19' 0,00909
To sine M	= 134 59 9,84961
So tan. given ∠. B =	62 52 10,29034
To tan. req. ∠. C =	125 21 10,14904

And is obtuse, being unlike the given angle, because M is greater than BC, the side adjacent to the required angle.

To find the angle A. (150)

As Rad	= 90° 00' 10,00000
To cof. given ∠. B =	62 52 9,65902
Sot. S. op. re. ∠. BC =	56 40 10,18196
To tan. of M	= 34 44 9,84098

Acute, being like side op. req. ∠, the given angle being acute.

Take the difference between M and BA, and it is 79° 46'. call it N.

As sine N	= 79° 46' 0,00696
To sine M	= 34 44 9,75569
So tan. given ∠. B =	62 52 10,29034
To tan. req. ∠. A =	48 29 10,05299

And is acute, being like the given angle, as M is less than AB, the side adjacent to the required angle.

To find the other side AC. (151)

As Rad	= 90° 00' 10,00000
To cof. given ∠. B =	62 52 9,65902
So tan. eith. S. A.B =	114 30 10,34129
To tan. M	= 134 59 10,00031

As cof. M	= 134° 59' 0,15064
To cof. N	= 78 19 9,30643
So cof. S. used A.B =	114 30 9,61773

To cof. S. req. AC = 83 11 9,07480

Obtuse being like AB the side used, because the given angle is acute.

Then the difference of M and BC, or 78° 19' = N.

And is acute, being like N, because the given angle is acute.

166. **EXAMPLE IV.** In the oblique angled spheric triangle ABC.

Given the angle
the angle

$$\begin{array}{l} BCA = 125^{\circ} 20' \\ BAC = 48^{\circ} 31' \end{array}$$

And the included side AC = 83 13 } Required the rest.

CONSTRUCTION.

To put the given side on the primitive circle.

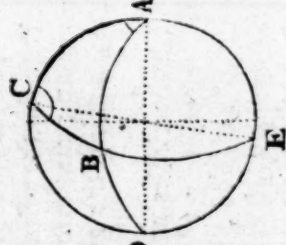
Describe the primitive circle; draw the diameter AD; and through A describe the great circle ABD, making the given $\angle BAC = 48^{\circ} 31'$. (75)

Make AC equal to the given side = $83^{\circ} 13'$. (70)

Draw the diameter CE, and through c describe the great circle CBE, making the given angle BCA = $125^{\circ} 20'$ (75), cutting ABD in B.

Then is ABC the triangle sought.

And the parts required are measured by art. 70, 72.



COMPUTATION.

To find the side AB. (152)

As Rad	=	90° 00'	10,00000	
To cof. gn. side AC	=	83 13	9,07230	
Sota. $\angle$ op. r. S. c	=	125 20	10,14941	
To cot. m	=	99 28	9,22171	

Obtuse, being like $\angle$ op. side req. the given side being acute.
Take the diff. between m and $\angle$ A, and it is $50^{\circ} 57'$, call it n.

As cof. n.	=	50° 57'	0,20066	
To cof. m	=	99 28	9,21610	
Sota. gn. side AC	=	83 13	10,92464	
To tan. req. sid. AB	=	114 30	10,34140	

And is obtuse, being unlike n, because the angle opposite to the side required is obtuse.

To find the side BC. (152)

As Rad	=	90° 00'	10,00000	
To cof. gn. sid. AC	=	83 13	9,07230	
So tan. $\angle$ op. r. S. A	=	48 31	10,05345	
To cot. of m	=	82 23	9,12575	

Acute, being like $\angle$ op. side required, the given side being acute.
Take the diff. between m and $\angle$ C, and it is $42^{\circ} 57'$, call it n.

As cof. n	=	42° 57'	0,13552	
To cof. m	=	82 23	9,12236	
So tan. gn. sid. AC	=	83 13	10,92464	
Totan. req. sid. BC	=	56 41	10,18252	

And is acute, being like n, because the angle A opposite to BC, the side required, is acute.

To find the other angle B. (153)

As Rad	=	90° 00'	10,00000	
To cof. gn. sid. AC	=	83 13	9,07230	
Sota. either $\angle$ C	=	125 20	10,14941	
To cot. m	=	99 28	9,22171	

As fine m = $99^{\circ} 28' 0,00595'$
To fine B = $50 57 9,8019$
So cof. $\angle$ used, c = $125 20 9,76218$

To cof. req. $\angle$ B = $62 53 9,65832$
And is acute, being unlike the angle c here used, as m is greater than the other angle A.

Obtuse, being like $\angle$ c here used, because the given side is acute.
Take difference of m and $\angle$ A, viz. $42^{\circ} 57'$, and call it n.

167. **EXAMPLE V.** In the oblique angled spheric triangle *ABC*.

Given the side $AB = 114^{\circ} 30'$ } Required the rest.
 the side $AC = 83^{\circ} 13'$
 the side $BC = 56^{\circ} 40'$

CONSTRUCTION.

To put either side, as *AC*, on the primitive circle.

Describe the primitive circle, and from any point in the circumference, as *A*, set off one of the given sides; as $AC = 83^{\circ} 13' (70)$; and draw the diameters *AD*, *CE*.

About *C*, as a pole, and at a distance equal to the given side $BC_2 = 56^{\circ} 40'$, describe a small circle *nB*. (68)

About *A*, as a pole, and at a distance equal to the given side *AB* (when *AB* is less than 90°) describe another small circle *mB* (68), cutting the former in *B*: But when

the side, as $AB = 114^{\circ} 30'$, is greater than 90° ; then about *D*, the opposite pole to *A*, describe a small circle with the supplement of *AB*, as *mB*, cutting the former small circle *nB* in *B*.

Thro' the points *A*, *B*, *D*, and *C*, *B*, *E*, describe the great circles *ABD*, *CBE*. Then is *ABC* the triangle sought, and the angles are measured by art. 72.

COMPUTATION.

To find the angle *C*. (154)

Here $AC = E = 83^{\circ} 13'$
 $CB = F = 56^{\circ} 40'$

$AB = G = 114^{\circ} 30'$
 $E - F = D = 26^{\circ} 33'$

$G + D = 141^{\circ} 03' 70^{\circ} 31' = \frac{1}{2} \text{sum}$

$G - D = 87^{\circ} 57' 43' 58\frac{1}{2}' = \frac{1}{2} \text{diff.}$

Ar. Co. sine *E* $= 83^{\circ} 13'$ 0,00305
 Ar. Co. sine *F* $= 56^{\circ} 40'$ 0,07806
 Sine $\frac{1}{2}$ sum $= 70^{\circ} 31\frac{1}{2}'$ 9,97441
 Sine $\frac{1}{2}$ diff. $= 43^{\circ} 58\frac{1}{2}'$ 9,84158

Sum of the four Logs 19,89710

$\frac{1}{2}$ sum gives $62^{\circ} 39\frac{1}{2}'$ 9,94855

Which doubled gives $125^{\circ} 19' = \angle C$.

To find the angle *A*. (154)

Here $AB = E = 114^{\circ} 30'$
 $AC = F = 83^{\circ} 13'$

$BC = G = 56^{\circ} 40'$
 $E - F = D = 31^{\circ} 17'$

$G + D = 87^{\circ} 57' 43^{\circ} 58\frac{1}{2}' = \frac{1}{2} \text{sum}$

$G - D = 25^{\circ} 23' 12' 41\frac{1}{2}' = \frac{1}{2} \text{diff.}$

Ar. Co. sine *E* $= 114^{\circ} 30'$ 0,04098
 Ar. Co. sine *F* $= 83^{\circ} 13'$ 0,03005
 Sine $\frac{1}{2}$ sum $= 43^{\circ} 58\frac{1}{2}'$ 9,84158
 Sine $\frac{1}{2}$ diff. $= 12^{\circ} 41\frac{1}{2}'$ 9,34184

Sum of four Logs 19,22745

$\frac{1}{2}$ sum gives $24^{\circ} 13\frac{1}{2}'$ 9,61372

Which doubled gives $48^{\circ} 31' = \angle A$.

To find the angle *B*

Here $AB = E = 114^{\circ} 30'$
 $BC = F = 56^{\circ} 40'$

$AC = G = 83^{\circ} 13'$
 $E - F = D = 57^{\circ} 50'$

$G + D = 141^{\circ} 03' 70^{\circ} 31' = \frac{1}{2} \text{sum}$

$G - D = 25^{\circ} 23' 12' 41\frac{1}{2}' = \frac{1}{2} \text{diff.}$

Ar. Co. sine *E* $= 114^{\circ} 30'$ 0,04098

Ar. Co. sine *F* $= 56^{\circ} 40'$ 0,07806

Sine $\frac{1}{2}$ sum $= 70^{\circ} 31\frac{1}{2}'$ 9,97441

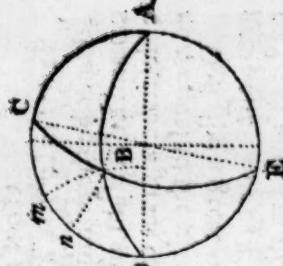
Sine $\frac{1}{2}$ diff. $= 12^{\circ} 41\frac{1}{2}'$ 9,34184

Sum of four Logs 19,43529

$\frac{1}{2}$ sum gives $31^{\circ} 18'$ 9,71764

Which doubled gives $62^{\circ} 56' = \angle B$.

168. Ex-



168. EXAMPLE VI. In the oblique angled spheric triangle ABC.

Given the angle $A = 48^{\circ} 31'$ } Required the rest.
 the angle $B = 62^{\circ} 52'$ }
 the angle $C = 125^{\circ} 20'$ }

CONSTRUCTION.

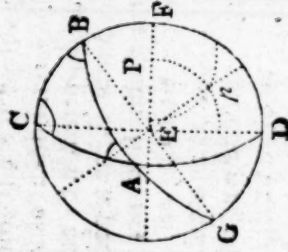
To put either two angles, as C and B , at the primitive.

Describe the primitive circle; draw the diameters CD and EF at right angles to one another; and thro' C describe a great circle CAD ; making the angle BCA equal to the given angle $C = 125^{\circ} 20'$. (75)

Describe a great circle BAG , cutting the given great circles CFD , CAD , in the given angles $B = 62^{\circ} 52'$, and $A = 48^{\circ} 31'$. (81)

Then is ABC the triangle sought.

Wherein the sides are measured by art. 70.



COMPUTATION.

To find the side AB. (155)

Here $\angle B = E = 62^{\circ} 52'$
 $\angle A = F = 48^{\circ} 31'$

Ar. Co. sine $E = 62^{\circ} 52'$ 0,05064
 Ar. Co. sine $F = 48^{\circ} 31'$ 0,12543
 Sine $\frac{1}{2}$ sum $= 34^{\circ} 30\frac{1}{2}'$ 9,75322
 Sine $\frac{1}{2}$ diff. $= 20^{\circ} 09\frac{1}{2}'$ 9,53733

Sup. $\angle C = G = 54^{\circ} 40'$
 $E - F = D = 14^{\circ} 21'$

$E + D = 69^{\circ} 01'$ $34^{\circ} 30\frac{1}{2}' = \frac{1}{2}$ sum
 $G - D = 40^{\circ} 19'$ $20^{\circ} 09\frac{1}{2}' = \frac{1}{2}$ diff.

Sum of the four Logs $19,46662$
 $\frac{1}{2}$ sum gives $32^{\circ} 45\frac{1}{2}'$ $9,73331$
 The sup. of its double is $114^{\circ} 29' = AB$.

To find the side AC. (155)

Here $\angle C = E = 125^{\circ} 20'$
 $\angle A = F = 48^{\circ} 31'$

Ar. Co. sine $E = 125^{\circ} 20'$ 0,08842
 Ar. Co. sine $F = 48^{\circ} 31'$ 0,12543
 Sine $\frac{1}{2}$ sum $= 96^{\circ} 58\frac{1}{2}'$ 9,99677
 Sine $\frac{1}{2}$ diff. $= 20^{\circ} 09\frac{1}{2}'$ 9,53733

Sup. $\angle B = G = 117^{\circ} 08'$
 $E - F = D = 76^{\circ} 49'$

$E + D = 193^{\circ} 57'$ $96^{\circ} 58\frac{1}{2}' = \frac{1}{2}$ sum
 $G - D = 40^{\circ} 19'$ $20^{\circ} 09\frac{1}{2}' = \frac{1}{2}$ diff.

Sum of four Logs $19,74795$
 $\frac{1}{2}$ sum given $48^{\circ} 25\frac{1}{2}'$ $9,87397$
 The sup. of its double is $83^{\circ} 09' = CA$.

To find the side BC. (155)

Here $\angle C = E = 125^{\circ} 20'$
 $\angle B = F = 62^{\circ} 52'$

Ar. Co. sine $E = 125^{\circ} 20'$ 0,08842
 Ar. Co. sine $F = 62^{\circ} 52'$ 0,05064
 Sine $\frac{1}{2}$ sum $= 96^{\circ} 58\frac{1}{2}'$ 9,99677
 Sine $\frac{1}{2}$ diff. $= 34^{\circ} 30\frac{1}{2}'$ 9,75322

Sup. $\angle A = G = 131^{\circ} 29'$
 $E - F = D = 62^{\circ} 28'$

$E + D = 193^{\circ} 57'$ $96^{\circ} 58\frac{1}{2}' = \frac{1}{2}$ sum
 $G - D = 69^{\circ} 01'$ $34^{\circ} 30\frac{1}{2}' = \frac{1}{2}$ diff.

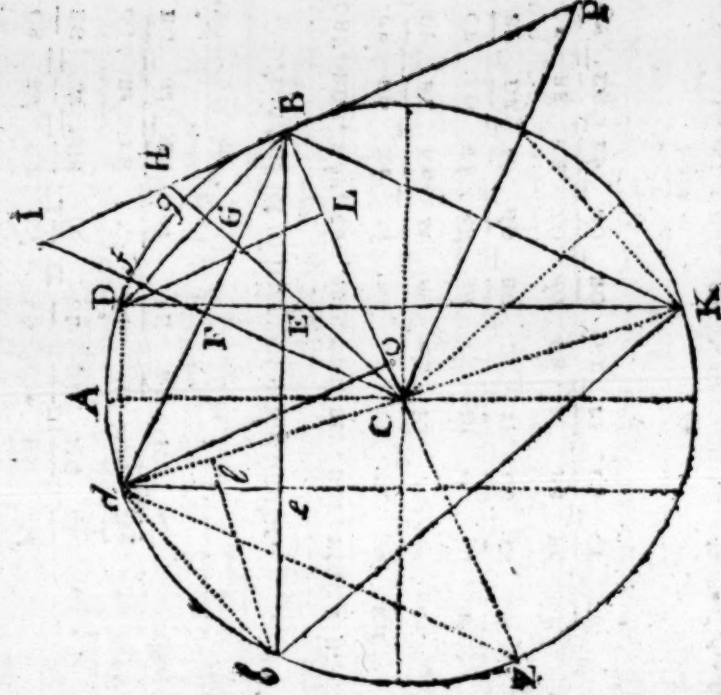
Sum of four Logs $19,88995$
 $\frac{1}{2}$ sum gives $51^{\circ} 39'$ $9,94452$

The sup. of its double is $56^{\circ} 42' = BC$.

SECTION

SECTION. X.

169. The principles already delivered have been shewn sufficient for deriving methods for the solution of all the cases in spherical Trigonometry : yet as there are many other useful and curious particulars which appertain to the subject, it was thought proper to add some of them for the entertainment of speculative readers. The chief of these relations cannot, perhaps, be better investigated, than by imitating the method of the late William Jones, Esq. who published in the year 1747, in the Philosophical Transactions, N^o 483, some properties of Goniometrical lines ; which properties are mostly derived from a general figure which Mr. Jones improved from one communicated to him by the great Dr. Halley. See *Synopsis Palmariorum Mathematicos*, p. 245.



Let AB, AD ; or Ab, Ad , be any two arcs, each less than 90 degrees.
 Be and BE , or bE and bE , be the sum and difference of their right lines.
 KE and DE , the sum and difference of their co-sines.

Let the arcs Bd, BD ; or bD, bD , be the sum and difference of arcs AB, AD .
 dO, DL , sines of the arcs Bd, BD , the sum and difference of arcs AB, AD ;
 BO, BL , the versed sines of that sum and difference.
 $zO, zL=KL$, the versed sines of the supplements of their sum and diff.

N 2

Let

Let the arcs Bf , Bg , be the half sum, and half diff. of the arcs AB , AD .
 Bf , Bg , the lines

Cf , Cg , the co-fines

Bi , Bh , the tangents

Ci , Ch , the secants

Bd , Bd , twice the lines

Kb , Kb , twice the co-fines

PB , PC , the co-tangent and the co-secant of the half sum of the arcs AB , AD .

} of the half sum and half diff. of AB and AD .

Now the following set of triangles being similar,

viz. CBG , Bde , KbE , Kbl , DBL , CHB , BHG , Kdb .

$$\text{Then } \frac{CB}{CG} = \frac{Bd}{Be} = \frac{KB}{KE} = \frac{Kb}{Kl} = \frac{BD}{DL} = \frac{CH}{CB} = \frac{BH}{BG} = \frac{Kd}{Kb}$$

$$\frac{CB}{BG} = \frac{Bd}{de} = \frac{KB}{BE} = \frac{Kb}{bl} = \frac{BD}{BL} = \frac{CH}{BH} = \frac{BH}{HG} = \frac{Kd}{db}$$

$$\frac{CG}{BG} = \frac{Be}{de} = \frac{KE}{BE} = \frac{Kl}{bl} = \frac{DL}{BL} = \frac{CB}{BH} = \frac{BG}{HG} = \frac{Kb}{db}$$

Also this set of triangles being similar,

viz. CBF , BdE , KbE , ZdO , dBO , CIB , BIF , PCB , PIG , KdB .

$$\text{Then } \frac{CB}{CF} = \frac{Bd}{Bf} = \frac{Kb}{KE} = \frac{Zd}{ZO} = \frac{dD}{dO} = \frac{CB}{CO} = \frac{BI}{BF} = \frac{PC}{PB} = \frac{PI}{PC} = \frac{Kd}{BK}$$

$$\frac{CB}{BF} = \frac{Bd}{DE} = \frac{Kb}{Eb} = \frac{Zd}{dO} = \frac{dD}{BO} = \frac{BI}{IF} = \frac{CB}{CI} = \frac{dK}{dB}$$

$$\frac{CF}{BF} = \frac{BE}{DE} = \frac{KE}{Eb} = \frac{ZO}{dO} = \frac{dO}{BO} = \frac{CB}{BI} = \frac{BF}{IF} = \frac{PC}{CB} = \frac{CI}{CI} = \frac{BK}{Bd}$$

'Then from the third line of both sets will be collected the following equal ratios, arising from equal antecedents.

$$\text{viz. } \frac{dO}{BO} = \frac{Be}{de} = \frac{BE}{DE} = \frac{BL}{BL} = \frac{ZO}{dO} = \frac{KE}{BE} = \frac{Kl}{Eb} = \frac{CF}{BF} = \frac{CG}{BG}$$

170. Now the several values of the radius CB being collected, are placed as in the annexed table; where the letters s, t, f, v , stand for the sine, tangent, secant, versed sine; and the letters $\dot{s}, \dot{t}, \dot{f}$, the co-sine, co-tangent, co-secant, of the arcs A, a ; or of the arcs $\frac{1}{2}A+a, \frac{1}{2}A-a$; and v , the versed sine of the supplement.

Goniometrical Properties.			
(171) $\frac{\dot{s}, A + \dot{s}, a}{\dot{s}, A - \dot{s}, a} \times \dot{t}, \frac{1}{2}A + a.$	(172) $\frac{\dot{s}, A + \dot{s}, a}{\dot{s}, A - \dot{s}, a} \times \dot{t}, \frac{1}{2}A - a.$	(173) $\frac{2\dot{s}, \frac{1}{2}A - a}{\dot{s}, A + \dot{s}, a} \times \dot{s}, \frac{1}{2}A + a.$	(174) $\frac{2\dot{s}, \frac{1}{2}A - a}{\dot{s}, A + \dot{s}, a} \times \dot{s}, \frac{1}{2}A - a.$
(175) $\frac{\dot{s}, A + \dot{s}, a}{\dot{s}, A - \dot{s}, a} \times \dot{t}, \frac{1}{2}A + a.$	(176) $\frac{\dot{s}, A - \dot{s}, a}{\dot{s}, A - \dot{s}, a} \times \dot{t}, \frac{1}{2}A - a.$	(177) $\frac{\dot{s}, A + \dot{s}, a}{2\dot{s}, \frac{1}{2}A - a} \times \dot{s}, \frac{1}{2}A + a.$	(178) $\frac{\dot{s}, A + \dot{s}, a}{2\dot{s}, \frac{1}{2}A - a} \times \dot{s}, \frac{1}{2}A - a.$
(179) $\frac{\dot{s}, A - \dot{s}, a}{\dot{s}, A - \dot{s}, a} \times \dot{t}, \frac{1}{2}A + a.$	(180) $\frac{\dot{s}, A + \dot{s}, a}{\dot{s}, A - \dot{s}, a} \times \dot{t}, \frac{1}{2}A - a.$	(181) $\frac{2\dot{s}, \frac{1}{2}A - a}{\dot{s}, A - \dot{s}, a} \times \dot{s}, \frac{1}{2}A + a.$	(182) $\frac{2\dot{s}, \frac{1}{2}A - a}{\dot{s}, A - \dot{s}, a} \times \dot{s}, \frac{1}{2}A - a.$
(183) $\frac{\dot{s}, A - \dot{s}, a}{\dot{s}, A - \dot{s}, a} \times \dot{t}, \frac{1}{2}A + a.$	(184) $\frac{\dot{s}, A - \dot{s}, a}{\dot{s}, A - \dot{s}, a} \times \dot{t}, \frac{1}{2}A - a.$	(185) $\frac{\dot{s}, A - \dot{s}, a}{2\dot{s}, \frac{1}{2}A - a} \times \dot{s}, \frac{1}{2}A + a.$	(186) $\frac{\dot{s}, A - \dot{s}, a}{2\dot{s}, \frac{1}{2}A - a} \times \dot{s}, \frac{1}{2}A - a.$
(187) $\frac{\dot{s}, \frac{1}{2}A + a}{\dot{s}, \frac{1}{2}A - a} \times \dot{t}, \frac{1}{2}A + a.$	(188) $\frac{\dot{s}, \frac{1}{2}A - a}{\dot{s}, \frac{1}{2}A - a} \times \dot{t}, \frac{1}{2}A - a.$	(189) $\frac{2\dot{s}, \frac{1}{2}A + a}{\dot{s}, A + a} \times \dot{s}, \frac{1}{2}A + a.$	(190) $\frac{2\dot{s}, \frac{1}{2}A - a}{\dot{s}, A - a} \times \dot{s}, \frac{1}{2}A - a.$
(191) $\frac{\dot{v}, A + a}{\dot{s}, A - a} \times \dot{t}, \frac{1}{2}A + a.$	(192) $\frac{\dot{v}, A - a}{\dot{s}, A - a} \times \dot{t}, \frac{1}{2}A - a.$	(193) $\frac{2\dot{s}, \frac{1}{2}A + a}{\dot{v}, A + a} \times \dot{s}, \frac{1}{2}A + a.$	(194) $\frac{2\dot{s}, \frac{1}{2}A - a}{\dot{v}, A - a} \times \dot{s}, \frac{1}{2}A - a.$
(195) $\frac{\dot{v}, A + a}{\dot{v}, A - a} \times \dot{t}, \frac{1}{2}A + a.$	(196) $\frac{\dot{v}, A - a}{\dot{v}, A - a} \times \dot{t}, \frac{1}{2}A - a.$	(197) $\frac{2\dot{s}, \frac{1}{2}A + a}{\dot{v}, A + a} \times \dot{s}, \frac{1}{2}A + a.$	(198) $\frac{2\dot{s}, \frac{1}{2}A - a}{\dot{v}, A - a} \times \dot{s}, \frac{1}{2}A - a.$
(199) $\frac{\dot{v}, A + a}{\dot{v}, A - a} \times \dot{t}, \frac{1}{2}A + a.$	(200) $\frac{\dot{v}, A - a}{\dot{v}, A - a} \times \dot{t}, \frac{1}{2}A - a.$	(201) $\frac{\dot{s}, \frac{1}{2}A + a}{\dot{t}, \frac{1}{2}A + a} \times \dot{s}, \frac{1}{2}A + a.$	(202) $\frac{\dot{s}, \frac{1}{2}A + a}{\dot{t}, \frac{1}{2}A + a} \times \dot{s}, \frac{1}{2}A - a.$
(203) $\frac{\dot{v}, A + a}{\dot{v}, A - a} \times \dot{t}, \frac{1}{2}A + a.$	(204) $\frac{\dot{s}, \frac{1}{2}A - a}{\dot{t}, \frac{1}{2}A - a} \times \dot{s}, \frac{1}{2}A - a.$	(205) $\frac{\dot{s}, \frac{1}{2}A + a}{\dot{t}, \frac{1}{2}A + a} \times \dot{s}, \frac{1}{2}A + a.$	(206) $\frac{\dot{s}, \frac{1}{2}A + a}{\dot{t}, \frac{1}{2}A + a} \times \dot{s}, \frac{1}{2}A - a.$
(207) $\frac{\dot{v}, A + a}{\dot{v}, A - a} \times \dot{t}, \frac{1}{2}A + a.$	(208) $\frac{\dot{s}, \frac{1}{2}A - a}{\dot{t}, \frac{1}{2}A - a} \times \dot{s}, \frac{1}{2}A - a.$	(209) $\frac{\dot{s}, \frac{1}{2}A + a}{\dot{t}, \frac{1}{2}A + a} \times \dot{s}, \frac{1}{2}A + a.$	(210) $\frac{\dot{s}, \frac{1}{2}A + a}{\dot{t}, \frac{1}{2}A + a} \times \dot{s}, \frac{1}{2}A - a.$

From the preceding table a very great number of properties are readily deduced; some of which are here annexed, as examples of its use; where analogies are, in general, expressed by equal ratios.

211. The $\frac{\text{sum of the fines of two arcs}}{\text{diff. of the fines of those arcs}} = \frac{\text{tan. of half the sum of those arcs}}{\text{tan. of half the diff. of the arcs}}$ (171, 172)

212. The $\frac{\text{sum of the co. sin. of two arcs}}{\text{diff. of the co. sin. of those arcs}} = \frac{\text{co-tan. of half the sum of the arcs}}{\text{tan. of half the diff. of the arcs}}$ (175, 180)

213. The $\frac{\text{fine of the sum of two arcs}}{\text{line of the diff. of those arcs}} = \frac{\text{sum of the tan. of those arcs}}{\text{diff. of the tan. of those arcs}}$,

For $\frac{s, A + s, a}{s, A - s, a} = \frac{t, \frac{1}{2}A + a}{t, \frac{1}{2}A - a}$ (211.) And $\frac{s, A + s, A + s, A - s, a}{s, A + s, a - s, A + s, A} = \frac{t, \frac{1}{2}A + a + t, \frac{1}{2}A - a}{t, \frac{1}{2}A + a - t, \frac{1}{2}A - a}$ by Composition.

Then $\frac{t, \frac{1}{2}A + a + t, \frac{1}{2}A - a}{t, \frac{1}{2}A + a - t, \frac{1}{2}A - a} = \left(\frac{s, A + s, A}{s, a + s, a} \right)$ (III. 47, 48) $= \frac{2s, A}{2s, a} = \frac{s, A}{s, a}$.

Here the arcs A, a , are the sum and diff. of the arcs $\frac{1}{2}A + a, \frac{1}{2}A - a$.

214. The $\frac{\text{cof. of sum of two arcs}}{\text{cof. of diff. of the arcs}} = \frac{\text{diff. of tan. of one and cot. of other}}{\text{sum of tan. of one and cot. of other}}$ taking the tan. of the same arc.

For $\frac{t, \frac{1}{2}A + a}{t, \frac{1}{2}A - a} = \frac{s, A + s, a}{s, a - s, A}$ (212.) And $\frac{t, \frac{1}{2}A + a - t, \frac{1}{2}A - a}{t, \frac{1}{2}A + a + t, \frac{1}{2}A - a} = \frac{s, A + s, a - s, a - s, A}{s, A + s, a + s, a - s, A}$.

Then $\frac{t, \frac{1}{2}A + a - t, \frac{1}{2}A - a}{t, \frac{1}{2}A + a + t, \frac{1}{2}A - a} = \left(\frac{2s, A}{2s, a} \right) = \frac{s, A}{s, a}$.

Here the arcs A, a , are the sum and diff. of the arcs $\frac{1}{2}A + a, \frac{1}{2}A - a$.

215. The fine of the sum of two arcs, into Radius; is equal to the sum of the products, of the fine of the greater by the co-fine of the less, and the fine of the less by the co-fine of the greater. And,

The fine of the difference of two arcs, into Radius; is equal to the difference of the products, of the fine of the greater by the co-fine of the less, and the fine of the less by the co-fine of the greater.

For $\left\{ \begin{array}{l} R \times \frac{1}{2}s, A + \frac{1}{2}s, a = s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a \text{ (173).} \\ R \times \frac{1}{2}s, A - \frac{1}{2}s, a = t, \frac{1}{2}A - a \times s, \frac{1}{2}A + a \text{ (182).} \end{array} \right\}$ Here $\frac{1}{2}A + a$ and $\frac{1}{2}A - a$ are the arcs.

Hence $\left\{ \begin{array}{l} R \times s, A \text{ (the sum)} = s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a + s, \frac{1}{2}A - a \times s, \frac{1}{2}A + a, \\ R \times s, A \text{ (the diff.)} = s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a - s, \frac{1}{2}A - a \times s, \frac{1}{2}A + a. \end{array} \right.$

216. The

216. The co-fine of the sum of two arcs, into Radius; is equal to the difference, between the product of the co-fines, and product of the fines, of those arcs.

The co fine of the difference of two arcs, into Radius; is equal to the sum, of the product of the co-fines, and product of the fines, of those arcs.

For $\left\{ \begin{array}{l} R \times \frac{1}{2} s, A + \frac{1}{2} s, A = s, \frac{1}{2} A + a \times s, \frac{1}{2} A - a \text{ (174).} \\ R \times \frac{1}{2} s, a - \frac{1}{2} s, A = s, \frac{1}{2} A + a \times s, \frac{1}{2} A - a \text{ (181).} \end{array} \right\} \frac{1}{2} A + a \text{ and } \frac{1}{2} A - a \text{ being the arcs.}$

Then $\left\{ \begin{array}{l} R \times s, A \text{ (the sum)} = s, \frac{1}{2} A + a \times s, \frac{1}{2} A - a - s, \frac{1}{2} A + a \times s, \frac{1}{2} A - a. \\ R \times s, a \text{ (the diff.)} = s, \frac{1}{2} A + a \times s, \frac{1}{2} A - a + s, \frac{1}{2} A + a \times s, \frac{1}{2} A - a. \end{array} \right.$

Radius, lets the co-fine of an arc = Square the tan. of half that arc
217. Radius, more the co-fine of an arc = Square of the Radius .

$$(195)$$

$$R - s, A + a = \left(\frac{t, \frac{1}{2} A + a}{R} \right)^2 \times s, A + a.$$

For $R + s, A + a = \left(\frac{t, \frac{1}{2} A + a}{R} \right)^2 \times s, A + a.$ (191)

$$\text{Then } \frac{R - s, A + a}{R + s, A + a} = \frac{t, \frac{1}{2} A + a}{R}$$

218. The sum of the fine & co-fine of an arc = Radius
The diff. of the fine & co-fine of that arc = tan. of diff. of that arc & 45° .

The sum of Rad. and tan. of an arc = Radius

The diff. of Rad. and tan. of that arc = tan. of diff. of that arc & 45° .

For, if $A + a = 90^\circ$; then $\frac{1}{2} A = 45^\circ - \frac{1}{2} a$; and $\frac{1}{2} a = 45^\circ - \frac{1}{2} A$.

Also $s, a = s, A$; $s, a = s, A$: and $s, A = t, A \times s, A \div R$. (III. 33)

Then $\frac{s, A + s, A}{s, a - s, A} = \frac{t, \frac{1}{2} A + a}{t, \frac{1}{2} A - a}.$ (212)

$$\text{Or } \frac{s, A + s, A}{s, A - s, A} = \left(\frac{t, 45^\circ - \frac{1}{2} a + \frac{1}{2} a}{t, \frac{1}{2} A - 45^\circ + \frac{1}{2} A} \right) = \frac{R}{t, A \tan 45^\circ}.$$

$$\text{Again, } \frac{s, A + s, A}{s, A - s, A} = \left(\text{III. 33} \right) \frac{t, A \times s, A \div R + s, A}{t, A \times s, A \div R - s, A} = \frac{t, A \times s, A + s, A}{t, A \times s, A - s, A}.$$

$$\text{Then } \left(\frac{s, A + s, A}{s, A - s, A} \right) = \frac{t, A + t, A \times s, A}{t, A - t, A \times s, A} = \frac{R + t, A}{R \tan t, A} = \frac{R}{t, A \tan 45^\circ}.$$

• This table shew; the difference of the values it stands between.

219. The difference of the co-fines of two arcs, is equal to the difference of the verfed fines of those arcs.

220. The product of the fines of two arcs, is equal to the product of half the Radius into the difference of the co-fines, of the sum and difference of those arcs.

$$\text{That is, } s, \frac{1}{2}A + a \times s, \frac{1}{2}A - a = \frac{1}{2}R \times s, a - s, a = s, \frac{1}{2}A + a - \frac{1}{2}A - a = s, \frac{1}{2}A + a + \frac{1}{2}A - a. \quad (181)$$

$$\text{Or, } s, z \times s, x = \frac{1}{2}R \times s, z + x - s, z - x. \text{ Putting } z = \frac{1}{2}A + a; x = \frac{1}{2}A - a.$$

221. The product of the fines of two arcs, is equal to the product of half the Radius into the difference between the verfed fines, of the sum and difference of those arcs.

$$\text{That is, } s, z - x \times s, x = (\frac{1}{2}R \times s, z + x - s, z - x)(220) = \frac{1}{2}R \times s, z - x + \frac{1}{2}R. \quad (219)$$

$$222. \text{ Half the Radius} = \frac{\text{square of the fine of an arc}}{\text{verfed fine of twice that arc}}. \quad (193)$$

$$= \frac{\text{square of the co-fine of an arc}}{\text{sup-verfed fine of twice that arc}}. \quad (197)$$

$$= \frac{\text{product of the fines of two arcs}}{\text{diff. of ver. fines of the sum and diff. of those arcs}}. \quad (221)$$

$$223. \text{ The sq. of Rad.} = \frac{\text{prod. of the squares, of the fine and cot. of an arc}}{\text{square of the co-fine of that arc}} = \frac{\text{prod. of the squares of the co-fine & tan of an arc}}{\text{square of the fine of that arc}}.$$

$$\text{For } R = \frac{s, A \times t, A}{s, A} = \frac{s, A \times t, A}{s, A} \quad (187.) \text{ Then } RR = \frac{ss, A \times tt, A}{ss, A} = ss, A.$$

224. The product of Radius, and the co-fine of an arc, is equal to the difference of the squares, of the fine and co-fine of half that arc.

$$\text{For } \frac{s, \frac{1}{2}A + a}{ss, \frac{1}{2}A + a} = \left(\frac{v, A + a}{v, A + a} \right) (193, 197) = \frac{R + s, A + a}{R - s, A + a}. \text{ Put } z = \frac{1}{2}A + a.$$

$$\text{Then } \frac{R + s, z - R + s, z}{R + s, z + R - s, z} = \frac{ss, \frac{1}{2}z - ss, \frac{1}{2}z}{ss, \frac{1}{2}z + ss, \frac{1}{2}z}. \text{ By composition.}$$

$$\text{Or } \frac{ss, z}{2R} = \frac{ss, \frac{1}{2}z - ss, \frac{1}{2}z}{RR} \quad (\text{II. III.}) \text{ And } R \times s, z = ss, \frac{1}{2}z - ss, \frac{1}{2}z.$$

$$= s, \frac{1}{2}z + s, \frac{1}{2}z \times s, \frac{1}{2}z - s, \frac{1}{2}z. \quad (\text{II. 119})$$

ln

In any spheric triangle ABC, if in the side CB produced, be taken BE, BD, each equal to BA, and BG be drawn at right angles to CA.

Then $CE = BC + BA$ is the sum of the legs including the angle at B.

$CD = BC - BA$ is the diff. of the legs including the angle at B.

CG and AG are the segments of the

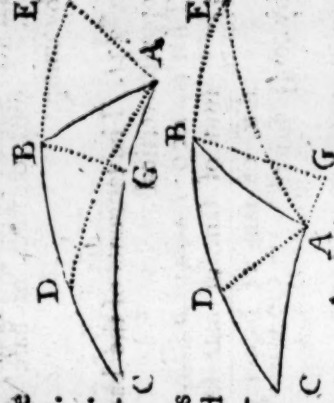
basis, or side opposite to the angle B.

∠A and ∠C are called base angles.

∠CBG = a, ∠ABG = c are the ver-

tical angles.

Now a very great number of relations may be formed between the sides and angles; some of which are here enumerated.



225. The sines of the legs, are as the sines of the opposite base angles.

That is, $s, BC : s, BA :: s, A : s, C$.

(110)

sum of the sines of the legs = sum of the sines of the base angles

Hence $\frac{\text{diff. of the sines of the legs}}{\text{sum of the sines of the legs}} = \frac{\text{diff. of the sines of the base angles}}{\text{sum of the sines of the base angles}}$ by composition.

226. The co-sines of the base angles, are as the sines of the vertical angles.

That is, $s, C : s, A :: s, a : s, c$.

(3d 122, and 2d of 124)

sum of co-sines of base angles = sum of sines of vertical angles

Hence $\frac{\text{diff. of co-sines of base angles}}{\text{sum of co-sines of base angles}} = \frac{\text{diff. of sines of vertical angles}}{\text{sum of sines of vertical angles}}$ by composition.

227. The co-sines of the legs, are as the co-sines of the adjacent segments of the base.

That is, $s, BC : s, BA :: s, CG : s, AG$.

(3d of 121, and 2d of 123)

sum of co-sines of the legs = sum of co-sines of base segments

Hence $\frac{\text{diff. of co-sines of the legs}}{\text{sum of co-sines of the legs}} = \frac{\text{diff. of co-sines of base segments}}{\text{sum of co-sines of base segments}}$ by composition.

228. The co-tangents of the legs, are as the co-sines of the adjacent vertical angles.

That is, $t, BC : t, BA :: s, a : s, c$.

(2d of 121, and 2d of 124)

sum of cot. of the legs = sum of co-sines of vert. angles

Hence $\frac{\text{diff. of cot. of the legs}}{\text{sum of cot. of the legs}} = \frac{\text{diff. of co-sines of vert. angles}}{\text{sum of co-sines of vert. angles}}$ by comp.

229. The tangents of the legs, are as the cof. of the adjacent vertical angles reciprocally.

sum of tan. of the legs = sum of cof. of vert. angles by comp.

Hence $\frac{\text{diff. of tan. of the legs}}{\text{sum of tan. of the legs}} = \frac{\text{diff. of cof. of vert. angles}}{\text{sum of cof. of vert. angles}}$ by comp.

230. The

230. The sines of the base segments, are as the tangents of the adjacent base angles reciprocally.

That is, $s, CG : s, AG :: (t, C : t, A ::) t, A : t, C$. (2d of 122, and 1st of 123)
Hence $\frac{\text{sum of sines of base segments}}{\text{diff. of sines of base segments}} = \frac{\text{sum of tan. of base angles}}{\text{diff. of tan. of base angles}}$
by comp.

231. The tangents of the base segments. are as the tangents of the opposite vertical angles.

That is, $t, CG : t, AG :: t, a : t, c$. (108)
Hence $\frac{\text{sum of tan. of base segments}}{\text{diff. of tan. of base segments}} = \frac{\text{sum of tan. of vert. angles}}{\text{diff. of tan. of vert. angles}}$
by composition.

232. The $\frac{\text{tan of half the sum of the legs}}{\text{tan. of half the diff. of the legs}} = \frac{\text{tan. of half the sum of the base ang.}}{\text{tan. of half the diff. of the base ang.}}$

$$\text{For } \frac{s, BC + s, BA}{s, BC - s, BA} = \frac{s, A + s, C}{s, A - s, C} \quad (225) = \frac{t, \frac{1}{2}BC + BA}{t, \frac{1}{2}BC - BA} = \frac{t, \frac{1}{2}A + C}{t, \frac{1}{2}A - C} \quad (211)$$

233. The $\frac{\text{tan. of } \frac{1}{2} \text{ sum of base segments}}{\text{tan. of } \frac{1}{2} \text{ sum of the legs}} = \frac{\text{tan. of } \frac{1}{2} \text{ diff. of the legs}}{\text{tan. of } \frac{1}{2} \text{ diff. of the base segments}}$

$$\text{For } \frac{s, BA + s, BC}{s, BA - s, BC} = \frac{s, GA + s, GC}{s, GA - s, GC} \quad (227) = \frac{t, \frac{1}{2}BC + BA}{t, \frac{1}{2}BC - BA} = \frac{t, \frac{1}{2}CG + GA}{t, \frac{1}{2}CG - GA} \quad (212)$$

$$\text{Then } \frac{t, \frac{1}{2}BC - BA}{t, \frac{1}{2}CG - GA} = \left(\frac{t, \frac{1}{2}BC + BA}{t, \frac{1}{2}CG + GA} \right) \quad (\text{II. 145}) = \frac{t, \frac{1}{2}CG + GA}{t, \frac{1}{2}BC + BA} \quad (\text{III. 37})$$

234. The $\frac{\text{fine of sum of legs}}{\text{fine of diff. of legs}} = \frac{\text{co-tan. of } \frac{1}{2} \text{ sum of vert. angles}}{\text{tan. of } \frac{1}{2} \text{ diff. of vert angles}}$
 $= \frac{\text{co-tan. } \frac{1}{2} \text{ diff. of vert. angles}}{\text{tan. of } \frac{1}{2} \text{ sum of vert. angles}}$

$$\text{For } \frac{s, EC + BA}{s, EC - BA} = \left(\frac{t, BC + t, BA}{t, BC - t, BA} \right) \quad (213) = \frac{s, c + s, a}{s, c - s, a} \quad (226) = \frac{t, \frac{1}{2}a + c}{t, \frac{1}{2}a - c} = \frac{t, \frac{1}{2}a - c}{t, \frac{1}{2}a + c} \quad (112)$$

235. The $\frac{\text{cot. of } \frac{1}{2} \text{ sum of vert. angles}}{\text{tan. of } \frac{1}{2} \text{ sum of the base angles}} = \frac{\text{tan. of } \frac{1}{2} \text{ diff. of base angles}}{\text{tan of } \frac{1}{2} \text{ diff. of vert. angles}}$

$$\text{For } \frac{t, \frac{1}{2}A + C}{t, \frac{1}{2}A - C} = \left(\frac{s, C + s, A}{s, C - s, A} \right) \quad (212) = \frac{s, a + s, c}{s, a - s, c} \quad (226) = \frac{t, \frac{1}{2}a + c}{t, \frac{1}{2}a - c} \quad (211)$$

$$\text{Hence } \frac{t, \frac{1}{2}A - C}{t, \frac{1}{2}a - c} = \left(\frac{t, \frac{1}{2}A + C}{t, \frac{1}{2}a + c} \right) \quad (\text{II. 145}) = \frac{t, \frac{1}{2}a + c}{t, \frac{1}{2}A + C} \quad (\text{III. 37})$$

236. The $\frac{\text{fine of sum of the legs}}{\text{fine of diff. of the legs}} = \frac{\text{square of cot. of } \frac{1}{2} \text{ sum of vert. angles}}{\tan. \frac{1}{2} \text{ sum, into tan. } \frac{1}{2} \text{ diff. of base } \angle}$.

$$\text{For } \frac{t_{\frac{1}{2}A+C} \times t_{\frac{1}{2}A-C}}{t_{\frac{1}{2}A+C}} = t_{\frac{1}{2}A-C}. \quad (235)$$

$$\text{Then } s_{\frac{1}{2}BC+BA} : s_{\frac{1}{2}BC-BA} :: t_{\frac{1}{2}A+C} : \frac{t_{\frac{1}{2}A+C} \times t_{\frac{1}{2}A-C}}{t_{\frac{1}{2}A+C}}. \quad (234)$$

$$:: t_{\frac{1}{2}A+C} : t_{\frac{1}{2}A+C} \times t_{\frac{1}{2}A-C}. \quad (\text{II. 151})$$

237. The $\frac{\text{fine of } \frac{1}{2} \text{ the sum of legs}}{\text{fine of } \frac{1}{2} \text{ the diff. of legs}} = \frac{\text{co-tan. of } \frac{1}{2} \text{ the sum of vert. angles}}{\tan. \text{ of } \frac{1}{2} \text{ the diff. of the base angles}}$.

$$\text{For } \frac{t_{\frac{1}{2}A+C} \times t_{\frac{1}{2}A-C}}{t_{\frac{1}{2}A+C} \times t_{\frac{1}{2}A-C}} = \frac{s_{\frac{1}{2}BC+BA}}{s_{\frac{1}{2}BC-BA}} \quad (236). \text{ And } \frac{t_{\frac{1}{2}A+C}}{t_{\frac{1}{2}A-C}} = \frac{t_{\frac{1}{2}BC+BA}}{t_{\frac{1}{2}BC-BA}}. \quad (232)$$

$$\text{Then } \frac{t_{\frac{1}{2}A+C} \times t_{\frac{1}{2}A-C}}{t_{\frac{1}{2}A+C} \times t_{\frac{1}{2}A-C} \times t_{\frac{1}{2}A-C}} = \frac{s_{\frac{1}{2}BC+BA} \times t_{\frac{1}{2}BC+BA}}{s_{\frac{1}{2}BC-BA} \times t_{\frac{1}{2}BC-BA}}. \quad (\text{II. 156})$$

$$\text{And } \frac{t_{\frac{1}{2}A+C}}{t_{\frac{1}{2}A-C}} = \frac{2s_{\frac{1}{2}BC+BA}}{2s_{\frac{1}{2}BC-BA}} \quad (195, 193). \text{ Then } \frac{s_{\frac{1}{2}BC+BA}}{s_{\frac{1}{2}BC-BA}} = \frac{t_{\frac{1}{2}A+C}}{t_{\frac{1}{2}A-C}}$$

238. The $\frac{\text{cof. of } \frac{1}{2} \text{ sum of the legs}}{\text{cof. of } \frac{1}{2} \text{ diff. of the legs}} = \frac{\text{cot. of } \frac{1}{2} \text{ the sum of vertical angles}}{\tan. \text{ of } \frac{1}{2} \text{ the sum of the base angles}}$.

$$\text{For } \frac{t_{\frac{1}{2}A+C} \times t_{\frac{1}{2}A-C}}{t_{\frac{1}{2}A+C} \times t_{\frac{1}{2}A-C}} = \frac{s_{\frac{1}{2}BC+BA}}{s_{\frac{1}{2}BC-BA}} \quad (236). \text{ And } \frac{t_{\frac{1}{2}A-C}}{t_{\frac{1}{2}A+C}} = \frac{t_{\frac{1}{2}BC+BA}}{t_{\frac{1}{2}BC-BA}}. \quad (232)$$

$$\text{Then } \frac{t_{\frac{1}{2}A+C} \times t_{\frac{1}{2}A-C}}{t_{\frac{1}{2}A+C} \times t_{\frac{1}{2}A-C} \times t_{\frac{1}{2}A+C}} = \frac{s_{\frac{1}{2}BC+BA} \times t_{\frac{1}{2}BC+BA}}{s_{\frac{1}{2}BC-BA} \times t_{\frac{1}{2}BC-BA}}. \quad (\text{II. 156})$$

$$\text{And } \frac{t_{\frac{1}{2}A+C}}{t_{\frac{1}{2}A+C}} = \frac{2s_{\frac{1}{2}BC+BA}}{2s_{\frac{1}{2}BC-BA}} \quad (191, 197). \text{ Then } \frac{s_{\frac{1}{2}BC+BA}}{s_{\frac{1}{2}BC-BA}} = \frac{t_{\frac{1}{2}A+C}}{t_{\frac{1}{2}A+C}}$$

The two last propositions solve the problem where two sides and the included angle, of a spheric triangle, are given, to find the other angles.

Or where two angles and the included side are given, to find the other sides, using the word, angles for legs; the given side for sum of vertical angles; the other sides for base angles.

In art. 237, 238, the conclusions were gained from this principle, namely, that *the sides of proportional sines, are in the same proportion as the squares.*

239. The

239. The co-sine of an angle, is to Radius :

As the Radius into cof. of the opposite side, less the product of the cosines of the including sides,

To the product of the fines of the including sides,

$$\text{For } s, CG = (s, AC - AC) = s, AC \times s, AG + s, AC \times s, AG \div R. \quad (216)$$

$$\text{And } (s, CG =) \frac{s, BC \times s, AG}{s, AB} \quad (227) = s, AC \times s, AG + s, AC \times s, AG \div R. \quad (\text{II. 46})$$

$$\text{Therefore } s, BC \times s, AG = \frac{s, AB}{R} \times s, AC \times s, AG + \frac{s, AB}{R} \times s, AC \times s, AG.$$

$$\text{Therefore } \frac{R \times s, BC - s, AB \times s, AC}{R} \times s, AG = \frac{s, AB \times s, AC}{R} \times s, AG.$$

$$\text{Then } \frac{R \times s, BC - s, AB \times s, AC}{s, AB \times s, AC} = \left(\frac{s, AG}{s, AG} (\text{II. 163}) = \right) \frac{s, AG}{R}. \quad (187)$$

$$\text{But } s, AG = \frac{s, A}{R} \times s, AB \quad (131). \text{ And } \frac{s, AG}{R} = \left(\frac{s, A}{R R} \times s, AB = \right) \frac{s, A}{R R} \times \frac{s, AB \times R}{s, AB}. \quad (187)$$

$$\text{Then } \frac{s, A}{R} \times \frac{s, AB}{s, AB} = \frac{R \times s, BC - s, AB \times s, AC}{s, AB \times s, AC}. \text{ And } \frac{s, A}{R} = \frac{R \times s, BC - s, AC \times s, AB}{s, AC \times s, AB}.$$

$$240. \text{ Hence } R \times s, BC = \frac{s, A \times s, AC \times s, AB}{R} + s, AC \times s, AB.$$

241. The verfed sine of an angle, is to the square of Radius ;

As the diff. of the verfed fines, of op. side, and diff. of including sides,

To the product of the fines of the sides including that angle.

$$\text{For } (239) \frac{R \times s, BC - s, AC \times s, AB}{s, AC \times s, AB} = \left(\frac{s, A}{R} = \right) \frac{R - v, A}{R}.$$

$$\text{Therefore } R R \times s, BC - R \times s, AC \times s, AB = R \times s, AC \times s, AB - s, AC \times s, AB \times v, A.$$

$$\text{Therefore } R R \times s, BC + s, AC \times s, AB \times v, A = \frac{s, AC \times s, AB + s, AC \times s, AB \times R}{= s, AC - AB \times R R}. \quad (216)$$

$$\text{Then } s, AC \times s, AB \times v, A = (R R \times s, CD - R R \times s, BC =) s, CD - s, BC \times R R.]$$

$$\text{And } \frac{v, A}{R R} = \left(\frac{s, CD - s, BC}{s, AC \times s, AB} = \right) \frac{v, CB - v, CD}{s, AC \times s, AB}. \quad (219)$$

242. Hence

242. Hence $\frac{v, A}{2R} = \frac{v, BC - v, CD}{v, CE - v, CD}$. Or $\frac{1}{2} v, A = \frac{v, BC - v, CD}{v, CE - v, CD}$, when $R = 1$.

For $(s, AC \times s, AB =) \frac{1}{2} R \times \overline{v, CE - v, CD} (222) = \overline{v, CB - v, CD} \times \frac{RR}{v, A}$. (241)

Then $\frac{v, CB - v, CD}{v, CE - v, CD} = \left(\frac{\frac{1}{2} R \times v, A}{RR} = \right) \frac{v, A}{2R}$.

243. The versed sine of the sup. of an angle, is to the square of Radius;
As the diff. of the versed sines, of the opposite side, and sum of
the including sides,

To the product of the sines of the sides including that angle.

For (239) $\frac{R \times s, BC - s, AC \times s, AB}{s, AC \times s, AB} = \left(\frac{s, A}{R} = \right) \frac{v, A - R}{R}$.

Therefore $RR \times s, BC - R \times s, AC \times s, AB = s, AC \times s, AB \times v, A - R \times s, AC \times s, AB$.

Therefore $RR \times s, BC - s, AC \times s, AB \times v, A = R \times \overline{s, AC \times s, AB - s, AC \times s, AB}$
 $= (RR \times s, AG + AB =) RR \times s, CE. (216)$

Then $RR \times s, BC - RR \times s, CE = s, AC \times s, AB \times v, A$.

And $\frac{v, A}{RR} = \left(\frac{s, BC - s, CE}{s, AC \times s, AB} = \right) \frac{v, CE - v, BC}{s, AC \times s, AB}$.

244. Hence $\frac{v, A}{2R} = \frac{v, CE - v, CB}{v, CE - v, CD}$. Or $\frac{1}{2} v, A = \frac{v, CE - v, CB}{v, CE - v, CD}$, when $R = 1$.

For $(s, AC \times s, AB =) \frac{1}{2} R \times \overline{v, CE - v, CD} (222) = \overline{v, CB - v, CD} \times \frac{RR}{v, A}$. (243)

Then $\frac{v, CE - v, CB}{v, CE - v, CD} = \left(\frac{\frac{1}{2} R \times v, A}{RR} = \right) \frac{v, A}{2R}$.

245. The square of the sine of half an angle, is to the square of the
Radius;

As $\frac{1}{2}$ Radius into the diff. of the versed sines, of the side opposite,
and diff. of the sides including that angle,

To the product of the sines of the sides including that angle.

For $(v, A =) \frac{s, \frac{1}{2} A}{\frac{1}{2} R} (222) = \frac{v, CB - v, CD}{s, AC \times s, AB} \times RR$. (241)

Then $\frac{s, \frac{1}{2} A}{RR} = \frac{v, CB - v, CD}{s, AC \times s, AB} \times \frac{1}{2} R = \frac{s, CD - s, CB}{s, AC \times s, AB} \times \frac{1}{2} R$. (219)

246. Hence

$$246. \text{ Hence } \frac{s^{\frac{1}{2}}, \frac{1}{2}}{RR} = \frac{s^{\frac{1}{2}}CB + CD \times s^{\frac{1}{2}}CB - CD}{s, AC \times s, AB} \quad (181)$$

$$= \frac{s^{\frac{1}{2}}CB + AC - AB \times s^{\frac{1}{2}}CB - AC + AB}{s, AC \times s, AB}; \text{ because } CD = AC - AB \\ = \frac{s, H - AC \times s, H - AB}{s, AC \times s, AB}. \text{ Putting } 2H = AC + AB + BC.$$

247. The squ. of the co-sine of half an angle, is to the squ. of Radius;
As $\frac{1}{2}$ Radius into the diff. of verfed sines, of the side opposite, and
sum of the included sides,
To the product of the sines of the sides including that angle.

$$\text{For } (v, A =) \frac{s^{\frac{1}{2}}, \frac{1}{2}}{\frac{1}{2}R} (222) = \frac{v, CE - v, CB}{s, AC \times s, AB} \times RR. \quad (243)$$

$$\text{Then } \frac{s^{\frac{1}{2}}, \frac{1}{2}}{RR} = \frac{v, CE - v, CB}{s, AC \times s, AB} \times \frac{1}{2}R = \frac{s^{\frac{1}{2}}CB - s^{\frac{1}{2}}CE}{s, AC \times s, AB} \times \frac{1}{2}R. \quad (219)$$

$$248. \text{ Hence } \frac{s^{\frac{1}{2}}, \frac{1}{2}}{RR} = \frac{s^{\frac{1}{2}}CE + CB \times s^{\frac{1}{2}}CE - CB}{s, AC \times s, AB}. \quad (177) \\ = \frac{s^{\frac{1}{2}}AC + AB + CB \times s^{\frac{1}{2}}AC + AB - CB}{s, AC \times s, AB}. \text{ For } CE = AC + AB. \\ = \frac{s, H \times s, H - CB}{s, AC \times s, AB}. \text{ Putting } 2H = AC + AB + BC.$$

249. The square of the tan. of half an angle, is to the square of Radius;
As the diff. of ver. sines, of the side op. and diff. of including sides,
To the diff. of ver. sines, of the side op. and sum of including sides.

$$\text{For } \frac{v, A}{v, A} = \frac{s^{\frac{1}{2}}, \frac{1}{2}}{s^{\frac{1}{2}}, \frac{1}{2}} (222) = \frac{t^{\frac{1}{2}}, \frac{1}{2}}{RR} (223) = \frac{v, CB - v, CD}{v, CE - v, CB}. \quad (241, 243)$$

$$250. \text{ Hence } \frac{t^{\frac{1}{2}}, \frac{1}{2}}{RR} = \frac{s^{\frac{1}{2}}CB + CD \times s^{\frac{1}{2}}CB - CD}{s^{\frac{1}{2}}CE + CB \times s^{\frac{1}{2}}CE - CB}. \quad (219, 181) \\ = \frac{s^{\frac{1}{2}}CB + AC - AB \times s^{\frac{1}{2}}CB - AC + CB}{s^{\frac{1}{2}}AC + AB + CB \times s^{\frac{1}{2}}AC + AB - CB} \\ = \frac{s, H - AB \times s, H - AC}{s, H \times s, H - BC}.$$

From the articles in the three last pages may be deduced many rules
for solving the problem, where the three sides are given to find an an-
gle; and thence, from the three angles given to find a side.

251. When two sides and the included angle are given to find the third side: Or when the three sides are given to find an angle; for these particular cases there have been given compendiums by Sir Jonas Moore, in his Mathematics, Vol. II. page 383; Also, by Nicholas Facio Duillier, in a small tract of his, which is very scarce; and by the learned Dr. Pemberton, in the Philosophical Transactions for the year 1760: The principle employed by each of them is the same as in Article 245; which will be here illustrated on account of its utility in some Astronomical Subjects.

In the triangle ABC, where $CD = AC \cos AB$.

Given AB, AC, $\angle A$; required BC.

$$\text{Now } \frac{S, \frac{1}{2} \angle A}{R^3} = \frac{AC \cos AB}{S, AC \times S, AB}$$

$$\text{And } \frac{S, \frac{1}{2} \angle A \times S, AC \times S, AB}{R^3} = \frac{S, CB \cos \frac{1}{2} \angle C D = N.$$

$$\text{Then } 2N - SCD = SCB.$$

Hence the following practical Rules.

252. I. To twice the log. sine of half the given angle,

Add the log. sines of the two containing sides;

The sum, abating three radii in the index, leaves the log. sine of an arc. From twice the nat. sine of that arc; take the nat. co-sine of the diff. of the given sides,

Leaves the nat. co-sine of the third side, or of its supplement.

253. II. But the side required may be found without the use of natural sines. Thus.

To twice the log. sine of half the given angle

Add the log. sines of the two containing sides;

From half the sum of these logs, subtract the log. sine of half the diff. of the sides

And the remainder is the log. tangent of an arc;

The log. sine of which arc, subtracted from the said half sum of logs,

Leaves the log. sine of half the required side.

Take the Example used in the fix cases of oblique spheric triangles.

Where $AC = 83^\circ 11'$, $AB = 56^\circ 40'$; $\angle A = 125^\circ 20'$.

Required BC, which was there found to be $114^\circ 30'$.

Given $\angle = 125^\circ 20'$

its half = $62^\circ 40'$

One side = $83^\circ 11'$

other side = $56^\circ 40'$

Are found = $40^\circ 53' \frac{2}{3}$ its nat. sine 65467 (256)

diff. sides = $26^\circ 31'$ its nat. cosine 89480

its double 130934

$65^\circ 30'$ the nat. cosine 41454 (257)

The side required $114^\circ 30'$



$$\begin{array}{l} \text{Log. sine } \{ 9,94858 \\ \quad \quad \quad 9,94858 \\ \text{Log. sine } 9,99692 \\ \text{Log. sine } 9,92194 \\ \hline 9,81602 \end{array}$$

The said Example wrought by the second Rule is as follows

Given $\angle$	=	125 30	
its half	=	62 45	log sine { 9,94858
One side	=	83 11	log sine { 9,94858
other side	=	56 45	log sine { 9,99692
Sum Logs			log sine { 9,92194
half sum.			39,81602
$\frac{1}{2}$ diff. sides	=	13 15 $\frac{1}{2}$	19,90801 half sum log. 19,90801
Arc	=	74 10	log. tan. 10,54753 Log. sine 74 10 9,98330
			Log. sine of 57 15 9,92481

The required side = 114 30

When the three sides are given to find an angle

254. I. *To the nat. cos. of the side opposite to the required angle, add the nat. cos. Of the diff. of the sides about that angle; half the sum is the nat. sine of an arc. To the log. sine of that arc, add the arith. comps. of the sides about the angle and also the radius.*

The half of this sum is the log. sine of half the angle sought.
Or without using the natural sines

255. II. *To the log. sine of half the diff. of sides about the angle, add the arith. comp. of the log. sine of half the base; the sum is the log. sine of an arc. To the log. cosine of this arc, add the log. sine of half the base, rejecting radius. To the double of this sum add the arith. comps of the log. sines of the containing sides.*

Half the sum is the log. sine of half the angle.

EXAM. Let the three sides be $BC=114\ 30$, $AC=80\ 11$, $AB=56^\circ\ 45'$. Required the angle A.

By I. Rule.

Base BC	=	114 30	
Its suppl.	65 30	nat. cosine =	41454 (256)
Diff. sides	26 31	nat. cos.	= 89480
Sum			130934
Half sum is the nat. sin. (217)		65467. arc 40° 53 $\frac{1}{2}$	log. sine 9,81602
		Ar. co. log. sine 83 11	0,00308
		Ar. co. log. sine 56 40	0,07806
			19,89710
			9,94858

By II. Rule

$\frac{1}{2}BC = 13\ 15\frac{1}{2}$	Log. sine	9,36048	
$\frac{1}{2}BC \pm 57\ 15$	Ar. Co. L. sin	0,07518	log. sine
Log. sine 15 49 $\frac{1}{2}$ an arc		9,43566	l. cos. 15 49 $\frac{1}{2}$
Log. sin. of $\frac{1}{2}BC \propto$ Log. f. $\frac{1}{2}CD$			9,90804
			2
2 L. f. $\frac{1}{2}A + L. f. AC + L. f. AB$			19,81608
Ar. Co. Log. sine AC			0,00308
Ar. Co. Log. sine AB			0,07806
Sum			19,89722
Half sum is Log. sine of 62 40			9,94861
Angle sought is			125 20 = $\angle A$.

As

As the natural sines of arcs, are not contained in this work, and are on some occasions necessary, it will be proper to shew how they are to be found from the Logarithmic tables contained herein.

256. *First. An arc being given, to find its natural sine to five places of figures.*

Rule. Take out the Log. sine of the arc, rejecting the Index;

Seek these figures among the logarithms of numbers;

The corresponding number is the natural sine of the given arc;

which is to be reckoned as a decimal fraction of the radius, or unity:

Prefixing the decimal comma (,) if the index of the log. sine was 9; But if the index was 8; 7; or 6; prefix 0,00; or ,000; whereby the left hand digit of the natural sine will stand in the place of firsts, seconds, thirds, or fourths.

Ex. I. *Required the natural sine and cosine of $4^{\circ} 22'$?*

Log. sines	sine	8,88161	Cosine	9,99874
Num. or nat. sines		0,07614		0,99710

Ex. II. *Required the natural sine and cosine of $28^{\circ} 35'$?*

Log. sines	sine	9,67982	Cosine	9,94355
Num. or nat. sines		0,47844		0,87812

If a given log. sine is found in the table of logs. of numbers, its natural number consisting of four places is seen at sight; and its right hand place is 0 when the index of the log. sine was 9.

But if a given log. sine is not found to every figure in the tables of log. numbers, its 5th, or right hand place is thus found.

Take the diff. between the log. num. next greater and less, than the given log. sine; and also the diff. between the given log sine and its next less log. numb.

Then, As 1st. diff. is to 2d. diff. so is 10, to the digit for the right hand place.

Thus to $4^{\circ} 22'$, the nat. sine is 0,07614; and cosine is 9,99710 -

But to $28^{\circ} 35'$, the log. sine and cos. does not appear exactly among the log. numb.

And the abovementioned two differences, for sine, are 9 and 3; for cos. are 5 and 1.

Then $9 : 3 :: 10 : 3$, the 5th. place And $5 : 1 :: 10 : 2$ the 5th place. 257. On the contrary. *A natural sine being given, its corresponding arc may be thus found.*

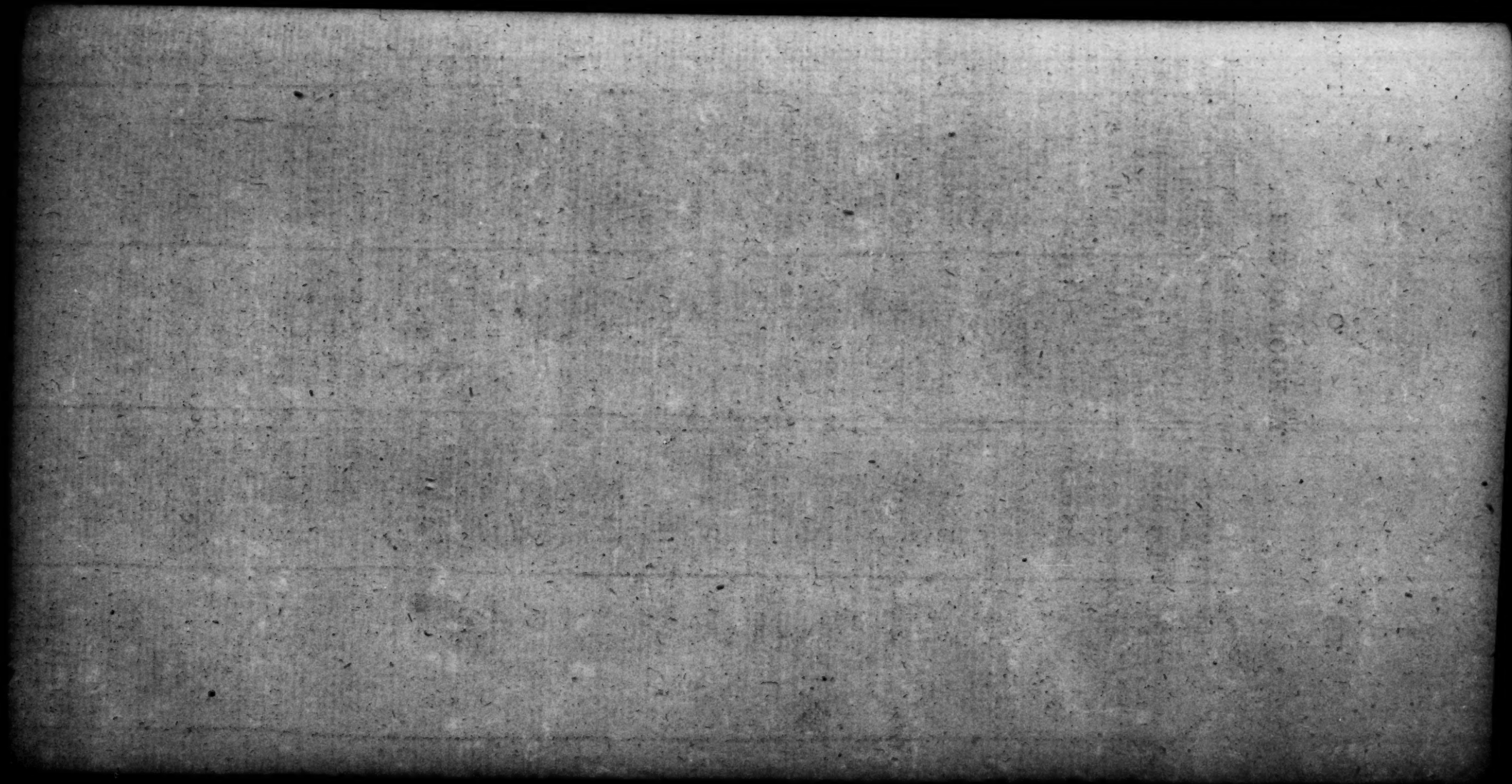
In the tables of num. and logs. enter with the natural sine as a num. and take out its log.

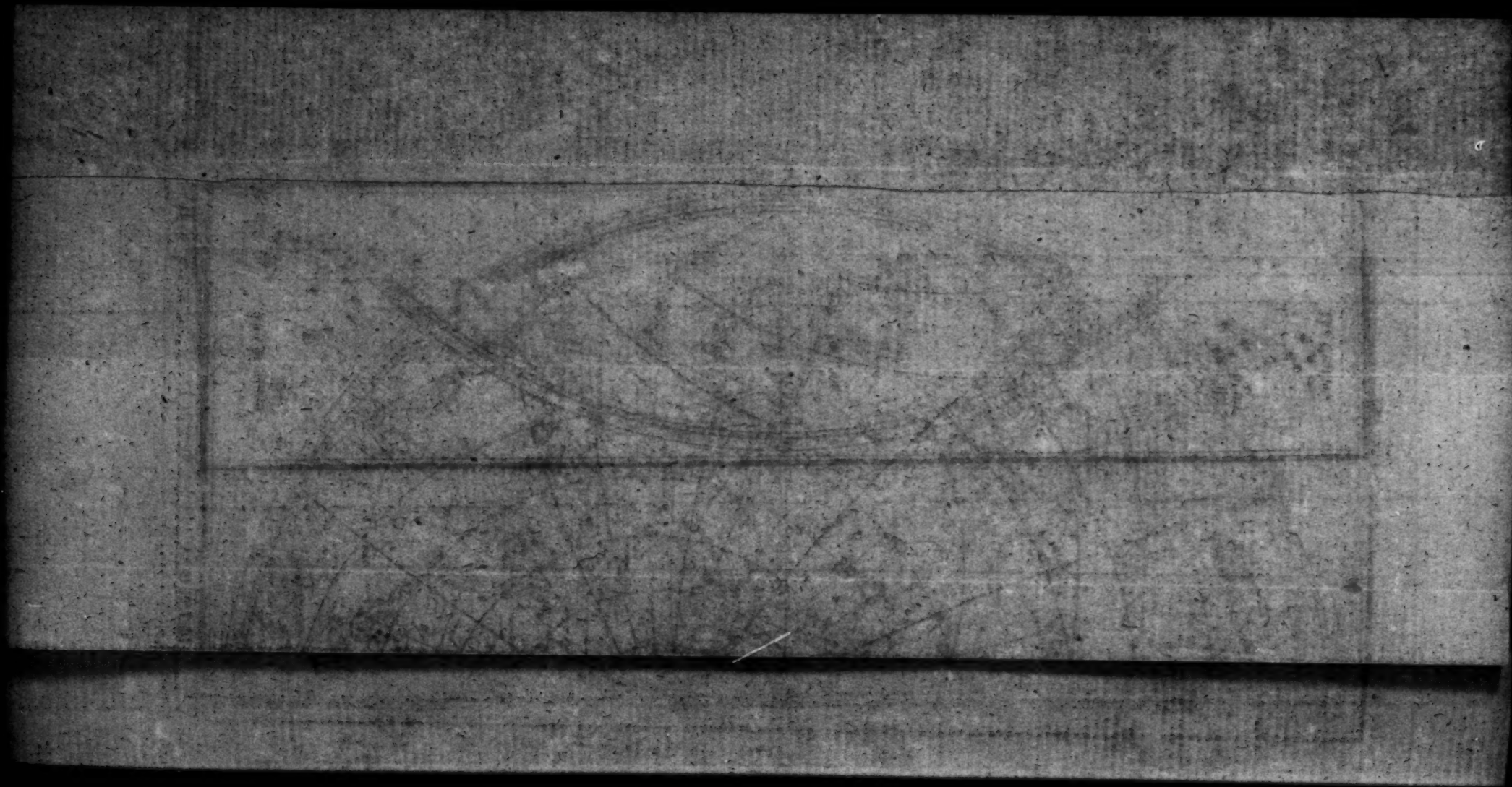
Seek this log. in the table of log. sines, and the corresponding degrees and minutes shew the arc required.

Prefixing the index 9, 8, 7, or 6; according as the left hand digit stood in the place of firsts, seconds, thirds, or fourths.

What has been said of the nat. and log. sines of arcs, is also applicable to the nat. and log. tangents of arcs.

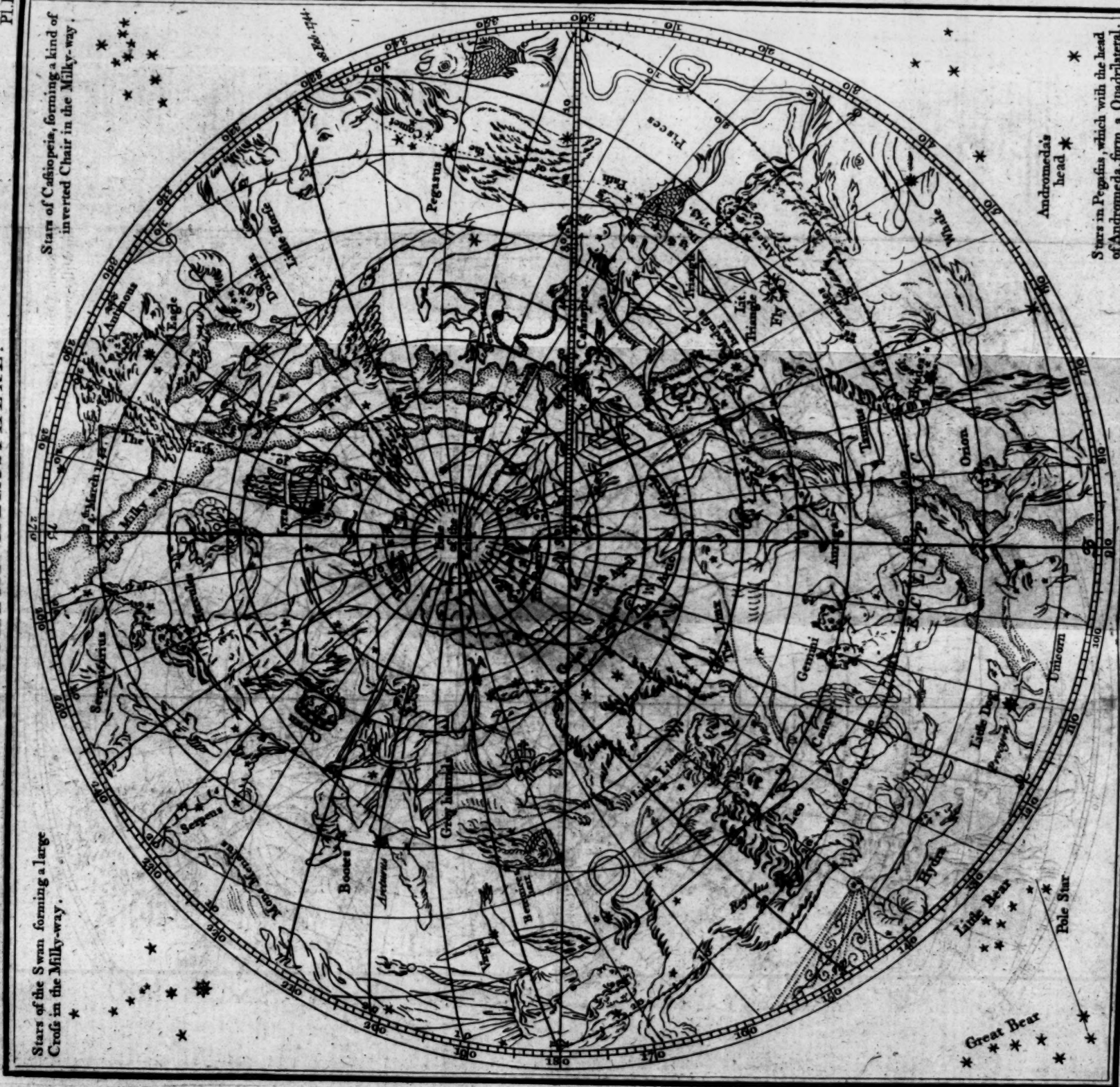
END OF BOOK IV.





Stars of the Swan forming a large Cross in the Milky-way.

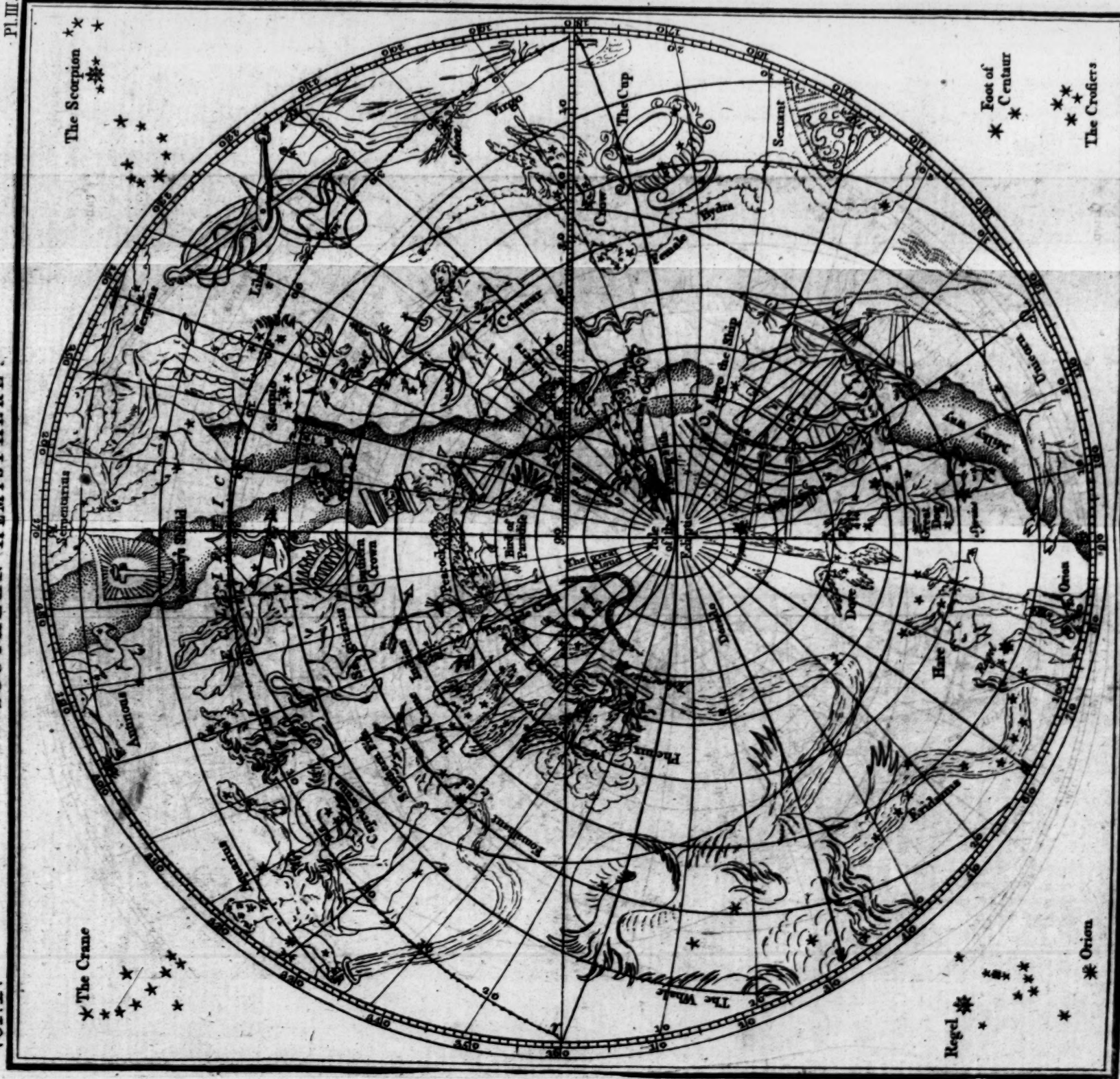
Stars of Cassiopeia, forming a kind of inverted Chair in the Milky-way.



Great Bear
Little Bear
Pole Star

Andromeda's head

Stars in Pegasus, which with the head of Andromeda, form a Quadrilateral.





THE
ELEMENTS
OF
NAVIGATION.

BOOK V.
OF ASTRONOMY.

SECTION I.

Of Solar Astronomy.

1. **A**STRONOMY is a science which treats of the motions and distances of the heavenly bodies, and of the appearances thence arising.

There have been a great variety of opinions among the philosophers of the preceding ages concerning the situation of the great bodies in the universe, or of the positions of the bodies which appear in the heavens: But the notion now embraced by the most judicious Astronomers, is, that the universe is composed of an infinite number of systems or worlds; in every system there are certain bodies moving in free space, and revolving at different distances around a SUN placed in or near the center of the system; and that these *suns* and other *bodies* are the stars which are seen in the heavens.

2. The **SOLAR SYSTEM**, so called by Astronomers, is that system in which our Earth is placed: and herein the notion of the Sun being fixed in or near the center, with several bodies revolving round him at different distances, is confirmed by all the observations hitherto made: This opinion is called the **COPERNICAN SYSTEM**; from *Nicholas Copernicus*,

a Polish Philosopher, who about the year 1500 revived this notion from the oblivion it had been buried in for many ages.

Stars are distinguished into two kinds, namely, *fixed and wandering*.

3. The **FIXED STARS** are the suns in the centers of their systems shining by their own light; and which are observed to preserve always the same situation in respect to one another.

4. The fixed stars appear of various sizes, doubtless occasioned by their different distances; these sizes are usually distinguished into six or seven classes, called **Magnitudes**: The largest, or brightest, are said to be of the **FIRST MAGNITUDE**; those in the next class, or degree of brightness, are called of the **SECOND MAGNITUDE**; and so on to the least, or those just discernible to the naked eye. But besides these, there are scattered throughout the heavens a great number of other stars, called **TELESCOPIC STARS**, as they cannot be seen but through that instrument. And indeed, it is to the assistance of that most admirable machine, that a great part of the modern Astronomy owes its rise, and which will undoubtedly transmit, with the greatest honour to the latest posterity, the name of **GALILEO**, among whose many inventions the telescope is ranked.

5. Although the visible fixed stars are probably at very unequal distances from the center of the *solar system*, yet Astronomers, for their ease of computation, reckon them as though they were all equally distant from our Sun, forming the surface of a sphere inclosing our system, and called the **CELESTIAL SPHERE**: Which supposition, with regard to the Solar System, may be strictly admitted, considering the immense distance of even the nearest of the fixed stars, from the center of the system.

6. A **CONSTELLATION** is a number of stars lying in the neighborhood of one another, on the surface of the *celestial sphere*, and which Astronomers, for their easy remembrance, suppose to be circumscribed by the outlines of some *animal*, or other *figure*; whereby the motions of the wandering stars are more readily described and compared.

The number of these constellations are 80, each containing several stars of different magnitudes: The number of stars of each magnitude, and also the constellation wherein they are ranged, are contained in the following table; where it may be observed, that the constellations are distinguished under three heads, namely, in the *zodiac*, and in the northern and southern hemispheres.

7. CONSTELLATIONS IN THE ZODIAC.

Names. Northern	Marks	Number	Magnitudes						Names. Southern,	Marks	Number	Magnitudes					
			I	II	III	IV	V	VI				I	II	III	IV	V	VI
Aries	♈	46	0	1	1	3	5	36	Libra	♎	33	0	2	1	8	3	19
Taurus	♉	109	1	1	3	9	24	71	Scorpio	♏	44	1	3	6	14	5	15
Gemini	♊	94	1	2	4	8	13	66	Sagittarius	♐	48	0	1	5	9	11	22
Cancer	♋	75	0	0	0	6	8	61	Capricornus	♑	58	0	0	0	2	5	9
Leo	♌	91	2	2	6	13	11	57	Aquarius	♒	93	0	0	4	7	28	54
Virgo	♍	93	1	0	5	11	24	52	Pisces	♓	110	0	0	1	7	28	74

8. NOR-

8. NORTHERN CONSTELLATIONS.

Names	q E N	Magnitudes						Names	q E N	Magnitudes					
		I	II	III	IV	V	VI			I	II	III	IV	V	VI
Little Bear	12	0	2	1	4	3	2	Camelopardalus	23	0	0	0	5	7	11
Great Bear	105	0	5	5	16	30	49	Serpent	50	0	1	7	6	5	31
Dragon	49	0	1	7	8	10	23	Serpentarius	67	0	1	6	12	17	31
Cepheus	40	0	0	3	7	10	20	Sobieki's Shield	8	0	0	0	2	3	3
Greyhounds	24	0	0	0	1	7	16	Eagle	29	1	0	5	1	4	18
Bootes	53	1	0	7	10	12	23	Antinous	34	0	0	0	5	2	7
Mens Menalus	11	0	0	0	1	0	10	Dolphin	18	0	0	0	0	2	10
Berenice's Hair	24	0	0	0	6	8	10	Colt	12	0	0	0	0	4	1
Charles's Heart	3	0	1	0	0	0	2	Arrow	13	0	0	0	0	4	1
Northern Crown	11	0	1	0	6	3	1	Andromeda	66	0	3	2	10	16	35
Hercules	92	0	0	12	12	28	40	Perseus	67	1	1	5	10	14	36
Cerberus	9	0	0	0	3	1	5	Pegasus	81	0	3	4	9	11	54
Harp	24	1	0	3	2	8	10	Auriga	46	1	1	1	9	9	25
Swan	73	0	1	5	15	20	32	Lynx	55	0	0	1	8	21	25
Fox	29	0	0	0	6	11	12	Little Lion	20	0	0	0	1	6	5
Goose	10	0	0	0	0	0	2	Great Triangle	10	0	0	0	0	3	1
Lizard	12	0	0	0	0	3	5	Little Triangle	5	0	0	0	0	0	5
Cassiopeia	52	0	0	5	7	7	33	Musca	6	0	0	1	2	2	1

9. SOUTHERN CONSTELLATIONS.

Names	q E N	Magnitudes						Names	q E N	Magnitudes					
		I	II	III	IV	V	VI			I	II	III	IV	V	VI
Whale	80	0	2	8	13	10	47	Peacock	14	0	1	3	5	4	1
Eridanus	72	1	0	10	24	19	18	Southern Crown	12	0	0	0	1	3	8
Phoenix	13	0	1	5	5	0	2	Crane	14	0	2	1	2	9	0
American Goose	9	0	0	4	2	3	0	Southern Fish	15	1	0	2	9	2	1
Orion	93	2	4	3	19	15	50	Hare	25	0	0	4	9	4	8
Monoceros	32	0	1	10	10	11	50	Noah's Dove	10	0	2	0	1	6	4
Little Dog	14	1	0	1	0	2	10	Charles's Oak	13	0	1	2	6	4	0
Hydra	53	0	1	1	14	13	22	Ship Argo	48	1	6	11	13	14	3
Sextan's Urania	4	0	0	0	0	0	4	Great Dog	29	1	5	1	4	10	8
Cup	11	0	0	0	0	8	2	Bee	4	0	0	0	2	2	0
Crow	8	0	0	2	2	2	2	Swallow	11	0	0	0	4	3	4
Centaur	36	2	6	6	14	8	0	Indus	12	0	0	0	4	6	2
Wolf	36	0	0	3	6	18	9	Chameleon	10	0	0	0	0	9	1
Altar	9	0	0	1	6	1	1	Flying Fish	7	0	0	0	0	6	1
Southern Triangle	5	0	1	2	0	2	0	Sword Fish	7	0	0	2	2	1	2

Constellations in the zodiac 12, contain

Northern constellations 36, contain

Southern constellations 32, contain

Number of stars in the 80 constellations

Stars	I	III	IV	V	VI
894	6	12	38	100	169
243	5	21	92	200	291
706	9	32	75	185	188
28,432	20	65	205	485	648
					1420

As these stars are found not to alter their situation in respect to one another, they serve Astronomers as fixed points, whereby the motions of other

other bodies may be compared; and therefore their relative positions have with great care been sought after for many ages, and catalogues of the observations have from time to time been published by those who have been at the pains to make them: Among these catalogues, the most copious, and, as generally esteemed, the best, is that called the *Historia Cœlestis* of our countryman FLAMSTEED.

10. The positions of the stars being obtained, their relative places may be delineated on a sphere or plane; and thus are the maps or charts of the heavens made, and the constellations drawn, inclosing their respective stars. There are two maps, usually called Celestial hemispheres, which are prefixed to this book; by the help whereof, a person may readily become acquainted with the positions and names of some of the principal fixed stars, thus:

On a clear night, let these prints be laid so as to correspond to the north and south parts of the heavens; then the observer sometimes looking on the stars, and then on the hemispheres, will with a little practice know some of the stars in the heavens, whose like positions and names he has observed on the prints.

11. The WANDERING STARS are those bodies within our system or celestial sphere, which revolve round the Sun; they appear luminous or bright, only by reflecting the light they receive from the Sun, and are of three kinds, namely, *primary planets*, *secondary planets*, and *comets*.

12. The PRIMARY PLANETS are those bodies which, in revolving round the Sun, respect him only as the center of their courses; where-in their motions are regularly performed in tracts or paths, that are by observations found to be, nearly circular and concentric to one another.

13. A SECONDARY PLANET, commonly called a SATELLITE or MOON, is a body which, while it is carried round the Sun, does also revolve round a primary planet, which it respects as its center.

14. COMETS, vulgarly called *blazing stars*, are bodies which also revolve round the Sun; probably in as regular order as the planets; but in much longer periods of time, from what is hitherto known of them: They are in number many more than all the planets; and their tracks or courses pass among the paths of the planets in a great variety of directions.

15. The ORBIT of a planet or comet is that track or path along which it moves.

There are six *primary planets*, which reckoned in order from the Sun, their names and marks are MERCURY ☿, VENUS ♀, the EARTH ♂ or ⊕, MARS ♂, JUPITER ♃, SATURN ♄.

Mars, *Jupiter*, and *Saturn*, are called SUPERIOR PLANETS, as their orbits include that of the *Earth*: But *Venus* and *Mercury*, whose orbits are contained within the *Earth's*, are called INFERIOR PLANETS.

16. There are found, by observations with telescopes, to be ten *secondary planets*; the Earth is attended by one called the Moon, Jupiter by four, and Saturn by five.

Saturn is also observed to have a kind of circle, called his RING, which surrounds the planet at some distance from his surface: And Jupiter has certain appearances, which seem like zones or girdles round him; and these are called JUPITER'S BELTS.

Every

Every primary planet is supposed to have two motions, namely, *annual* and *diurnal*.

17. The **ANNUAL MOTION** of a planet is that whereby the planet is carried in its orbit round the Sun; which in every one is found by observation to be from west to east.

This motion is discovered by the planets changing their places in the celestial sphere, upon whose surface they appear to move among the fixed stars; and in certain times return to the same stars from whence they were seen to depart; and so on continually.

18. The **DIURNAL MOTION** of a planet is that by which it turns or spins about its axis, and is also from west to east.

This motion is discovered by the spots that are seen, by telescopes, on the surfaces of the planets: These spots appear first on the eastern margin, or side of the planet, and gradually move from thence across it, till they disappear on the western side, or *limb*; and after a certain time appear again on the eastern side, and so on.

19. Each planet is observed always to pass through the constellations *Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, Pisces*; and it also appears, that every one has a track peculiar to itself; whereby, the paths of the six planets form among the stars a kind of road, which is called the **ZODIAC**, the middle path whereof, called the **ECLIPTRIC**, is the orbit described by the Earth, with which the orbits of the other planets, are compared.

As the *ecliptic* runs through twelve constellations, it is supposed to be divided into twelve equal parts, of 30 degrees each, called signs, having the same names with the twelve constellations they run through.

20. The **EQUINOCTIAL POINTS** are those two points of the *Ecliptic*, opposite to one another, through which the Earth passes, in its annual motion, when the lengths of the day and night are equal in all parts on the Earth. One of these points, called the **VERNAL EQUINOX**, answers to about the 20th of March; and the other, called the **AUTUMNAL EQUINOX**, answers to about the 22d of September.

21. The *Plane of the Ecliptic* is supposed to divide the celestial sphere into two equal parts, called the *northern* and *southern celestial hemispheres*; and any body in either of these hemispheres is said to have north or south latitude, according to the hemisphere it is in: So that the **LATITUDE** of a celestial object is its nearest distance from the *ecliptic*, taken on the sphere.

The Planes of the other five orbits are observed to lie partly in the northern, and partly in the southern hemisphere; so that every one cuts the *ecliptic* in two opposite points, called **NODES**: One called the **ASCENDING NODE**, marked thus, $\nearrow$, is that through which the planet passes when it moves out of the southern into the northern hemisphere; and the other called the **DSCENDING NODE**, marked thus, $\searrow$, is that through which the planet must pass in going out of the northern into the southern hemisphere.

The right line joining the two Nodes of any planet, is called the **LINE OF THE NODES**.

22. The names to most of the constellations were given by the ancient Astronomers, who reckoned that star in Aries, now marked γ , (ac-

cording

cording to *Bayer's* maps) to be the first point in the elliptic, this star being next the Sun when he entered the *Vernal Equinox*; and at that time each constellation was in the sign by which it was called: But observations shew, that the point marked in the heavens by the *Vernal Equinox* has been constantly going backward by a small quantity every year, whereby the stars appear to have advanced as much forwards; so that the constellation *Aries* is now almost removed into the sign *Taurus*, the said first star γ being got almost 30 degrees forwards from the equinox; which difference is called the PRECESSION OF THE EQUINOXES, whereof the yearly alteration is about 50 seconds of a degree, or about a degree in 72 years.

23. It was said in art. 12, that the planets revolved round the Sun in orbits nearly circular and concentric; for the several phenomena arising from their motions shew they are not strictly so; and the only curve they can move in, to reconcile all the various appearances, is found to be an Ellipsis: So that the orbits of the primary planets and comets are Ellipses of different curvatures, having one common focus, in which the Sun is fixed: But every secondary planet respects the primary planet round which it revolves, as the focus of its elliptic motion. For as no other suppositions can solve all the appearances that are observed in the motion of the planets, and as they agree with the strictest physical and mathematical reasoning; therefore they are now received as elementary principles.

24. The Line of the nodes of every planet passes through the Sun: For as the motion of every planet is in a plane passing through the Sun, consequently the intersections of these planes, that is, the lines of the nodes, must also pass through the Sun.

25. All the planets in their revolutions are sometimes nearer, sometimes farther from the Sun: This is a consequence of the Sun not being placed in the center of each orbit, and they being ellipses.

26. The APHELION, or SUPERIOR APSIS, is that point of the orbit where the planet is farthest from the Sun: And the PERIHELION, or INFERIOR APSIS, is that point where it is nearest the Sun: And the transverse diameter of the orbit, or the line joining the two apses, is called the LINE OF THE APSSES, or APSIDES.

27. The planets move faster as they approach the Sun, or come nearer to the perihelion, and slower as they recede from the Sun, or come nearer the aphelion: This is not only a consequence from the nature of the planet's motions about the Sun, but is confirmed by all good observations.

28. If a right line drawn from the Sun through any planet, usually called the *Vector Radius*, be supposed to revolve round the Sun with the planet, then this line will describe, or pass through every part of the plane of the orbit; so that the *Vector Radius* may be said to describe the area of the orbit.

29. There are two *chief laws* observed in the Solar System, which regulate the motions of all the planets; namely,

- I. *The planets describe equal areas in equal times*: That is, in equal portions of time the *Vector Radius* describes equal areas or portions of the space contained within the planet's orbit.

II. The

II. *The squares of the periodical times of the planets are as the cubes of their mean distances from the Sun:* That is, the square of the time which a planet A takes to revolve in its orbit, is to the square of the time taken by any other planet B to run through its orbit; so is the cube of the mean distance of A from the Sun, to the cube of the mean distance of B from the Sun.

30. The MEAN DISTANCE of a planet from the Sun, is its distance from him when the planet is at either extremity of the conjugate diameter; and is equal to half the transverse diameter.

31. The foregoing laws are the two famous laws of KEPLER, a great Astronomer, who flourished in Germany about the beginning of the 17th century, and who deduced them from a multitude of observations: But the first who demonstrated these laws was the incomparable Sir ISAAC NEWTON.

By the second law, the relative distances of the planets from the Sun are known; and was the real distance of any one known, the absolute distances of all the others would thereby be obtained.

32. Every thing already said of the planets is found in a great measure to be applicable also to the comets, as well from the observations that have been made of them, as from the physical and mathematical considerations of their motions.

33. Were the motions of the planets to be observed from the Sun, each of them would be ever seen to move the same way, though with different velocities; those nearer the Sun running their courses through the Zodiac in less time than those at greater distances: And hence it would happen, that some of them overtaking of the others would, in their passing by them, appear to be sometimes above, sometimes below, and sometimes as if they touched one another, according to the parts of the orbits those planets happened to be in with respect to their nodes.

34. When two planets are seen together in the same sign equally advanced therein, they are said to be in CONJUNCTION: But when they are in direct opposite parts of the Zodiac, they are said to be in OPPOSITION.

35. As the planes of the orbits are inclined to one another, therefore when two planets happen to be in conjunction at the time they come near a node of one of them, they would be seen from the Sun apparently to touch one another; and the farthest of those planets from the Sun would see the nearest moving over the face of the Sun like a black spot, being then directly between the Sun and the remoter planet, so the planet Venus was observed from the Earth, in the transits of the years 1761 and 1769. Also, should an opposition of two planets happen near a node of one of them, the Sun being then directly between them, would hide the light of one from the other. And these obscurations, or interceptions of the light of the planets one from the other, are called ECLIPSES.

36. The place that any planet appears to occupy in the celestial sphere, when seen by an observer supposed in the Sun, is called its *Heliocentric place*: And indeed all celestial appearances, as seen from the Sun, are called *Heliocentric phenomena*.

37. The following table exhibits at once some of the most material conclusions that have been deduced from the observations hitherto made, the mean distance of the Earth from the Sun being reckoned at 1000.

TABLE OF THE SOLAR SYSTEM.

Planets	Names	Marks	Mean dist. that of the Earth.	Eccentricity	Inclination of the orbits to the eclipt.	Times of the periodical revolutions	Diurnal ro- tations	True diameter.	No of moons
Mercury	☿	387	81	6° 52'	3 m. or	87 ^d 23 ^h	uncertain	0,32	0
Venus	♀	724.	53	23	8 m. or	224 17	23 ^h 0 ^m 0 ^s	0,87	0
Earth	♁	1000	17	0 00	1 y. or	365 6	23 56 4	1,00	1
Mars	♂	1524	14	1 52	2 y. or	686 23	24 40 0	0,73	0
Jupiter	♃	5201	25	0 1 20	12 y. or	4332 12	9 56 0	7,70	4
Saturn	♄	9528	54	2 12 20	30 y. or	10759 8	uncertain	4,19	5

38. By all the observations made on the secondary planets, it appears,

1st. That the satellites revolve round their superior planets from west to east, in curve lined orbits like ellipses, the primary planet being in the focus, and one of the orbit's diameters directed towards the Sun.

2d. The planes of the satellites orbits are inclined to the plane of their respective planet's orbit.

3d. That, like the primary planets, they describe equal areas in equal times; and the squares of the times of their revolutions are as the cubes of the mean distances from their primary planet.

In every revolution of a moon round its primary planet, there must be two conjunctions, betwixt planet, moon and Sun; namely once, when the moon is in that part of its orbit nearest the Sun; and once, when in that part of its orbit farthest from the Sun: And an eclipse may happen at either conjunction, according as the moon's nodes happen to be posited at those times. For the plane of a moon's orbit is inclined to that of its primary, and so makes two nodes: And whenever the Sun, planet, and moon happen at the same time in the line of the nodes, there must be an eclipse; which would occur at every conjunction, but for the inclination of the orbit.

One of the conjunctions, namely, that made by the moon's going beyond the primary, from the Sun, is called the SUPERIOR CONJUNCTION; and the other, made by the satellite on the side of the planet next the Sun, is called the INFERIOR CONJUNCTION.

The

The following table shews the times taken by each satellite in its revolution ; and also its mean distance from the primary in femidiameters thereof.

39	I			II			III			IV			V		
	d	h	m	d	h	m	d	h	m	d	h	m	d	h	m
Saturn's satell.	1	21	18 $\frac{1}{2}$												
Dist. from Sat.	8 $\frac{2}{3}$ f. diam.			2	17	41 $\frac{1}{2}$	3	12	25 $\frac{1}{2}$	15	22	41 $\frac{1}{4}$	79	7	48
				11 $\frac{1}{4}$ f. diam.			15 f. diam.			36 f. diam.			108 f. diam.		
Jupiter's satell.	1	18	28 $\frac{1}{2}$												
Dist. from Jup.	5 $\frac{2}{3}$ f. diam.			3	13	18 $\frac{1}{2}$	7	3	59 $\frac{1}{2}$	16	18	5 $\frac{1}{8}$			
				9 f. diam.			14 $\frac{1}{2}$ f. diam.			25 $\frac{1}{3}$ f. diam.					
The Moon	29	12	44 $\frac{1}{2}$												
													from the Earth 60 $\frac{1}{2}$ femidiameters.		

40.

Of the figure and light of the Planets.

That the Sun and planets are spherical bodies, is evident from all the observations that have been made of them ; and that the Earth is of like figure is not only deduced by analogy, but sufficiently confirmed by observation *. Now although Astronomers generally say that the planets are spherical, yet they do not mean a Geometrical sphere, but a figure called an oblate spheroid, which is something like the figure that a flexible sphere would be formed into by gently pressing it at its poles : Observations have determined this in Jupiter, and it is known that the Earth is of this figure both from observations and actual mensuration.

That the planets must have this oblate figure, is evident from this consideration ; that as they are of matter, and violently whirled on their axes, all the parts would endeavour to fly off, like water from a trundled mop ; those in the equator moving swiftest, have the greatest tendency to depart : And although the parts are retained in the sphere by the superior force of gravity, yet the equatorial diameters will be somewhat increased, and the polar lessened.

41. The planets are all opaque or dark bodies, and consequently shine only by the light they receive from the Sun : This is known, by observing, that those bodies are not visible when they are in such parts of their orbits as are between the Sun and Earth, or partly so. Now, as all the planets sometimes appear with a strong light, therefore the rays they receive from the Sun must convey to them a degree of warmth proportional, in some measure to their distance ; which proportion is reciprocally as the squares of the distances ; and this must be readily inferred from the heat which the inhabitants of the Earth receive from the Sun.

42. As a planet revolves on its axis, every part of its surface will be turned towards the Sun, and thereby enjoy its light and heat.

* Observations shew, that in eclipses of the Moon, the darkened part is bounded by a circular curve ; and consequently the body, which casts the shade, or obstructs the light, must be bounded by a like curve : But as these observations are caused by different parts of the Earth, consequently its surface must be limited by a circular figure, that is, it must be globular.

S E C.

SECTION II.

Of Terrestrial Astronomy.

43. TERRESTRIAL ASTRONOMY is that which considers the motions of the celestial bodies as seen from the Earth, which is also in motion.

The motions described in the preceding section were such as would appear to an observer viewing the heavens from the Sun : But was he placed in one of the planets, suppose the Earth, and there observed the motions of the rest, it would appear to him that the Sun, and other planets, revolved round the Earth as a center ; but the Sun would be the only one that moved uniformly the same way : For the other planets would seem to move sometimes from west to east, and then to stand still ; then they would seem to run from east to west ; and after standing some time, they would again move from west to east, and so on continually.

44. The place in the celestial sphere that any planet appears in, when seen from the center of the Earth, is called its **GEOCENTRIC PLACE**.

45. The **DIRECT MOTION** of a planet is that by which it appears to move from west to east, and this motion is said to be *according to the order of the signs*, or in *consequentia*. When the planet appears to stand still, it is said to be **STATIONARY** ; and when its motion is apparently from east to west, it is then called **RETROGRADE**, or has a motion in *antecedentia*, or contrary to the order of the signs. These different appearances follow partly in consequence of the observer being himself in motion while he is viewing those of the planets, and partly because he is not in the center of the motions which he observes.

46. *The Phenomena of the Inferior Planets. See Fig. 1. Plate IV.*

Let ABC represent an arc of the celestial sphere ; EOP the Earth's orbit ; LNIG the orbit of an inferior planet, as of *Venus* ; and s the Sun : Let the earth at first be supposed to stand still in its orbit at E : Now it is evident that the Sun will appear at the point B, and the planet always within the arc AC. Whilst the planet moves in its orbit from I through Q to N, it will seem to move from B to A in *consequentia* : But passing from N to L, it will seem to an eye at E to return back from A to B, or be *retrograde*. While the planet is at, or near, the point N, being betwixt moving on from B to A, and returning from A to B, and moving, as it were, in a right line towards the Earth, it will for some time seem to stand still near A, that is, to be *stationary*.

47. When the planet is in that part of her orbit N or G, which is contiguous to the tangent EA or EC, she will then appear at A or C, her greatest distance from the Sun, and is said to be in her greatest **ELONGATION** : This elongation is measured by the angle SEG. The more distant a planet is from the Sun the greater will its angle of elongation

tion be; that of Venus is about 48 degrees, and of Mercury about 28 degrees.

In the space of a revolution, the two inferior planets will, with respect to the Earth, undergo two conjunctions; one when it is beyond the Sun at I, the other when it is at L between the Sun and the Earth; the former is called the *superior*, and the latter the *inferior conjunction*.

48. Whilst Venus goes from her superior conjunction, she appears in the arc BA always to the eastward of the Sun, and therefore sets some time after the Sun, and is called the *evening star*: But during the time she is going from her inferior to her superior conjunction, she will be seen somewhere in the arc BC to the westward of the Sun, and so will set before him in the evening, and rise before him in the morning; hence she is called the *morning star*.

Hitherto the planet only has been supposed to move while the Earth stood still: But when both move, the foregoing phenomena will be much the same, only the planet will be more direct in the farthest part of the orbit, and less retrograde in the nearest; the former arising from the sum of their motions, and the latter from the difference.

49. Of the Phenomena of the Superior Planets.

The *direct*, *stationary*, and *retrograde* motions of the superior planets are explained much after the same manner as those of the inferior ones, but with these differences.

1st. The retrograde motions of the superior planets happen oftner the slower their motions are, as the retrogradations of the inferior planets happen oftner the swifter their angular motions: Because the retrograde motions of the superior planets depend upon the motions of the Earth, but those of the inferior on their own angular motions. A superior planet is retrograde once in each revolution of the Earth; an inferior one, in every revolution of its own.

2d. The superior planets do not always accompany the Sun as the inferior do, but are often in opposition to him; which necessarily follows from the orbit of the Earth being included within the orbits of the superior planets.

50. Of the apparent motion of the Sun.

As a spectator in the Sun, would see the Earth revolve through the signs in the ecliptic; so to a spectator in the Earth, the Sun apparently revolves the same way, but is always in the opposite point of the ecliptic: (For it is well known to every one, especially to those who use the sea, that fixed objects appear to change their place by the motion of the observer:.) So that the heliocentric place of the Earth, and the geocentric place of the Sun, are always in direct opposite points of the ecliptic.

Now although it is the motion of the Earth that really causes a great variety in the apparent motions of the other planets; yet as the motion of the Sun being known, gives that of the Earth, therefore do Astronomers

mers speak of the motion of the Sun ; and in their computations use the quantities of those motions as if they were real.

51. Beside the various appearances that arise from the annual motion of the Earth, there are many resulting from its diurnal motion : For that the Earth has a daily motion round its axis must necessarily be inferred from the most strict reasoning on the motions of the planets : Besides the notion of bodies so immensely distant as are the stars, really revolving round the Earth in 24 hours, is now treated as a great absurdity by every one who has rightly considered these things : However, as the motions are apparent, and the speaking of them as real, is customary and no way affecting the conclusions ; therefore Astronomers treat of these motions as they appear.

52. Any sphere revolving as on an axis, must have two points on its surface, at the extremities of its axis, that do not revolve at all ; these points, with respect to the Earth, are called its *poles*.

53. By the Earth's rotation on its axis from west to east in a day, the surface of the celestial sphere appears to move from east to west in the same time ; and all the celestial objects appear to describe circles in the heavens, which are greater or less, according as they are farther from, or nearer to, the apparent centers of those motions : For there are two points in the heavens which are apparently fixed, and the nearer any stars are to these points, the slower are their motions. These points are called the *CELESTIAL POLES* ; the right line joining them is called the *Axis OF THE SPHERE*, and passes through the poles of the Earth. That circle in the heavens, equally distant from the poles of the celestial sphere, is called the *EQUINOCTIAL* ; the corresponding circle on the Earth is called the *EQUATOR*, which is equally distant from both the poles of the Earth.

54. As the Sun's rays falling on any sphere enlightens one half of its surface ; therefore one half of the Earth is always illuminated at once, and consequently the enlightened part is bounded by a great circle, which may be called the *TERMINATOR*, from its property of terminating, or bounding the verges of light and darkness : Now, by the rotation of the Earth on its axis once in 24 hours, there will be a constant succession of light in all parts on its surface as they are turned towards the Sun, and of darkness in these parts as they move out of his rays ; and hence arise the vicissitudes of *DAY* and *NIGHT*.

55. Did the plane of the equator coincide with the plane of the ecliptic, and the axis of the earth stand perpendicular to it, the terminator would always pass through the poles of the Earth, and there would be a constant equality of day and night in every part of its surface, except at the two poles, where there would be constant day : But the contrary of this is known to every one, and observations shew that the Earth's axis is inclined to the plane of the ecliptic in an angle of about $66\frac{1}{2}$ degrees ; therefore the poles of the ecliptic and equator are about $23\frac{1}{2}$ degrees distant from one another ; consequently the ecliptic and equinoctial, which in the heavens intersect one another in the opposite points of Aries and Libra, make at those intersections angles of about $23\frac{1}{2}$ degrees (IV. 33) : This angle is called the *OBLIQUITY OF THE ECLIPTIC*.

The axis of the Earth being thus inclined to the plane of the ecliptic, and moving parallel to itself in all points of the *ANNUAL ORBIT*, or ecliptic,

ecliptic, is the occasion of the inequality of days and nights, and of the different seasons of the year; which two phenomena are explained as follows.

56. It must be observed, that the Sun will appear to be vertical to that part of the Earth, which is cut by a straight line joining the centers of the Sun and Earth.

57. Now when the Earth is at $\mathcal{E}$, Fig. 3. Plate IV. the Sun appearing then in $\mathcal{S}$, will be vertical to that point of the terrestrial ecliptic, it lying in the right line joining the centers of the Sun and Earth: And this point being in the Earth's northern hemisphere, all those who live therein will enjoy summer, or the hottest time of the year, the solar rays falling more copiously, and more perpendicularly upon their hemisphere at that time.

58. At the same time the inhabitants of the southern hemisphere will have winter, the rays of the Sun falling more obliquely, and in less quantity on them, and consequently affording them less heat.

59. Again, the inhabitants of the northern hemisphere will have their days longer than their nights, in proportion as they are more distant from the equator; whilst those who live under the equator will have an equal share of day and night all the year round: For in this position the terminator, which is always at right angles to the plane of the ecliptic, will pass $23\frac{1}{4}$ degrees beyond the north pole, and consequently will cut all the circles parallel to the equator, it meets with, into two unequal parts. Those that are in north latitude will have the greater portions of those parallels in the enlightened hemisphere. But the terminator being a great circle, will cut the equator into two equal parts; therefore half the equator is always illuminated.

60. Hence it necessarily follows, that those who live under the equator will have their days and nights equal; Those who live within the limits of $23\frac{1}{4}$ degrees round the north pole, will have no night: And the inhabitants between this limit and the northern neighbourhood of the equator, will have their nights shorter than their days. In the mean time, those who live in the southern hemisphere will have their nights longer than their days, in proportion as they approach nearer to the south pole; and those regions contained within the limits of $23\frac{1}{4}$ degrees round the south pole, will have no day.

61. Suppose the Earth now to move in its orbit from $\mathcal{V}$ through the signs $\mathcal{X}$ to $\mathcal{Q}$, the Sun will seem to run through the signs Ω , $\mathcal{A}$ to $\mathcal{E}$; and this will be the place of the Sun in autumn.

While the Earth it in $\mathcal{Q}$, the days and nights will be equal in both hemispheres, and the season is a medium between summer and winter: For at that time the Sun will appear vertical to the equator, because a right line joining the centers of the Sun and Earth will then cut the surface of the Earth in the equator; so that the terminator, whose plane is always at right angles to the said line, will pass through the poles; consequently, all the Earth will then have an equal share of day and night. And because the rays of the Sun then fall perpendicularly upon the axis of the Earth, it will follow, that they must fall with an equal obliquity, and in equal number upon either hemisphere; therefore they must enjoy an equal degree of heat and cold.

Now

Now suppose the Earth to move from φ to ω , and the Sun will seem to move from ϵ to ν , where it will be in its nearest approach to the south pole; and at this time of the year it will be winter in the northern hemisphere. For to this hemisphere the like phenomena will now happen, as did before to the southern, when the Earth was in ν ; and by a parity of reason, when the Earth has got as far as ϵ , and the Sun is apparently in φ , the northern hemisphere will enjoy spring, and the southern will have autumn.

62. The four points of the ecliptic, wherein the Earth has been considered in summer, autumn, winter, and spring, are called the four **CARDINAL POINTS**; ν and ω are called **SOLSTITIAL POINTS**; ϵ and φ , **EQUINOCTIAL POINTS**.

63. The first point of Cancer is called the **SUMMER SOLSTICE**; because when the Sun enters it, which is about the 21st of June, he is then got to his greatest extent northwards, and being about to return towards the equator, he seems for a day or two to be at a stand. And for the same reason, the first point of Capricorn, which the Sun enters about the 21st of December, is called the **WINTER SOLSTICE**, with respect to the northern hemisphere.

64. The first points of Aries and Libra are called the **VERNAL** and **AUTUMNAL EQUINOCTIAL POINTS**, from the equality of days and nights all over the surface of the Earth, when the Sun enters those points.

65.

Of the Rising and Setting of the Stars.

There is only one half of the celestial sphere visible at one time to any observer on the surface of the Earth, the other half being hid by the Earth itself: Now the apparent plane on which the observer stands, seems to be extended to, and cuts the heavens, and there marks out a circle that divides the visible from the invisible hemisphere; this circle is called the **HORIZON**, above which all the celestial motions are seen. When this horizon is a great circle of the celestial sphere, it is called the **RATIONAL HORIZON**: But when by the particular situation of the observer, he sees more or less than half the celestial sphere, then the circle bounding his view is called the **SENSIBLE HORIZON**.

The horizon is one of the most useful circles in Astronomy, for to this circle, which is the only apparent one, almost all the celestial motions are referred. It is the common termination of day and night; it marks out the times of the rising and setting of the Sun and stars, and many other particulars, of which hereafter.

66.

Of Parallaxes.

The **PARALLAX** of any object is the difference between the places that object is referred to in the celestial sphere, when seen at the same time from two different places within that sphere: Or, it is the angle under which any two places in the inferior orbits are seen from a superior planet, or even from the fixed stars: But the parallaxes used by Astronomers are those arising from seeing the object from the centers

ters of the Earth and Sun; from the surface and center of the Earth; and from all three compounded.

67. The difference between the heliocentric and geocentric place of a planet, is called the *parallax of the annual orbit* (namely, that of the Earth); that is, the angle at any planet, subtended by the distance between the Sun and Earth, is called the parallax of the Earth's, or annual orbit.

68. The difference in the two longitudes is called the parallax of longitude; and that of the two latitudes is called the parallax of latitude.

69. In the SYZIGIES, that is, in the *oppositions or conjunctions*, the Sun and planet being either equally advanced in the same sign, or in like places in opposite signs, the parallax of latitude is then greatest.

70. And when the planet is in its QUADRATURES, that is, when it is 90 degrees distant from the Sun, the parallax of longitude is then the greatest.

71. To explain the parallaxes which respect the Earth only. Fig. 2. Plate IV.

Let HSW represent the Earth, where T is the center; O Part of the Moon's orbit, *rvg* part of a planet's orbit, and *zAA* part of a great circle in the celestial sphere: Now to a spectator at *s* upon the surface of the Earth, let the Moon appear in *G*, that is, in the sensible horizon of *s*, and it will be referred to *A*; but if viewed from the center T, it will be referred to the point D, which is its true place.

The arc AD will be the Moon's parallax; the angle SGT the parallactic angle: Or the parallax is expressed by the angle under which the semidiameter TS of the Earth is seen from the Moon.

If the parallax be considered with respect to different planets, it will be greater or less, as those objects are more or less distant from the Earth. Thus the parallax AD of *G* is greater than the parallax Ad of *g*. If it be considered with respect to the same planet, it is evident that the horizontal parallax (or the parallax when the object is in the horizon) is greatest of all; and diminishes gradually as the body rises above the horizon, until it comes to the zenith, where the parallax vanishes, or becomes equal to nothing. Thus AD and Ad, the horizontal parallaxes of *G* and *g*, are greater than *as* and *ab*, the parallaxes of *R* and *r*; and the objects *O* or *P*, seen from *s* or *T*, appear in the same place *z*, or the zenith.

72. By knowing the parallax of any celestial object, its distance from the center of the Earth may be easily obtained by Trigonometry. Thus if the distance of *G* from *T* be sought; in the triangle STG, the side ST being known, and the angle SGT determined by observation, the side TG is thence known.

The parallax of the Moon may be determined by two persons observing her from different stations at the same time, she being vertical to the one, and horizontal to the other; and it is generally concluded to be about 57 minutes of a degree; consequently her mean distance TG is about 60 semidiameters of the Earth, or 60 times TS.

But the parallax the most wanted is that of the Sun, whereby his absolute distance from the Earth would be known; and thence the absolute

lete distances of all the other planets would be obtained from their relative distances found by the second *Keplerian Law*.

Before the year 1761 some Astronomers reckoned the Sun's parallax at $12\frac{1}{2}$ seconds, others at 10; these different parallaxes gave very different distances between the Sun and the Earth; the former making the distance near 8270 diameters of the earth, and the latter 10313 diameters.

But in each of the years 1761 and 1769, the planet Venus passed between the Earth and the Sun; and was seen like a black spot moving over the face of the Sun, and called *a transit over the Sun's disk*. These very rare Phenomena (not happening in more than an 100 years) were observed by many Astronomers, from different parts of the Earth, and the result of their observations makes the Sun's mean parallax about $8\frac{1}{2}$ seconds; and hence the mean distance between the Sun and Earth comes to very near 11900 diameters of the Earth: And, from what was shewn many years ago by the excellent Dr. Halley, if these observations were made with the accuracy he supposed, the distance between the Sun and the Earth might be obtained to within a 500th part of the whole distance.

73.

Of the Measure of the Earth.

The relative distances of the planets are discovered by the 2d *Keplerian Law*, and their relative magnitudes are gathered from the angles which they appear under (when viewed with very accurate instruments) compounded with their distances: Now as these distances and magnitudes can by means of the parallaxes be compared with the diameter of the Earth, consequently this diameter being accurately known, would serve as a measure wherewith the magnitudes and distances of all the other planets might be compared.

To find the measure of the Earth is a problem of such importance in Astronomy, that it has been attempted by some of the most considerable men in almost all the preceding ages: But its solution was not brought to any degree of accuracy till the year 1635, when it was very nearly ascertained by our countryman RICHARD NORWOOD, an eminent mathematician at that time. The principle he proceeded upon was, that as 360 degrees were contained in every great circle, both of the celestial sphere, and of the Earth. and that these circles being considered as concentric to the center of the Earth; therefore was the measure of the length known on a great circle of the Earth, corresponding to one or more degrees of a great circle of the heavens, then, by analogy, the whole circumference of a great circle of the Earth would be known in that measure, and consequently its diameter would be obtained.

(IL. 197)

NORWOOD solved this problem in the following manner: He chose two distant places which were known to lie nearly north and south one of the other, as London and York; and by a method like that of Traverse sailing (explained in Book VII.) he found their difference of latitude, or the distance between the parallels of latitude passing through those places; or which is the same thing, the length of that arc of the terrestrial meridian: He also with a good instrument found the distance between

between the zeniths of those places, and consequently he thence knew the quantity of the celestial arc answering to the measured terrestrial one: Then saying, As that celestial arc, is to a great circle of the celestial sphere, or 360 degrees; so is the arc of the terrestrial great circle measured in feet, to the circumference of a great circle on the Earth, in feet measure.

And thus he found that about $69\frac{1}{2}$ English miles answered to one degree; whereby the circumference of the Earth appears to be 25020 miles, and its diameter about 8000 miles.

By the same kind of reasoning, the distances found in art. 72. from the parallaxes, were obtained.

For $12^{\circ}\frac{1}{2} : 360^{\circ} :: 1 \text{ semi-diam.} : 103680$ } the { Circumference of the
 $10 : 360 :: 1 \text{ semi-diam.} : 129600$ } Earth's orbit in semi-
 $8\frac{2}{3} : 360 :: 1 \text{ semi-diam.} : 149538$ } diameters of the Earth.

And $6,283185 : 103680 :: 1 : 16539,5$ } the { Mean dist. of the Earth
 $6,283185 : 129600 :: 1 : 20626,4$ } from the Sun, in semi-
 $6,283185 : 149538 :: 1 : 23799,8$ } d. of the Earth. (II. 197)

Then $16539,5 \times 4000 = 66158000$ } the { Mean distance of the
 $20626,4 \times 4000 = 82505600$ } Earth from the Sun, in
 $23799,8 \times 4000 = 95199200$ } miles.

74.

Of the Moon.

The Moon revolves in her orbit from west to east round the Earth, and with it is carried perpetually through the annual orbit round the Sun, making in the space of one year 13 *periodical*, and 12 *synodical revolutions*.

75. A *PERIODICAL MONTH*, or *REVOLUTION*, is the time the Moon takes up in revolving from one point of her orbit to the same point again, and consists of 27 d. 7 h. 43 m.

76. A *SYNODICAL MONTH*, or *REVOLUTION*, is the time the Moon spends in passing from one conjunction with the Sun to another which is 29 d. 12 h. 44 m.; being 2 d. 5 h. 1 m. longer than the *Periodical Month*: For whilst the Moon is passing from her former conjunction with the Sun round to it again; the Earth has proceeded forwards in its annual course, as it were leaving the Moon behind it; so that, in order to complete her next conjunction with the Sun, she must not only come round to her former point again, but also go beyond it.

77. Besides this monthly motion of the Moon round the Earth, she has also a motion round her axis, which is performed exactly in the same time with her periodical revolution: From whence it comes to pass, that the same face of the Moon is always turned towards the Earth, her diurnal motion turning just as much of her face to us, as her periodical from us.

78. Though the same side of the Moon be ever turned towards us, yet it is not always visible, but daily seems to put on different appearances

ances, called PHASES: For the Moon being an opaque body like the rest of the planets, it borrows its light from the Sun, having always one hemisphere enlightened by the solar rays.

When the enlightened hemisphere is wholly turned from the Earth, as at her change or time of new-moon, the planet then being betwixt us and the Sun, the Moon's whole enlightened face, or *disk*, must needs be invisible to the Earth. When passing from this state, and turning some little portion of the illuminated half to us, it must appear horned, the CUSPS or points being turned from the Sun towards the east. When the Moon is in her quadratures, or at 90 degrees from the Sun, then half the illuminated face becomes visible: She afterwards continues to shew more than half the enlightened disk, until she comes in opposition to the Sun or time of full-moon, when the whole of the illuminated orb is presented to us; from whence receding, she must put on the like phases as before; but in an inverse order, the cusps being now turned towards the west.

79.

Of solar and lunar Eclipses.

Eclipses of the Sun and Moon can only happen about the times of the conjunctions and oppositions: Those of the Sun fall out at the conjunctions, when the Moon intercepts the light of the Sun from the Earth; and those of the Moon occur in the oppositions, when the Earth getting between the Sun and Moon, the latter loses her light during the time of that interposition.

The reason why there is not an eclipse in every syzigie is, because of the inclination of the plane of the Moon's orbit to that of the ecliptic, which is about $5^{\circ} 18'$: For it is certain, that unless the Sun, Earth and Moon be all in the plane of the ecliptic, or nearly so, the shadows of the Earth and Moon can never fall on one another, but must be directed either above or below. Now they can never be in the same plane, and in one right line, but when the Moon is in her nodes, and the nodes and Sun's center are in the same right line.

80. The solar and lunar eclipses do not happen every year in the same places of the zodiac, but in succeeding years, they fall in places gradually removed backwards, or towards the antecedent signs: For since the nodes are found to go continually backwards, the eclipses must also observe the same order.

81. Eclipses of the Moon are either *total* or *partial*: The total happens when the node falls in or near the center of the shadow: And the partial, when the node happens to be on either side the center, within or without the shadow. Now the longer the duration is of a partial eclipse, so much the greater is that part of the Moon which enters into the shadow of the Earth.

82. Hence it is usual to conceive the Moon's diameter as divided into 12 parts, called DIGITS, by which the greatness of partial eclipses are measured, they being said to be of so many digits as there are parts covered by the Earth's shadow: Thus if 5 of the 12 parts are covered, it is called an eclipse of 5 digits.

83. As the planet Mars is never eclipsed by the Earth, it is plain the shadow of the latter does not reach so far as the orbit of the former, but

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but tapers to a point at a less distance; and consequently the Earth's shadow must be a cone, whose vertex is extended beyond the orbit of the Moon. It naturally follows from hence, that the Sun is a much larger body than the Earth; and is indeed, in diameter, above 100 times that of the Earth.

84. If a person was placed just at the vertex, or point of this shadow, he would see nothing of the Sun but a small rim of light round his disk; and the farther the observer was removed from the vertex, the larger would the rim of light appear, and consequently the fewer rays would be intercepted by the opaque body, till at last it would appear only as a spot in the Sun; in like manner as the planets Venus and Mercury appear when they are seen to pass over the Sun's disk.

85. What has hitherto been said of the shadow, includes that of the atmosphere surrounding the Earth: for in lunar eclipses, the shadow of the atmosphere is to be considered: And hence it is that the Moon is visible in eclipses, the shadow cast by the atmosphere being not near so dark as that cast by the Earth.

86. The Moon always enters the western side of the shadow with her eastern limb, and quits it with her western limb; and in her approach to, and recess from the shadow, she must pass through a PENUMBRA, or imperfect shade, which is caused by the Earth itself.

87. In the same manner, in which it has been shewn that the Moon must come into the shadow of the atmosphere when she is at full, and at or near a node, it may also be shewn that her shadow must fall upon the Earth at the time of new Moon, provided she be in or near a node: But the penumbra of the Moon's shadow is much more sensible in solar eclipses, than that accompanying the shadow of the Earth in lunar ones.

88. It is observed, that to determinate parts of the Earth, solar eclipses are not seen so oft as lunar ones; which is owing to the shadow of the Moon being less than that of the Earth: For the Earth's shadow often covers all the Moon; but that of the Moon cannot cover all the Earth; and as it sometimes falls on one part, sometimes on another, it causes solar eclipses, in general, to be more frequent than lunar ones; yet to any determinate place on the Earth, there are more eclipses of the Moon visible, than of the Sun.

What has hitherto been said, may suffice to give beginners a general idea of the motions of the bodies in the solar system, and of some of the phenomena thence arising; those who desire to be farther acquainted with particulars, may find them fully treated of in M. de la Caille's Elements of Astronomy, published in English a few years since*; and also in the works of Gregory, Keil, and others.

* Translated by the Author of these Elements.

SECTION III.

The Astronomy of the Sphere.

DEFINITIONS and PRINCIPLES.

89. By the Astronomy of the sphere, is meant the finding, from proper things given, the measure of certain arcs and angles formed on the surfaces of the celestial and terrestrial spheres, by the apparent motions of the bodies which are seen in the heavens.

The surfaces of those spheres are supposed to be concentric, to the center of the Earth, and to have correspondent circles described on both spheres.

90. GREAT CIRCLES are those which divide either sphere into two equal parts.

LESSER CIRCLES, those which divide the sphere into unequal parts. The POLES of a circle are those points on the sphere equally distant from that circle.

An AXIS is a right line supposed to connect the poles.

The CELESTIAL AXIS is that right line about which the heavens seem to revolve.

The NORTH and SOUTH POLES of the world are those two points where the axis cuts the celestial sphere.

91. The EQUINOCTIAL, or EQUATOR, is that great circle of the sphere equally distant from the poles of the world.

92. MERIDIANS, or HOUR CIRCLES, or CIRCLES of RIGHT ASCENSION, or CIRCLES of TERRESTRIAL LONGITUDE, are great circles perpendicular to the equator, and passing through the poles of the world.

93. The ECLIPTIC is a great circle inclined to the equator in an angle of about $23\frac{1}{2}^{\circ}$, and cutting it in two points diametrically opposite. The ecliptic is supposed to be divided into 12 equal parts, called SIGNS, beginning from one of its intersections with the equator; each sign containing 30 degrees, named and noted thus.

<i>Aries</i>	<i>Taurus</i>	<i>Gemini</i>	<i>Cancer</i>	<i>Leo</i>	<i>Virgo</i>
♈	♉	♊	♋	♌	♍
<i>Libra</i>	<i>Scorpio</i>	<i>Sagittarius</i>	<i>Capricornus</i>	<i>Aquarius</i>	<i>Pisces</i>
♎	♏	♐	♑	♒	♓

The first six are called *northern*, and the latter six *southern* signs.

94. The CARDINAL POINTS of the ecliptic are the four first points of the signs ♈, ♌, ♐, ♍; those of ♈ and ♌ are called EQUINOCTIAL POINTS, and those of ♐ and ♍ are called SOLSTITIAL POINTS.

95. The EQUINOCTIAL COLURE is a meridian passing through the *equinoctial points*; and the SOLSTITIAL COLURE is another meridian passing through the *solstitial points*. The colures cut one another at right angles in the poles of the world.

96. CIRCLES OF CELESTIAL LONGITUDE are great circles perpendicular to the ecliptic.

97. The LATITUDE of any point in the heavens is an arc of a circle of longitude intercepted between that point and the ecliptic, and is called north or south latitude, as the point is on the north or south side of the ecliptic.

98. PARALLELS OF CELESTIAL LATITUDE are small circles parallel to the ecliptic.

99. The LONGITUDE of any point in the heavens is an arc of the ecliptic intercepted between the first point of Aries and a circle of longitude passing through that point.

100. The RIGHT ASCENSION of any point of the celestial sphere is an arc of the equator, contained between the first point of Aries and a meridian passing through that point: Or, it is the angle formed by the equinoctial colure, and the meridian passing over that point.

101. The DECLINATION of any point, is an arc of a meridian contained between that point and the equinoctial: If the point is on the north side of the equinoctial, it is called *north declination*; but if on the south side, it is called *south declination*.

102. The OBLIQUITY OF THE ECLIPTIC is the angle made by the intersection of the equator and ecliptic, and is measured by the Sun's greatest declination; which, according to the modern observations, is about $23^{\circ} 28'$.

103. PARALLELS OF DECLINATION are small circles parallel to the equinoctial. The TROPIC OF CANCER is a parallel of declination at $23^{\circ} 28'$ distant from the equinoctial in the northern hemisphere; and the TROPIC OF CAPRICORN is that parallel of declination as far distant in the southern hemisphere:

104. The ARCTIC POLAR CIRCLE is a parallel of declination at $23^{\circ} 28'$ distant from the north pole; and the ANTARTIC POLAR CIRCLE is that parallel of declination as far distant from the south pole.

105. The ZENITH is that point of the heavens directly over a place; and the NADIR is the point directly underneath.

106. The HORIZON is that great circle of the sphere which is equally distant from the *zenith* and *nadir* of any place, and divides the sphere into the upper and lower hemispheres.

107. The RISING of a celestial object, is when its center appears in the eastern part of the horizon; and its SETTING is when its center disappears in the western quarters of the horizon.

108. AZIMUTH, or VERTICAL CIRCLES, are great circles perpendicular to the horizon, passing through its poles, which are the *zenith* and *nadir*.

109. The PRIME VERTICAL is that vertical circle which passes through the east and west points of the horizon, and is at right angles to the *meridian of the place*, which is a vertical circle passing through the north and south points of the horizon.

* In the solution of the spherical problems following, the obliquity used is $23^{\circ} 29'$.

110. As the meridian of a place is called the *twelve o'clock hour circle*, so the hour circle at right angles to the meridian is called the *six o'clock hour circle*.

111. The *AZIMUTH* of any celestial object, is an angle at the zenith formed by the meridian of any place, and a vertical circle passing through that object when it is above or below the horizon: And, it is measured by the arc of the horizon intercepted between those vertical circles.

112. The *AMPLITUDE* of any object in the heavens, is usually taken as an arc of the horizon contained between the east or west points thereof, and the center of the object at its rising or setting; Or it may be taken as an angle at the zenith included between the meridian of a place and a vertical circle passing through the object at its rising or setting.

113. The *ALTITUDE* of any point in the heavens, is an arc of a vertical circle intercepted between that point and the horizon.

114. The *ZENITH DISTANCE* of any object, is an arc of a vertical circle contained between that point and the zenith.

The altitude and zenith distance are complements one of the other.

115. The *MERIDIAN ALTITUDE*, or *MERIDIAN ZENITH DISTANCE*, is the altitude or zenith distance when the object is on the meridian of the place.

116. The *CULMINATING* of any celestial object, is the time it *transits*, or comes to the meridian. And the *MEDIUM COELI*, or *MID-HEAVEN*, to any place, is that degree of the ecliptic, or part of the heavens, over the meridian of that place, at any time. Or the *MID-HEAVEN* is the distance of the meridian from the first point of Aries.

117. The *Nonagesimæ degree*, is the 90th. degree of the Ecliptic reckoned from its intersection with the eastern point of the horizon, at any given time.

Consequently the altitude or height of the nonagesimæ degree above the horizon is equal to the distance of the poles of the Ecliptic and Horizon; and is the measure of the angle which the ecliptic makes with the horizon.

118. *ALMICANTHERS*, or *PARALLELS OF ALTITUDE*, are small circles parallel to the horizon.

119. A *PARALLEL SPHERE*, is that position of the sphere wherein the circles, apparently described by the diurnal rotation, are parallel to the horizon; which can happen only at the poles.

120. A *RIGHT SPHERE*, is that wherein the diurnal motions are at right angles to the horizon: Thus it appears in all places under the equator.

121. An *OBLIQUE SPHERE* has all the diurnal motions oblique to the horizon: And thus the motions appear to all parts of the Earth, except under the poles and equator.

122. *DIURNAL ARCS*, are those parts of the parallels of declination of celestial objects apparently described between the times of the rising and setting of those objects: And *NOCTURNAL ARCS*, are the parts of those parallels apparently described from the time of setting to the time of rising.

123. *SEMI-DIURNAL* and *SEMI-NOCTURNAL ARCS*, or the halves of diurnal and nocturnal arcs, are the parts of the parallels between the meridian

meridian and their intersection with the horizon. The corresponding part of the equator answering to the semi-diurnal arc, gives the time between noon and the rising or setting; and the equatorial part answering to the semi-nocturnal arc, shews the time between midnight and the time of setting or rising.

124. The **OBLIQUE ASCENSION** of any point in the heavens, is the degree of the equator which rises with that point in an oblique sphere: And the **OBLIQUE DESCENSION**, is the degree which sets with that point.

125. The **ASCENSIONAL DIFFERENCE** belonging to any celestial object is the difference between its *right* and *oblique ascensions*. In the Sun, it is the time that he rises or sets before or after the hour of fix.

126. The **LATITUDE** of any place on the Earth, is an arc of a terrestrial meridian contained between that place and the equator; being north or south, according to the side of the equator it is on.

127. The **LONGITUDE** of any place on the Earth, is an arc of the equator contained between the meridian of that place and the meridian which is chose for the first, where the reckoning of longitude begins: Or, it is the angle at the pole formed by the first meridian, and that of the place.

128. **REFRACTION**, in an astronomical sense, is the difference between the true and apparent altitudes of celestial objects; they appearing more elevated above the horizon than they really are, on account of the density of the Earth's atmosphere, or air and vapours surrounding it.

129. The **TWILIGHT** is that medium between light and darkness, which happens in the morning before sun-rise, and in the evening after sun-set.

This is occasioned by the atmosphere's refracting the solar rays upon any place, although the Sun is below the horizon of that place, and by observation it is found to begin and end at about 18° below the horizon.

130. The **CREPUSCULUM** is a small circle parallel to the horizon at 18° below it, where the twilight begins and ends.

131. The **LATITUDE OF A PLACE** is expressed by an arc of the meridian, shewing the distance between the zenith of that place and the equinoctial; or, by an arc of the meridian, shewing the height of the pole above the horizon.

For under the pole, or in the latitude of 90 degrees, the pole is in the zenith, or is 90 degrees above the horizon; so that, in this case, the horizon coincides with the equinoctial.

And as many degrees as the observer goes from the pole towards the equator, so many degrees does his horizon go below the equator on one side, and approach the pole on the other side.

Therefore the pole approaches the horizon just as much as the zenith approaches the equator; that is, the height of the pole above the horizon is equal to the distance of the zenith from the equinoctial, which is equivalent to the distance of the observer from the equator, or is equal to the latitude.

132. ASTRONOMICAL TABLES, in general, contain numbers shewing, either the measure of the distances of the heavenly bodies from certain limits which are used to represent remarkable *times* and *places*: Or, the times when those bodies had, or will have, given positions relative to those limits.

Some of the chief astronomical tables are, Solar and lunar tables, for finding the places of those luminaries at given times.

Tables for finding the places of the other planets.

Stellar tables, for finding the places of the stars.

Tables shewing the Sun's place, declination, and right ascension for given times.

Tables of refractions, for correcting observations on altitudes.

Tables of the equation of time; or the difference between the times shewn by a sun-dial and a well regulated clock. &c.

The astronomical tables chiefly wanted in this work, are placed at the end of this book; and are preceded by an account of their construction and use.

133. As the Earth makes one revolution on its axis in a common day of 24 hours; therefore every point of the equator will describe the circle of 360 degrees in 24 hours; and consequently, if 360 degrees give 24 hours, any other number of degrees will give its proportional hours: And if 24 hours give 360 degrees, any other number of hours will give its proportional number of degrees.

And hence are derived methods for converting arcs of circles into measures of time, and measures of time into arcs of circles.

To reduce degrees, minutes, &c. to time.

Multiply by 24, and divide by 360; or multiply by 4, and divide by 60:

Or, Divide the given degrees by 15 for hours; multiply the remainder by 4 for minutes, adding to the product 1 minute for every 15' of a degree; the overplus minutes of a degree, multiplied by 4, gives seconds of time, &c.

Or thus: Let the quotient of the given degrees by 60 stand for the first name; the remaining degrees for the second name; and the other given names in order following: Then this number multiplied by 4, will give the hours, minutes, seconds, &c. in order.

To reduce time into degrees.

Multiply the given hours by 15 gives degrees, to which add 1° for every 4 minutes of time; for every overplus minute reckon 15' of a degree; and for every second of time take 15" of a degree.

Or thus: Divide the time by 4, carrying by sixties, the quotient will be in order, sixties of degrees, degrees, minutes, seconds. &c.: Then sixties of degrees and degrees being reduced, will give the degrees, &c. required.

Robinson's Elem^{ts} of Navigation Book. V. pc: 218

To Convert Degrees into Time

Multiply the Number by 4. & then Count the degrees as minutes of time, the minutes of a degree as seconds of time, & the seconds of a degree as thirds of time - as for Example

Ex: I

$$\begin{array}{r} 69 : 20 : 45. \text{ into its corresponding time.} \\ \text{Multiply by } 4 \\ \hline 277 : 23 : 0 \\ \hline \end{array}$$
 i.e. in time $277 : 23 : 0$
 277 Reduced into Hours divide by 60 $4 : 37 : 23 : 0$

Or by another Method.

Divide the Number of Degrees by 60. placing the quotient in Sexagenes & the remainder in degrees - i.e. if the Number of degrees is under 60 - write 0 - but if it exceeds 60 - divide by that Number and write the quotient in Sexagenes & degrees.

Double this Number, Double that double - and the total will be Hours, Min: Sec: &c. as for Ex.

Ex: III
 to Convert $297 : 44 : 37$ - to its equivalent time divide by 60

$$\begin{array}{r} 3 : 57 : 44 : 37 \\ \hline 3 : 57 : 44 : 37 \\ \hline 7 : 55 : 29 : 14 \\ \hline 7 : 55 : 29 : 14 \\ \hline 15 : 50 : 58 : 28 \end{array}$$

Total. in time

also when it is less than 60 as to Convert $37 : 48 : 55$
 write $0 : 37 : 48 : 55$

$$\begin{array}{r} 0 : 37 : 48 : 55 \\ \hline 1 : 15 : 37 : 50 \\ \hline 1 : 15 : 37 : 50 \\ \hline 2 : 31 : 15 : 40 \end{array}$$

in time

To Convert Time into Degrees

First reduce the Hours & minutes of time all into minutes; Half it & then take the half of that moiety; writing Degrees in the place of minutes of time, Minutes of a Degree for seconds of time, & seconds of a Degree for thirds of time - as for Example

Ex: II. to Convert $4 : 37 : 23 : 0$ its Correspond. Degrees
 Reduce the H: to Min - $60 :$
 Total. minutes &c. $277 : 23 : 0$

The Staff

$$\begin{array}{r} 138 : 41 : 30 \\ \hline 69 : 20 : 45 \end{array}$$

Ex: IV - to Convert $15 : 50 : 58 : 28$ to Degrees

$$\begin{array}{r} 15 : 50 : 58 : 28 \\ \hline 60 : \end{array}$$

The Hours reduced to minutes $950 : 58 : 28$

The Staff $475 : 29 : 14$

The 2 half is Degrees &c. $237 : 44 : 37$

Thus to find the Longitude of any place by the Eclipses of Satellites &c. to find the Immersion at the place of Observation - $8 : 14 : 58 : 0$
 Immersion calculated for the R. Observatory $5 : 49 : 28 : 0$
 The difference in time $2 : 25 : 30 : 0$
 Reduce the Hours into Minutes $60 :$
 produces in Minutes $145 : 30 : 0$

the Staff
 the difference of degrees of the Equator or Longitude East because the time of Observation is more than that at the Royal Observatory at Greenwich

$$\begin{array}{r} 72 : 45 : 0 \\ \hline 36 : 22 : 30 \end{array}$$

EXAM. I. Reduce $69^{\circ} 20', 45''$, to its corresponding time.

$$\begin{array}{r} 15) \underline{69^{\circ} 20' 45''} \quad (4^h 37^m 23^s \\ 9 \times 4 + 1 = 37 \\ \hline 5 \times 4 + 3 = 23 \end{array}$$

EXAM. II. Reduce $4^h 37^m 23^s$, to its corresponding degrees.

$$\begin{array}{r} 4^h 37^m 23^s \\ 15 \quad \hline 60^{\circ} 0' 0'' \text{ for 4 hours.} \\ 9 \quad 15 \quad 0 \text{ for 37 minutes.} \\ 5 \quad 45 \text{ for 23 seconds.} \\ \hline 69^{\circ} 20' 45'' \end{array}$$

EXAM. III. Reduce $237^{\circ}, 44'$, to its equivalent time.

$$\begin{array}{r} 60) \underline{237^{\circ}} \\ \text{Let } 60 = 1^h \\ \hline 3^h 57^m 44^s \\ 4 \\ \hline \text{Answer } 15^h 50^m 58^s 28^s \end{array}$$

EXAM. IV. Reduce $15^h 50^m 58^s 28^s$, to its equivalent degrees.

$$\begin{array}{r} 4) \underline{15^h 50^m 58^s 28^s} \\ \hline 3^h 57^m 44^s 37'' \\ \hline \text{Or } 237^{\circ} 44' 37'' \text{ Answer} \end{array}$$

As the Sun is constantly changing his place, the tables of his right ascension shew for every day at noon (when he comes to the meridian of the place for which the tables are made), what part of the equator is intercepted between that meridian and the equinoctial point γ . The tables for the stars, shew the equatorial arcs contained between the point γ and the sections of circles of right ascension, passing through those stars: The measures of the arcs of right ascension are reduced to time.

There are few days but one or more stars come to the meridian with the Sun, and then have the same right ascension with him: Also; at some time of the year, the Sun must have the same right ascension as any proposed star has; when at other times he may have a less, and so precedes, or comes to the meridian, before the star; or a greater, and so follows the star, and comes to the meridian later.

And hence is derived the following method,

TO FIND THE CULMINATING OF THE STARS.

134. CASE I. To find the time when any star in the table will be upon the meridian.

RULE. Seek the right ascension of the given star, and also that of the Sun for the proposed day; their difference will shew the difference between the times of their culminating; which will be afternoon if the Sun's is least; but before noon if the Sun's is greatest.

EXAM. I.

EXAM. I. *At what time will the star Arcturus come to the meridian of London on the 1st of September, 1775?* Ex. II. *On the 25th of Feb. 1775, at what hour will the star Virgin's Spike be on the meridian of London?*

Arcturus's right ascension	= $14^h 09^m$	Sun's right ascension	= $22^h 34'$
Sun's right ascension	= $10 42$	Virgin's Spike's right ascen.	= $13 12$

Star culm. afternoon	= $3 21$	Star transits before noon	= $9 22$
----------------------	----------	---------------------------	----------

That is, the star Arcturus will be on the meridian of London at 21 minutes after 3 in the afternoon. So the star will be on the meridian of London at 38 minutes after 2 in the morning.

CASE II. *To find if any star in the table will be on, or near the meridian at a given time, reckoned from the preceding noon.*

RULE. To the given time, add the Sun's right ascension for that time; the sum (rejecting 24 hours, if above) is the right ascension of the mid-heaven; which seek among those of the stars, will shew what star will be on, or near the meridian at the time proposed.

EXAM. I. *What star will be on the meridian of London about 10 o'clock at night on the 25th January, 1774?* Ex. II. *On the 10th May, 1774, what star will be on the merid. of Lond. about 30 min. after 4 in the morning?*

Given time 10 hours P. M.	$10^h 0^m$	Given time	$16^h 30'$
Sun's right ascension at noon	$20 32$	Sun's right ascension at noon	$3 09$
And for 10 hours more	2	And for 16 hours more	3

Sum (abating 24 hours)	$6 34$	Right ascension mid-heaven	$19 42$
answers to Syrius.		Answers nearly to Altair.	

SECTION IV.

Of the Projection of the Sphere.

135.

PROBLEM I.

To project the sphere upon the plane of the solstitial colure, or upon the plane of the meridian of any place; those planes being supposed to coincide.

For this projection, the eye is supposed to be in the first point of co , or the common intersection of the equator, ecliptic, and equinoctial circle; that being the pole of the plane of projection, or primitive circle. Pl. IV. Fig. 4.

1st. With the chord of 60 degrees describe a circle $PESQ$ to represent the solstitial colure, whose center φ is its pole. (IV. 62)

2d. A diameter EQ will be the equator, and another PS at right angles to it will shew the equinoctial colure (IV. 60), or the axis of the world, whose extremities P, S , will be the north and south poles.

3d. *For*

3d. *For the parallels of declination.* On the primitive circle, beginning at E and Q, apply the chords of the given degrees of declination, suppose every 10 degrees, and also the distances of the *tropics* and *polar circles* from the equator, namely, $23\frac{1}{2}^{\circ}$ and $66\frac{1}{2}^{\circ}$: Then from the center Ψ in the axis P s produced, apply the respective secants of the complements of the degrees laid on the primitive (IV. 58), and these will give the centers of the corresponding parallels of declination; from which centers, with the extents to the several divisions in the circumference, describe the small circles 10, 10; 20, 20; 30, 30; and these will be the parallels of declination required: Among which $a\epsilon\zeta$, $b\gamma\delta$, are the tropics of Cancer and Capricorn; and ca , dd , the arctic and antarctic and polar circles.

4th. *For the circles of right ascension, or hour circles.* In the diameter EQ produced, lay off from the center Ψ , both ways, the tangents of 15° , 30° , 45° , 60° , 75° , respectively, and they will give the centers of circles to be described through P and s, and cutting the equator in the points representing the 24 hours; the solstitial colure being the 12 o'clock, and the equinoctial colure, Ps, the 6 o'clock hour circles. And in like manner may any other of these kind of circles be drawn. (IV. 75)

5th. *The ecliptic* $\epsilon\gamma\delta$ is drawn, making with the equator an angle of $23\frac{1}{2}^{\circ}$; whose poles c , d , are the intersections of the polar circles with the solstitial colure.

6th. *Parallels of celestial latitude* are drawn parallel to the ecliptic, in the same manner as the circles of declination are drawn parallel to the equator.

7th. *Circles of celestial longitude* are described through c , d , the poles of the ecliptic, in the same manner as the circles of right ascension were described through P, s, the poles of the equator; and thus were the divisions of the ecliptic found that are marked with the signs.

8th. *The horizon* is represented by drawing a diameter HR making an angle with the axis Ps, equal to the latitude of the place; and the poles of the horizon z, N, the zenith and nadir, are at 90° dist. from the circle HR.

9th. *Azimuths*, or *vertical circles*, making any angle with the meridian, are described like circles of right ascension: Thus zN is the prime vertical, and zAN is another azimuth, 45° from the south.

10th. *Almicuths*, or *parallels of altitude*, are in this projection drawn parallel to the horizon, in like manner as were the circles of declination drawn parallel to the equator.

136. PROBLEM II.

To project the sphere upon the plane of the horizon.

In this projection, the eye is supposed in the nadir one of the poles of the horizon, or plane of projection. Plate IV. Fig. 5.

1st. *The horizon* is represented by the primitive circle, where the upper XII is the north the lower XII the south, E the west and Q the east points.

2d. *The azimuth circles* are represented by diameters drawn through z, the center or pole of the horizon: Thus the diameter XII, XII is for the meridian, and EQ for the prime vertical; and other azimuth circles, forming any angle with the meridian, are readily drawn, by laying off their distances in the primitive from the north or south points.

3d. *Pa-*

3d. *Parallels of altitude* are concentric to the primitive, and are described about the pole z with the half tangent of their distance from it: Thus the small circle, whose diameter is ab , is a parallel of altitude 10° above the horizon, or at 80° distant from its pole z .

4th. *The equinoctial's* distance from the zenith, is equal to the latitude of the place, and therefore this circle makes with the horizon an angle, which is measured by the complement of the latitude; then setting off from the center z in z XII continued, the tangent of 50° (the latitude in this example being 40°), it will give the center of the circle EAQ , representing the equinoctial; and the half tangent of 50° , set the same way from z , will give P , the pole of the world.

5th. *The fix o'clock hour circle* passes through the poles of the world, making with the horizon an angle equal to the measure of the latitude; therefore taking in the meridian, from z towards A , the tangent of the latitude 40° , it gives G , the center of the fix o'clock hour circle EPQ .

6th. *The hour circles* pass through the poles of the world, and make with one another angles of 13 degrees: Therefore (IV. 55) in a line DE , drawn through G at right angles to the meridian, set off on both sides of G the tangents 15° , 30° , 45° , 60° , 75° , to the radius PG , and they will give the centers of the several hour circles passing through P , cutting the horizon and equinoctial in the hour points.

7th. *The polar circles, tropics, and other circles of declination*, are described parallel to the equinoctial, about its pole P , at given distances from it, either by finding the centers of such parallels as shewn in art. IV. 66; or by setting off on each side of z the half tangents of their greatest and least distances from z ; then the middles of those intervals are the centers sought. Thus; the arctic circle is distant from P $23\frac{1}{2}^\circ$; then to, and from $zP = 50^\circ$, add and take $23\frac{1}{2}^\circ$; gives $73\frac{1}{2}^\circ$ and $26\frac{1}{2}^\circ$; the half tangents of these set off from z , gives p and q ; then a circle being described on the diameter pq is the arctic circle.

In like manner will the centers of the tropic of Cancer c and of Capricorn d be obtained.

8th. *The northern portion of the ecliptic* $\alpha\beta\Delta$ is described from a center distant from z towards P , the tangent of $73\frac{1}{2}^\circ = \angle$ the ecliptic makes with the horizon.

9th. *Circles of longitude* $\alpha p\Delta$, $p\delta$, $p\Pi$, $p\Omega$, $p\mathcal{M}$, are described thro' p , the pole of the ecliptic, from centers in the line BFC ; in like manner as the hour circles were described through P , the pole of the equator.

10th. *Circles of celestial latitude*, VIII qVI , are described about p , as the circles of declination were described about P , the pole of the equinoctial.

137.

PROBLEM III.

To project the sphere upon the plane of the equator.

In this projection the eye is supposed to be in one of the poles of the equator, suppose in the south pole, and projecting the north hemisphere. Plate IV. Fig. 6.

1st. *The equator* is represented by the primitive circle, whose center and pole is P .

2d. 72

2d. *The hour circles* are expressed by diameters making angles of 15° with one another; of which XII P-XI is the meridian, or solstitial colure, and VI P-VI the 6 o'clock circle, or equinoctial colure.

3d. *Circles of declination* are circles parallel and concentric to the equator, described from its center with radii equal to the half tangents of their several distances from the pole P, or half co-tangents of their degrees of declination: Thus *pq* the arctic circle, and *a æ* the tropic of Cancer, are described with the half tangents of $23\frac{1}{2}^{\circ}$ and $66\frac{1}{2}^{\circ}$ respectively; and so of the others.

4th. *The ecliptic* making an angle of $23\frac{1}{2}^{\circ}$ with the equator; therefore the tangent of these degrees laid from P towards *a* will give the center for describing the ecliptic *q æ æ*, whose pole *p* is in the polar circle.

5th. *Circles of longitude* are described through *p*, the pole of the ecliptic, in like manner as the hour circles were described through P, the pole of the equator in the last problem; and thus were the divisions δ , Π , Ω , ν , obtained.

6th. *Circles of celestial latitude* are projected in the same manner as the circles of declination in the last problem.

7th. *The horizon of any place*, suppose of London, being inclined to the equator in an angle equal to the co-latitude, $38^{\circ} 28'$; the tangent of this laid from P towards *æ*, and the half tangent laid from P to Z, will give the center, and Z the pole, of the horizon HOR.

8th. *The prime vertical* HZR making an angle with the equator equal to $51^{\circ} 32'$, the latitude of the place, its center is found by laying the tangent of $51^{\circ} 32'$ from P towards O.

9th. *Azimuth circles*, making given angles with the meridian ZO, are thus described: In a line drawn through the center of the prime vertical, at right angles to the meridian, take distances from that center, equal to the tangents of the proposed azimuth angles, the semidiameter of the prime vertical being the radius, those distances give the centers sought; and thus was the azimuth circle ZA described.

10th. *Parallels of altitude* are described about Z, the pole of the horizon, at the distances of the co-altitudes, in the same manner as were the circles of declination, in the last problem, described about P, the pole of the equator; and thus was the small circle *v b v* described at 10° distance from the horizon, or 80° distant from its pole Z.

138.

PROBLEM IV.

To project the sphere upon the plane of the ecliptic.

The eye is here supposed to be in one of the poles of the ecliptic, and thence seeing the northern hemisphere projected. Plate-IV. Fig. 7.

1st. *The ecliptic* is here represented by the primitive circle, whose center *p* is the pole of the ecliptic.

2d. *Circles of longitude* are herein shewn by diameters; those making angles of 30° with one another, being drawn through the divisions marked with the signs of the zodiac.

3d. Pa-

3d. *Parallels of celestial latitude*, are circles described about p concentric to the ecliptic; such is the small circle whose diameter is ab , representing the parallel of 10° of latitude.

4th. *The equator* making an angle with the ecliptic of $23\frac{1}{2}^\circ$; therefore the tangent of this inclination laid from p towards ss , will give the center of the equator q xii Δ ; and the half tangent of $23\frac{1}{2}^\circ$ laid from p the same way, gives p for the pole of the equator.

5th. *The equinoctial colure*, which here makes the *six o'clock circle*, makes an angle with the ecliptic of $66\frac{1}{2}^\circ$; therefore the tangent of $66\frac{1}{2}^\circ$ laid from p towards vs , gives the center of the 6 o'clock circle q p Δ .

6th. *Hour circles* passing through P , and making angles of 15° with one another, are described from centers, found in a right line passing through the center of q p Δ , and drawn at right angles to the solstitial colure q p vs ; by laying off in that line the tangents of 15° , 30° , 45° , 60° , 75° , reckoned from the center of q p Δ , on both sides, the semidiameter of this circle being the radius. These hour circles cut the equator in the hour points.

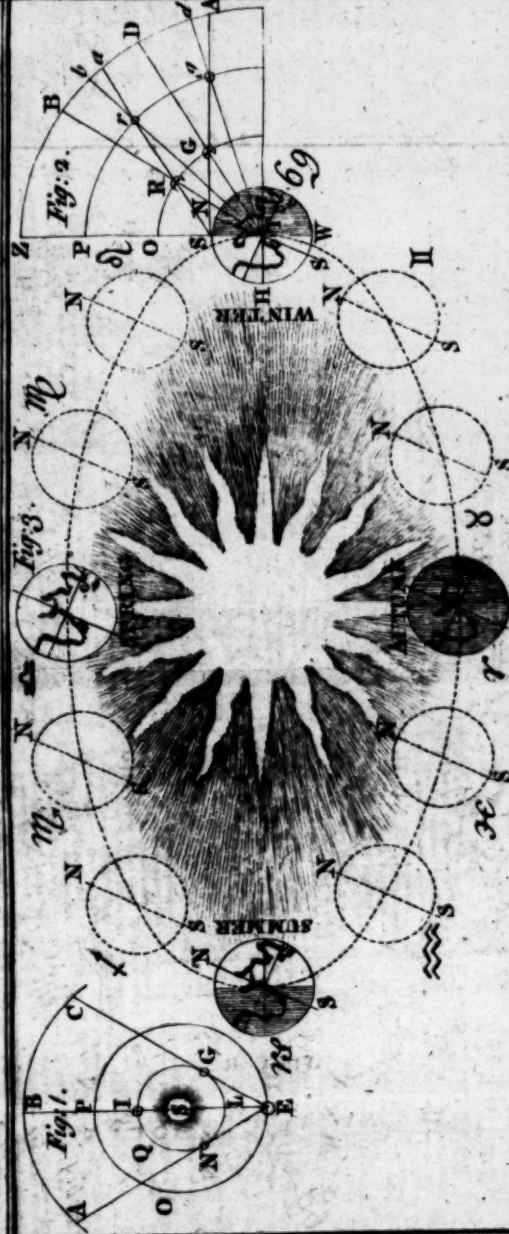
7th. *Parallels of declination*, such as the tropic of Cancer, and artic circle, whose diameters are 12, 12, and pq , are described, by laying off from p the half tangents of their greatest and least distances: Thus q being distant from p 47° , make $pq = \frac{1}{2}$ tangent of 47° , the middle of pq will be the center of the polar circle.

8th. *The horizon* HOR is to make an angle with the ecliptic equal to the difference between the co-latitude and the obliquity of the ecliptic, when P is projected to the north of p ; otherwise that angle is equal to the sum of those quantities. And for London, where the said difference $= (38^\circ 28' - 23^\circ 29') = 14^\circ 59'$, the tangent of $14^\circ 59'$ gives the center of HOR , and the half tangent gives z the zenith.

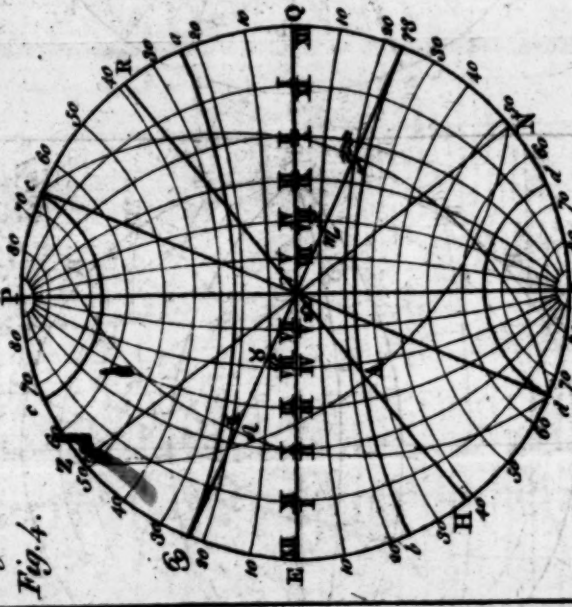
9th. *The prime vertical* HZR is described, by laying from p towards o the co-tangent of pz for a center.

10th. *Azimuth circles* are described through z , making given angles with the meridian zo , by finding their centers in a line drawn through the center of HZR , in the manner described for the hour circles, Prob. II.

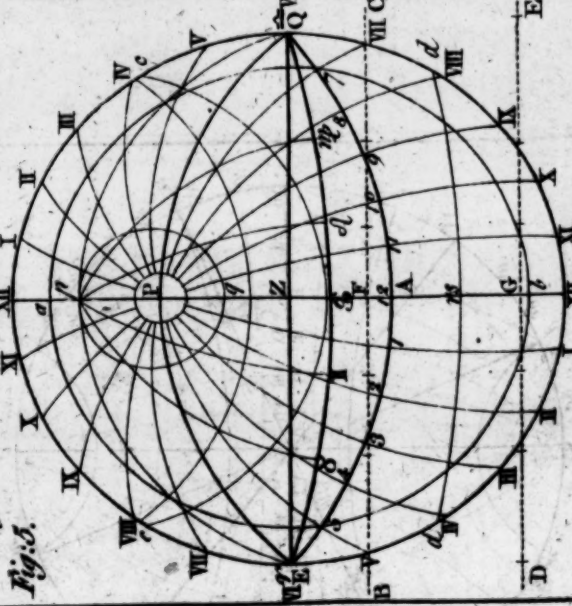
11th. *Parallels of altitude* are represented, by describing small circles parallel to the horizon HOR , at given distances from it; or, which comes to the same, describing small circles about the pole z , at distances equal to the complements of the given altitudes: And thus the circle *cdc* was described for a parallel of 33° of altitude.



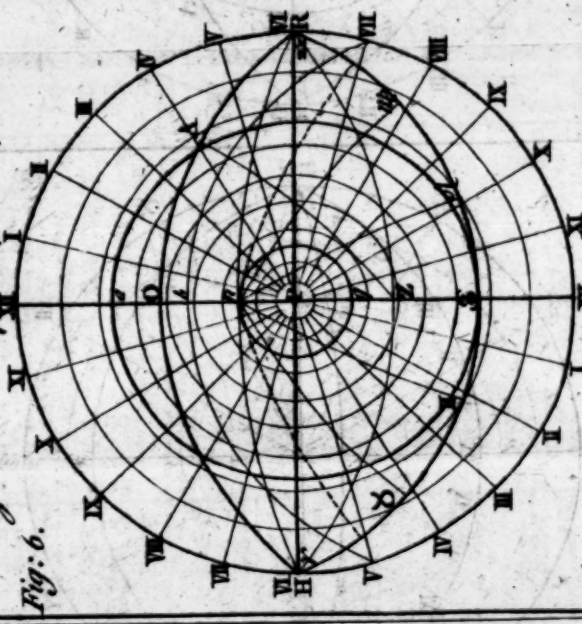
Projection on the Solstitial Colure or Meridian



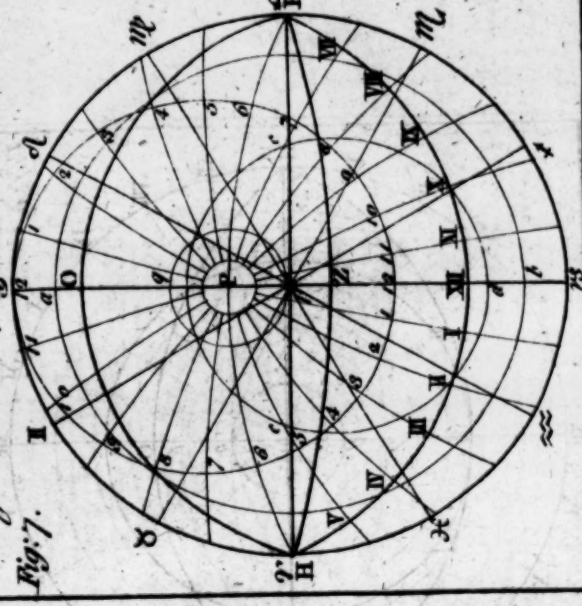
Projection on the plane of the Horizon.



Projection on the plane of the Equator.



Projection on the plane of the Ecliptic.



SECTION V.

Problems of the Sphere.

139.

PROBLEM V.

Pl. V.

Given the Sun's longitude, and the obliquity of the ecliptic; Required the Sun's right ascension and declination.

EXAM. *Let the obliquity of the ecliptic, or the Sun's greatest declination, be $23^{\circ} 29'$, and the Sun's place be $13^{\circ} 16'$ in Taurus: Required the rest?*

CONSTRUCTION.

In the primitive circle PESQ representing the plane of the solstitial colure, whose center is φ , draw a diameter EQ for the equator, and at right angles to EQ draw a diameter PS for the equinoctial colure: Make $ES = 23^{\circ} 29'$, and draw a diameter $\varpi \wp$ for the ecliptic, in which (IV. 71.) take $\varphi \odot = 43^{\circ} 16'$ for the Sun's distance from the point φ : Through P $\odot$ s describe a circle of right ascension.

COMPUTATION. See Book IV. art. 130, 131.

In the right angled spheric triangle $\varphi \odot B$.

Given Sun's longitude $\varphi \odot = 43^{\circ} 16'$ } Req. right ascen. φB .
greatest declinat. $\angle \odot \varphi B = 23^{\circ} 29'$ } pref. declin. $B \odot$.

To find the present declination.

To find the right ascension.

As Radius	= R	10,00000	As Radius	= R	10,00000
To f. Sun's lon.	=	$43^{\circ} 16'$	To t. Sun's lon.	=	$43^{\circ} 16'$
So f. great decl.	=	$23^{\circ} 29'$	So cos. obl. ecl.	=	$23^{\circ} 29'$
To f. pref. decl.	=	$15^{\circ} 51'$	To t. rt. ascen.	=	$40^{\circ} 48'$
					$9,93616$

140. While the Sun is moving from φ to ϖ , or is in the first quadrant of the ecliptic, the given longitude is the hypothenuse in the triangle $\varphi \odot B$, the declination $B \odot$ is north, and φB is the right ascension.

When the sun has past the solstice ϖ , and is descending towards ϖ , he is then said to be in the second quadrant, and his longitude or distance from φ being taken from 180° , the remainder $\varpi \odot$ becomes the hypothenuse, and the declination is still north; but the arc $B \varpi$ found for the right ascension, is only the supplement, and must therefore be taken from 180° .

The Sun having past the point ϖ , and descending towards $\wp$, is got into the third quadrant; the longitude then, reckoned from φ , will be greater than 180° : In this case the excess above 180° , or the distance the Sun is removed from ϖ , will be the hypothenuse $\varpi \odot$; the declination will be south; and the arc ϖA , found for the right ascension, must be added to 180° , to give the right estimation estimated from φ .

When the Sun has past the solstice $\wp$, and is ascending towards φ , he is then in the fourth quadrant; therefore the longitude is greater than 270° , and must be taken from 360° , to give the hypothenuse $\varpi \odot$. Herein the declination is south, and the right ascension ϖA , found by the proportion, must be taken from 360° , to give the right ascension from φ .

At equal distances from the equinoctial points φ or ϖ , the Sun will have equal quantities of declination; but will be of different names, according as it is on the north or south sides of the equinoctial.

141. PRO-

141.

PROBLEM VI.

Pl. V.

Given the obliquity of the ecliptic, and the Sun's present declination; Required the Sun's longitude and right ascension.

EXAM. *The obliquity of the ecliptic, being $23^{\circ} 29'$, what is the Sun's longitude and right ascension when he has $20^{\circ} 43'$ of north declination?*

CONSTRUCTION.

Having described the solstitial colure, drawn the equator EQ , the axis PS , and the ecliptic es 12° , as before; make en , qn , equal to the given declination, and (3d 133) describe the parallel of declination nn , its intersection with the ecliptic gives $\odot$ the Sun's place; through P , $\odot$, S , describe the circle of right ascension POS .

COMPUTATION.

In the right angled spheric triangle $\varphi \odot B$.

Given Sun's greatest decl. $\angle \odot \varphi B = 23^{\circ} 29'$ } Required Sun's lon. $\varphi \odot$:
present decl. $\odot B = 20^{\circ} 43'$ } rt. ascen. φB .

To find the Sun's longitude.

To find the Sun's right ascension.

As $f. \odot$ great decl. $= 23^{\circ} 29'$	$0,399,59$	As Radius	$= R$	$10,00000$
To $\sin$. pref. decl. $= 20^{\circ} 43'$	$9,54869$	To cot. gr. decl. $= 23^{\circ} 29'$	$10,36204$	
So Radius	$= R$	So tan. pref. decl. $= 20^{\circ} 43'$	$9,57772$	

To $\sin. \odot$ longit. $= 62^{\circ} 35'$	$9,94828$	To $\sin$. rt. ascen. $= 60^{\circ} 31'$	$9,93976$
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Therefore the Sun is in $\Pi 2^{\circ} 35'$, or in $\Xi 27^{\circ} 25'$, according as the time of the year is before or after the summer solstice.

142.

PROBLEM VII.

Pl. V.

Given the obliquity of the ecliptic, and the Sun's right ascension; Required the Sun's longitude and present declination.

EXAM. *When the Sun's right ascension is $60^{\circ} 31'$, what is his longitude and present declination, the greatest declination being $23^{\circ} 29'$?*

CONSTRUCTION.

The solstitial colure, equator, axis, and ecliptic being described, as before, make φB = given right ascension (4th 133), and describe the circle PBS , cutting the ecliptic in $\odot$ the Sun's place.

COMPUTATION.

In the right angled spheric triangle $\varphi \odot B$: the leg φB and $\angle \odot \varphi B$ being known, the hypoth. $\varphi \odot$, and other leg $\odot B$, are found as in art. 137, 138. Book IV.

As Rad. : cot. gr. decl. :: cot. rt. ascen. | As Rad. : tan. gr. decl. :: $\sin$. rt. ascen.
[: cot. $\odot$ long.] [: tan. pr. decl.]

As Rad. : cot. $23^{\circ} 29'$:: cot. $60^{\circ} 31'$ | As Rad. : cot. $23^{\circ} 29'$:: $\sin$. $60^{\circ} 31'$
[: cot. $62^{\circ} 35'$] [: tan. $20^{\circ} 43'$]

Three other problems may be formed out of the four things concerned, or greatest declination, present declination, longitude, and right ascension: But these being of little more importance than as an exercise for right angled spheric triangles, they are therefore omitted.

143. PRO-

143.

PROBLEM VIII.

Pl. V.

Given the latitude of the place, and the Sun's present declination; Required the Sun's altitude and azimuth at 6 o'clock.

EXAM. *At London, in lat. $51^{\circ} 32' N.$, on the longest day, when the Sun's declination is $23^{\circ} 29'$: Required the Sun's altitude and azimuth at 6 o'clock in the morning or evening?*

CONSTRUCTION.

Describe the meridian, draw the horizon HR , and prime vertical ZN ; make RP = latitude $51^{\circ} 32' N.$; draw the 6 o'clock hour circle PS , the equator EQ , the $23^{\circ} 29' N.$ parallel of declination nm , cutting the 6 o'clock hour circle PS in $\odot$; and through z , $\odot$, N , describe the azimuth circle $z \odot N$, cutting the horizon in A ; then the things given and required fall in either of the triangles $z \odot P$ or $\varphi \odot A$, they being supplemental triangles one to the other.

COMPUTATION.

In the spheric triangle $z \odot P$, right angled at P .

Given the co-latit. $zP = 38^{\circ} 28'$ } Required the co-altitude $z \odot$.
the co-decl. $\odot P = 66 \quad 31$ } the azimuth $\angle \odot zP$.

Or in the spheric triangle $\varphi A \odot$, right angled at A .

Given the latit. $A \varphi \odot = 51^{\circ} 32'$ } Required the altitude $A \odot$.
the decl. $\varphi \odot = 23 \quad 29$ } the co-azimuth φA .

To find the altitude $A \odot$.

As Radius	= R	10,000000
To fin. decl.	= $23^{\circ} 29'$	9,60041
So fin. lat.	= $51 \quad 32$	9,89375

To find the azimuth φA .

As Radius	= R	10,000000
To cos. lat.	= $51^{\circ} 32'$	9,79383
So tan. decl.	= $23 \quad 29$	9,63796

To fin. alt.	= 18	11	9,49416
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To cot. azim.	= 74	53	9,43179
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For the arc AR measures the $\angle RZA$, the azimuth.

(IV. 9)

144. On the shortest day at London, the parallel of S. declination cuts the 6 o'clock hour circle below the horizon; and as the triangles $\varphi A \odot$, $\varphi a \odot$, are congruous, the depression below the horizon, on the shortest day at 6 o'clock, will be equal to the altitude at the same hour on the longest day; and the azimuth will be also equal, if estimated from the south.

So that on the 21st of June, at London, the Sun will bear $N. 74^{\circ} 53' E.$ at 6 o'clock in the morning, and $N. 74^{\circ} 53' W.$ at 6 in the evening: But on the 21st of December, at the same hours, it will bear $S. 74^{\circ} 53' E.$, and $S. 74^{\circ} 53' W.$

From a due consideration of this Problem, it is evident, that as the declination increases, the altitude increases and the azimuth lessens; and the contrary happens while the declination is diminishing: So that on the days of the equinoxes, or when the Sun has no declination, the altitude at 6 o'clock will be nothing, or the Sun will be in the horizon; and the azimuth being then 90 degrees, the Sun will be due east in the morning, and west in the evening; that is, on the days of the equinoxes the Sun rises and sets at six, in the east and west points of the horizon.

Q²

145. PRO-

145.

PROBLEM IX.

Pl. V.

Given the latitude of the place, and the Sun's declination ;

Required the altitude and hour when the Sun is due east or west.

EXAM. *At London, in latitude $51^{\circ} 32' N.$, what is the Sun's altitude, and the hour when he is due east or west, on the longest day, or when the declination is $23^{\circ} 29' N.$?*

CONSTRUCTION.

Describe the primitive circle to represent the meridian of London, draw the horizon HR , and the prime vertical ZN ; make $RP = 51^{\circ} 32'$, the given latitude, draw the 6 o'clock hour circle PS , the equator EQ , the parallel of declination nm (3d 135), cutting the prime vertical in O , and through $P \odot$ s describe (II. 72) the hour circle POS , cutting the equator in A .

Here the things concerned in the Problem fall in either of the triangles $PZ \odot$ or $VA \odot$.

COMPUTATION.

In the spheric triangle $PZ \odot$, right angled at z .

Given the co-latit. $PZ = 38^{\circ} 28'$ } Required the co-altitude ZO .
the co-decl. $P \odot = 66^{\circ} 31'$ } the hour. fr. noon $\angle ZP \odot$.

Or in the spherical triangle $VA \odot$, right angled at A .

Given the latit. $\angle A V \odot = 51^{\circ} 32'$ } Required the altitude $V \odot$.
the decl. $A \odot = 23^{\circ} 29'$ } the hour after 6 $V A$.

To find the altitude $V \odot$.

As s. lat. $\angle A V \odot = 51^{\circ} 32'$ 0,10625
To sin. decl. $A \odot = 23^{\circ} 29'$ 9,60041
So Radius $= R$ 10,00000

To find the hour after 6.

As Radius $= R$ 10,00000
To cot. lat. $A V \odot = 51^{\circ} 32'$ 9,90009
So tan. decl. $A \odot = 23^{\circ} 29'$ 9,63796

To sin. alt. $V \odot = 30^{\circ} 36'$ 9,70666

To s. h. fr. 6. $A V = 20^{\circ} 12'$ 9,53805

Which $20^{\circ} 12'$ converted into time (132), gives 1 h. 20 m. 48 f. for the time after 6 in the morning, and before 6 in the evening, when the Sun will appear due east and west; which will be at 7 h. 20 m. 48 f. in the morning, and 4 h. 39 m. 12 f. in the afternoon.

Or, The compl. of $20^{\circ} 12'$, viz. $69^{\circ} 48'$ put into time, which gives 4 h. 39 m. 12 f., shews the time before and after noon, when the Sun will be due east or west.

146. This Problem worked for the shortest day, namely in the $\Delta VA \odot$, which is congruous to $VA \odot$, would give the Sun's depression at the time when he was east or west, which would be before 6 in the morning, and after 6 in the evening, by as much as was found above, viz. 1 h. 20 m. 48 f.

By this Problem it appears, that when the latitude of the place, and the Sun's declination, have the same name, then, the greater the declination and latitude, the greater the altitude and time from 6: And having contrary names, the same things happen; but with this difference, that in the former case the days lengthen on account of the increase of the latitude and declination; whereas, in the latter case, the days shorten on that account.

Given the latitude of a place, and the Sun's declination ;
Required his amplitude and ascensional difference.

EXAM. *At London, whose lat. is $51^{\circ} 32'$ N., on the 21st of June, being the longest day, when the Sun's declination is $23^{\circ} 29'$ N.: How far from the north does the Sun rise and set, at what time, and what is the length of the day and night?*

CONSTRUCTION.

Let the primitive circle represent the meridian of the place, and the diameter HR the horizon ; from R, the north point thereof, take $RP = 51^{\circ} 32'$ for the latitude, draw the axis or 6 o'clock hour circle PS, and at right angles to it, draw the equator EQ ; make $EN, QM = 23^{\circ} 29'$ the declination, and (3d 135) describe the parallel of declination nm , cutting the horizon in $\odot$ the place of the Sun at its rising and setting ; through which describe (II. 72) the hour circle $P\odot S$.

COMPUTATION.

Now as the arc $QR =$ co-latitude, measures the $\angle Q\odot R$.

In the spheric triangle $\odot\odot A$, right angled at A.

Given Sun's decl. $A\odot = 23^{\circ} 29'$ } Required the amplitude $\odot\odot$
co-latit. $\angle A\odot\odot = 38^{\circ} 28'$ } the ascen. diff. $\odot\odot A$.

To find the amplitude $\odot\odot$.

As f. co-l. $\angle A\odot\odot = 51^{\circ} 32'$ 0,20617

To sin. decl. $A\odot = 23^{\circ} 29'$ 9,60041

So Radius $= R$ 10,00000

To sin. amp. $\odot\odot = 39^{\circ} 50'$ 9,80658

This $39^{\circ} 50'$ is the amplitude reckoned from the east or west points of the horizon : But its complement $50^{\circ} 10'$ shews how far from the north the Sun rises or sets on the longest day at London.

To find the ascensional difference $\odot\odot A$.

As Radius $= R$ 10,00000

Tot. lat. $\angle A\odot\odot = 51^{\circ} 32'$ 10,99991

So tan. decl. $A\odot = 23^{\circ} 29'$ 9,63796

To f. asc. diff. $\odot\odot A = 33^{\circ} 09'$ 9,73787

Which $33^{\circ} 09'$ converted into time (132) gives 2 h. 12 m. 36 s. for the time which the Sun rises before, and sets after, the hour of fix on the longest day.

Suppose rs to be a parallel of declination as fh , as mn is north ; then the hour circle PBS , passing through $\odot$ the place of the Sun at its rising or setting, will form a triangle $\odot\odot B = \triangle \odot\odot A$, where the amplitude is to the southward of the east and west points.

148. Hence it is evident, that when the latitude and declination have the same name, the Sun rises before, and sets after 6 : But when of contrary names, the Sun rises after, and sets before 6.

149. And as the Sun describes the parallel of declination n m in 24 hours, being at n when it is noon, and at m when it is midnight; therefore the time in passing from m to $\odot$, or the time of rising, being doubled, gives the length of the night; and the time of setting being doubled, must give the length of the day.

Then to, and from	6 ^h 0 ^m 0 ^s
Add and subtract the ascen. differ.	2 12 36
Sum, gives $\odot$ setting	8 12 36
Diff. gives $\odot$ rising	3 47 24
Length of Day is	16 25 12
Length of night is	7 34 48

But when it is the shortest day at London, which is, when the Sun has $23^{\circ} 29'$ south declination; then the lengths of the day and night change places; the day being 7 h. 34 m. 48 f. long, and the night 16 h. 25 m. 12 f.

150. When the latitude and declination have the same name, the difference between the right ascension and the ascensional difference, is the oblique ascension; and their sum is the oblique descension.

But when they are of contrary names, their sum is the oblique ascension, and their difference is the oblique descension.

151. When the declination is equal to the co-latitude of any place (which can only happen to places within the polar circles), then the parallel of declination will not cut the horizon, and consequently the Sun will not set in those places during the time his declination exceeds the co-latitude: And the same may be said of the stars, every one of which, whose co-declination, or polar distance, is less than the latitude; or, which is the same thing, whose declination is not less than the co-latitude of a place, those stars will never descend below the horizon of that place. But this is to be understood only when the Sun or stars are in the same hemisphere with the given place; for when the Sun or stars are in a contrary hemisphere with any place, whose co-latitude does not exceed the declination of those celestial objects, then will they never rise above the horizon of that place, and consequently are never visible there.

152.

PROBLEM XI.

Pl. V.

Given the latitude of a place, the Sun's declination and altitude ;
Required the hour from noon, and the Sun's azimuth.

EXAM. *In the latitude of $51^{\circ} 32' N$. the Sun's altitude was observed to be $46^{\circ} 20'$, when his declination was $23^{\circ} 29' N$: What was the Sun's azimuth, and the hour when the observation was made?*

CONSTRUCTION.

Let the primitive circle $ZRNH$ represent the meridian of London, HR the horizon, ZN the prime vertical ; make $RP = 51^{\circ} 32'$ the height of the pole at London ; draw the axis PS , and the equator EQ . Lay off the declination En , Qm , $23^{\circ} 29' N$. ; the altitude HR , rs , $46^{\circ} 20'$; and (IV. 68) describe the parallel of declination nm , and the parallel of altitude rs , cutting one another in $\odot$ the place of the Sun at that time. Through z , $\odot$, N , describe an azimuth circle $z\odot N$, and through P , $\odot$, s , describe an hour circle $P\odot s$: Then the angles $\odot zP$, $\odot Pz$, being measured (IV. 72), will give the azimuth and hour from noon required.

COMPUTATION.

In the oblique angled spheric triangle $P\odot z$.

Given the co-latitude. $zP = 38^{\circ} 28'$ } Required the azim. $\angle \odot zP$.
the co-alt. or zen. dist $z\odot = 43^{\circ} 40'$ } and the h. fr. noon $\angle \odot Pz$.
the co-dec. or pol. dist. $\odot P = 66^{\circ} 31'$ } See art. 167. Book IV.

To find the azimuth $\angle \odot zP$.

$$\text{Here } z\odot = 43^{\circ} 40'$$

$$zP = 38 \quad 28$$

$$P\odot = 66 \quad 31$$

$$z\odot - zP = 5 \quad 12 = D.$$

$$\begin{array}{r} 71 \quad 43 \quad 35 \quad 51 \frac{1}{2} \\ 2) \quad \hline 61 \quad 19 \quad 30 \quad 39 \frac{1}{2} \end{array}$$

$$\begin{array}{l} \text{Then Co-ar. fin. co-lat.} = 38^{\circ} 28' \quad 0,20617 \\ \text{Co-ar. fin. co-alt.} = 43 \quad 40 \quad 0,16086 \\ \text{Sin. } \frac{1}{2} \text{ sum co-decl. \& n} = 35 \quad 51 \frac{1}{2} \quad 9,76774 \\ \text{Sin. } \frac{1}{2} \text{ diff. co-decl. \& n} = 30 \quad 39 \frac{1}{2} \quad 9,79750 \end{array}$$

The sum of the four logs $19,84227$

The $\frac{1}{2}$ sum gives $56^{\circ} 30'$ $9,92113$

Which doubled, gives $113^{\circ} 00'$ for the azimuth sought, reckoning from the north.

To find the hour from noon, $\angle \odot Pz$.

$$\text{Here } P\odot = 66^{\circ} 31'$$

$$zP = 38 \quad 28$$

$$\odot z = 43 \quad 40$$

$$P\odot - Pz = 28 \quad 03 = D.$$

$$\begin{array}{r} 71 \quad 43 \quad 35 \quad 51 \frac{1}{2} \\ 2) \quad \hline 15 \quad 37 \quad 7 \quad 48 \frac{1}{2} \end{array}$$

$$\begin{array}{l} \text{Then Co-ar. fin. co-decl.} = 66^{\circ} 31' \quad 0,03755 \\ \text{Co ar. fin. co-lat.} = 38 \quad 28 \quad 0,20617 \\ \text{Sin. } \frac{1}{2} \text{ sum co-alt. \& n} = 35 \quad 51 \frac{1}{2} \quad 9,76774 \\ \text{Sin. } \frac{1}{2} \text{ diff. co-alt. \& n} = 7 \quad 48 \frac{1}{2} \quad 9,13309 \end{array}$$

The sum of the four logs $19,14455$

The $\frac{1}{2}$ sum gives $21^{\circ} 56'$ $9,57227$

This doubled, gives $43^{\circ} 52'$ for the measure of the hour from noon, which is 2 h. 55 m. 28 f.

Hence it appears, that the observation was made either at 9 h. 4 m. 32 f. in the morning, or at 2 h. 55 m. 28 f. in the afternoon.

The azimuth being first found, the hour from noon might have been found by the proportion between opposite sides and angles.

Q₄

Had

Had the declination and latitude been of contrary names, the same kind of operation would have been used to find the things required; only the side $\odot P$ would have been obtuse, by adding the declinat. to 90° , instead of subtracting it as in the case of the lat. and decl. having like names.

153.

PROBLEM XII.

Pl. V.

Given the latitude of the place, and the Sun's declination;
Required the time when the twilight begins and ends.

EXAM. *At what time does the twilight begin and end at London on the first of May, the latitude being $51^\circ 32' N.$, and the declination at that time being $15^\circ 12' N.$?*

CONSTRUCTION.

Let the circle $zRNH$ represent the meridian of the place, HR the horizon, ZN the prime vertical, and ts the Crepusculum, or small circle parallel to the horizon described at 18 degrees below it (IV. 68); lay off the latitude RP , draw the axis PS , the equator EQ , and describe the parallel of declination nm , and where nm cuts ts in $\odot$, is the Sun's place at the time of the beginning or end of the twilight; through $\odot$ describe (II. 72) the vertical circle $z\odot N$, and the hour circle $P\odot S$; then the $\angle zP\odot$ being measured (IV. 72), will give the time before or after noon as required.

COMPUTATION.

In the oblique angled spheric triangle $z\odot P$.

Given the co-lat. $zP = 38^\circ 28'$ } Req. the hour from noon $= \angle zP\odot$
the polar dist. $P\odot = 74^\circ 48'$ } The manner of solution is the
the zenith dist. $z\odot = 108^\circ 00'$ } same as in last Problem.

Here $P\odot = 74^\circ 48'$

$Pz = 38^\circ 28'$

$\odot z = 108^\circ 00'$

$P\odot - Pz = 36^\circ 20' = D.$

$$\begin{array}{r} 144 \quad 20 \quad 72^\circ 10' \\ 2) \quad \hline 71 \quad 40 \quad 35 \quad 50 \end{array}$$

Then Co-ar. sin. polar dist. $= 74^\circ 48' 0.01547$
Co-ar. sin. co-latit. $= 38^\circ 28' 0.20617$
Sine $\frac{1}{2}$ sum. of zen. d. & $D = 72^\circ 10' 9.97861$
Sine $\frac{1}{2}$ diff. of zen. d. & $D = 35^\circ 50' 9.76747$

The sum of these four logs. 19.96772

The half sum gives $74^\circ 28'$ 9.98386

Which doubled, gives $148^\circ 56'$ for $\angle zP\odot$.

And $148^\circ 56'$ reduced to time, gives 9 h. 55 m. 44 f. either before or after noon; that is, the twilight begins at 2 h. 04 m. 16 f. in the morning, and ends at 9 h. 55 m. 44 f. in the evening, on the 1st of May, at London.

154. When the declination becomes greater than the difference between the co-latitude and 18 degrees, then the parallel of declination nm will not cut the parallel ts 18 degrees below the horizon, and consequently at that time there will be no night at that place, but the twilight will continue from Sun-setting to Sun-rising; and on this account it is, that from about the 22d of May to about the 21st of July there is no total darkness at London, The Sun's declination during that interval being greater than $20^\circ 28'$, which is the difference between 18° and $38^\circ 28'$, the complement of the latitude.

155. PRO-

155. PROBLEM XIII.

Pl. V.

Given the time of the year, the latitude of a place, and the altitude of a known fixed star;

Required the hour of the night when the observation was made.

EXAM. *Some time in the night, on the 27th of November, 1764, suppose the star Arcturus, whose declination $20^{\circ} 30' N$. should be observed at London to be $27^{\circ} 12'$ above the horizon; At what hour would the observation be made?*

CONSTRUCTION.

Describe the meridian of the place, draw the horizon HR, whose zenith and nadir are Z and N, and describe the parallel of altitude rs at $27^{\circ} 12'$ above the horizon; take P the north pole $51^{\circ} 32'$ above the horizon for the latitude of the place, and s the south pole as much below the horizon; draw the equator EQ, and describe (3d 135) the star's parallel of declination nm ; and where this parallel nm cuts the former rs in *, is the position of the star at the time of observation; describe (II. 72) the vertical circle $z*N$, and hour circle $P*s$, and the angle $zP*$ being measured (IV. 72), gives the hour from, or to, the time of the star's culminating.

COMPUTATION.

In the oblique angle spheric triangle $P*Z$.

Given the co-latitude $PZ = 38^{\circ} 28'$ } Required $\angle zP*$, or the hour
the co-altitude $z* = 62^{\circ} 48'$ } from culminating.
the polar dist. $*P = 69^{\circ} 30'$

$$\text{Here } P* = 69^{\circ} 30' \\ PZ = 38^{\circ} 28'$$

$$P* = 62^{\circ} 48'$$

$$P* - PZ = 31^{\circ} 02' = D.$$

$$\begin{array}{r} 93 \ 50 \ 469' \ 55 \\ 2) \hline 31 \ 46 \ 15 \ 53 \end{array}$$

$$\begin{array}{l} \text{Then Co-ar. fin. co-lat.} = 38^{\circ} 28' \ 0,20617 \\ \text{Co-ar. fin. pol. dist.} = 69 \ 30 \ 0,02841 \\ \text{Sin. } \frac{1}{2} \text{ sum zen. dist. \& D} = 46 \ 55 \ 9,86354 \\ \text{Sin. } \frac{1}{2} \text{ diff. zen. dist. \& D} = 15 \ 53 \ 9,43724 \end{array}$$

The sum of the four logs $19,53536$

The $\frac{1}{2}$ sum gives $35^{\circ} 51'$ $9,76768$

Which doubled, gives $71^{\circ} 42' = \angle zP*$.

This $71^{\circ} 42'$ turned into time (132) gives 4 h. 56 m. 48 f. for the time which must elapse before the star comes to the meridian.

Now, at the time of observation, November 27, 1764, (133)

The Sun's right ascension, found in the table *, is $16^h \ 13^m$

The star's right ascension, found in the table, is $14 \ 03$

difference of right ascensions

The star souths, or culminates

The time the star has to come to the meridian

The difference gives the hour of the night

Or, the sum gives the hour after midnight
Which is 2 h. 37 m. in the morning.

$$\begin{array}{r} 2 \ 10 \\ 12 \ 00 \\ 9 \ 50 \\ 4 \ 47 \\ \hline 5 \ 03 \text{ P.M.} \\ 14 \ 37 \end{array}$$

* Astronomical tables at the end of Book V.

And

And whether to subtract or add, will always be known by the star's being in the eastern part of the horizon, or ascending; or by being in the western part of the horizon, or descending.

156.

PROBLEM XIV.

Pl. V.

Given the obliquity of the ecliptic, and a star's right ascension and declination;

Required its latitude and longitude.

EXAM. *What is the latitude and longitude of a star, its right ascension being 16 h. 14 m., its declination $25^{\circ} 51' N.$, and the obliquity of the ecliptic, $23^{\circ} 29'$?*

CONSTRUCTION.

Let the primitive circle represent the solstitial colure, in which draw the equator EQ, mark its poles P, S, and describe (3d 135) the parallel of the star's declination $n m$.

The right ascension 16 h. 14 m. $= 243^{\circ} 30'$, which being $63^{\circ} 30'$ above 180° , falls in the third quadrant; therefore make (IV. 75) $\odot a = 63^{\circ} 30'$, describe (4th 135) the circle of right ascension, cutting the parallel $n m$ in $*$, the point of the heavens representing the star.

Make $E \varpi = 23^{\circ} 29'$, the obliquity of the ecliptic, draw the ecliptic $\varpi \nu$, find its poles p, q , and through $p, *, q$ describe a circle of longitude; then the arc $p *$ measured (IV. 70) will give the co-latitude, and the $\angle pp *$ will shew the longitude.

COMPUTATION.

In the oblique angled spheric triangle $pp *$:

Given the obliq. ecliptic $pp = 23^{\circ} 29'$ } Required the co-lat. $p *$.
the co-declination $p * = 64^{\circ} 09'$ } and the longit. $\angle pp *$.
the right ascen. $\angle pp * = 243^{\circ} 30'$ } See art. 150, 151. B. IV.

To find the latitude.

As Radius	= ∞	As cos. 4th arc	= $61^{\circ} 34'$	0,32227
To. cos. rt. asc.	= $26^{\circ} 30'$	To cos. 5th arc	= $38^{\circ} 05'$	9,86004
So tan. co-decl.	= $64^{\circ} 09'$	So sine decl.	= $25^{\circ} 51'$	9,63950
To tan. 4th arc	= $61^{\circ} 34'$	To sine lat.	= $46^{\circ} 07'$	9,85781
Obl. eclipt.	= $23^{\circ} 29'$			
Fifth arc	= $38^{\circ} 05'$			

Here the star's latitude is $46^{\circ} 07' N.$ because the declinat. is N., and greater than the obliquity of the ecliptic.

To find the longitude.

As sine 5th arc	= $38^{\circ} 05'$	0,20985	Here the longitude $144^{\circ} 36'$ being added to 90° , gives $234^{\circ} 36'$ for the star's longitude, reckoning from the first point of Aries.
To sine 4th arc	= $61^{\circ} 34'$	9,94417	
So tan. rt. asc.	= $26^{\circ} 30'$	9,69774	
To tan. longit.	= $144^{\circ} 36'$	9,85176	

For the right ascension being in the third quadrant, the star is there also. Now in $234^{\circ} 36'$, arc 7 signs $24^{\circ} 36'$; that is the stars place or longitude $24^{\circ} 36'$ in the 8th sign, or $24^{\circ} 36'$ in m .

By the precession of the equinoxes, the fixed stars, although they always keep the same latitudes, yet they are continually altering their longitudes

longitude; right ascension and declination; the alteration and longitude is uniformly 50 seconds yearly (22), but that of the right ascension and declination is constantly varying: So that many stars, which once had north declination, come to have south; while others change from S. to N. declination.

157. PROBLEM XV. PL. V.

Given the right ascensions and declinations of two fixed stars; Required their distance.

EXAM. *What is the distance between the fixed stars, Algol in the head of Medusa, and Aldebaran in Taurus; the former having 39° 59' N. declinat., with 2 h. 52 m. right ascension; and the other 15° 59' N. declinat. with 4 h. 22 m. of right ascension.*

CONSTRUCTION.

As Aldebaran precedes Algol in right ascension, let the primitive circle represent the circle of right ascension passing through Aldebaran; describe the circle of right ascension PA s, making with PA s an angle of 22° 30', (IV. 75) equal to the difference between the given right ascensions.

Describe the parallels of declination Bm , rs , at the given distances 15° 59' N., and 39° 59' N. (3d. 133); and the intersections B , of Aldebaran's declination, and A , of Algol's, with their respective circles of right ascension, will be the positions of those stars from one another: Then a great circle BAC , passing through B and A , the intercepted arc BA (measured by art. 70. Book IV.) shews the distance of those two stars.

COMPUTATION.

(IV. 151)

In the oblique angled spheric triangle PAB .

Given the co-decl. of Ald. $PB = 74^\circ 01'$ } Required their dist. AB .
the co-decl. of Algol $PA = 50^\circ 01'$
diff. of right ascen. $\angle APB = 22^\circ 30'$

As Radius	= R	10,00000	From 4th arc	=	72° 47'
To cos. diff. rt. as.	= 22° 30'	9,96562	Take Algol's co-decl.	=	50 01
So cot. Aldeb. dec.	= 15	59 10,54298	Remains 5th arc	=	22 46

To tan. 4th arc = 72 47 10,50860

And as cos. 4th arc = 72 47 0,52873

To cos. 5th arc = 22 46 9,96477

So sin. Aldeb. decl. = 15 59 9,43990

To cos. distance = 30 56 9,93340

The same result would have come out, had the declination of the star Algol been used in the proportions.

158. PROBLEM XVI. PL. V.

Given the latitudes and longitudes of two known fixed stars; Required their distance.

EXAM. *Aliaib, in the Great Bear, Lon. m 4° 55'. Lat. 54° 18' N. Arcturus, in Bootes, Lon. $\simeq$ 23 15. Lat. 30 57 N.*

The construction of this Problem is like that of the last; only instead of circles of right ascension, read circles of longitude, and use parallels of latitude instead of parallels of declination.

The computation is also like that of the last, there being given two co-latitudes and the included angle, which is the difference between the given longitudes: Thus Arcturus's long. is $6^{\circ} 20' 15''$, and Aliath's is $5^{\circ} 4^{\circ} 55'$; their difference is $1^{\circ} 15' 20''$, or $45^{\circ} 20'$.

Hence the distance will be found to be $39^{\circ} 42'$.

159.

PROBLEM XVII.

Pl. V.

Given the latitude and longitude of a fixed star, and also the obliquity of the ecliptic;

Required the right ascension and declination of that star.

EXAM. Suppose the latitude of a star is $7^{\circ} 09' N.$, its longitude $29^{\circ} 01'$: What is the right ascension and declination of that star, the obliquity of the ecliptic being $23^{\circ} 29'$?

The construction of this Problem is much like that of Prob. XIV; only here the intersection of a parallel of latitude cb with a circle of longitude paq , will give the place of the star.

The computation is also as in Prob. XIV; for here are given $rp = 23^{\circ} 29'$, $pa = 82^{\circ} 51'$, and the $\angle ppa = 60^{\circ} 59'$, the co-longitude: To find pa the co-declination, and $\angle ppa$ the right ascension.

The declination of the star will be found to be $17^{\circ} 49' N.$

And the right ascension will be $114^{\circ} 18'$: But as the star's place is in the first quadrant of the ecliptic, its right ascension must be in the first quadrant of the equator; therefore $24^{\circ} 18'$ is the right ascension reckoned from Aries.

160.

PROBLEM XVIII.

Pl. V.

Given the meridional altitude of any celestial object, suppose a comet, its distance from a known star, and the latitude of the place;

Required the declination and right ascension of that comet.

EXAM. Suppose a comet was observed on the meridian at London, when its altitude was $51^{\circ} 55'$, and its distance from the star Arcturus was $59^{\circ} 47'$: What was the declination and right ascension of the comet at that time?

CONSTRUCTION.

In the primitive circle, representing the meridian of the place, draw the horizon HR and prime vertical ZN ; lay off the given latitude $RP = 51^{\circ} 32'$, draw the axis PS , the equator EQ , and (3d 135) $n m$ Arcturus's parallel of declination $= 20^{\circ} 30'$. From the south point of the horizon lay off the given altitude of the comet $= 51^{\circ} 55'$ from H to o : About the point o as a pole, at the given distance between the comet and Arcturus, describe (IV. 68) a small circle aa cutting the parallel $n m$ in $*$, the position of Arcturus at that time: Describe the circle of right ascension $P * s$, and a great circle through o and $*$.

COMPUTATION.

Since $HE = 38^{\circ} 28'$, the co-lat., and the alt. $HO = 51^{\circ} 55'$, then $EO = (HO - HE =) 13^{\circ} 27'$, is the decl. sought; which is north as the altitude exceeds the co-lat.; consequently the polar distance $OP = 76^{\circ} 33'$.

Then

Then in the triangle $r * o$ are given the three sides to find the $\angle OP*$, the difference between the right ascensions of the comet and Arcturus. And (IV. 154) the $\angle OP*$ will be found $= 62^\circ 27' = 4^h. 9^m. 48^s$. which is the difference of their right ascensions: Now if Arcturus precedes the comet, the right ascension of the latter is $9^h. 53^m. 12^s$.: But if the comet precedes, its right ascension is $18^h. 12^m. 48^s$. the former being the difference, and the latter the sum of the right ascensions of Arcturus and the comet.

161.

PROBLEM XIX.

Pl. V.

Given the latitude of a place, the Sun's declination and azimuth; Required his altitude and the time of the observation.

EXAM. In the latitude of $13^\circ 30' N$., and when the Sun has $23^\circ 29' N$. declination: What is the Sun's altitude and time of the day, when he is seen on the ENE. azimuth circle?

CONSTRUCTION.

Let the primitive circle represent the meridian of the place, wherein HA represents the horizon, and ZN the prime vertical; make RP equal to the latitude, draw the 6 o'clock hour circle PS , the equator EQ , and (3d 153) the parallel of $23^\circ 29'$ of declination nm : The tangent of $67^\circ 30'$ being laid from the center a towards H , gives the center of the vertical circle zDN , which cuts the parallel nm in the points A and B ; and shews that at two distant times in the forenoon the Sun will have the azimuth proposed: Through the point A and B describe the hour circles PAS , PBS (II. 72); the angles zPA , zPB , shewing the times from noon, may be measured by art. 72. Book IV; and the altitudes DA , DB , by art. 70. Book IV.

COMPUTATION. See art. 146. Book IV.

In the spheric triangle PZA , or PZB , there are known,

The co-lat. $PZ = 76^\circ 30'$; the co-decl. PA , or $PB = 66^\circ 31'$; the azimuth. $\angle PZD = 67^\circ 30'$.

To find ZA , or ZB , and the $\angle ZPA$, or $\angle ZPB$.

As Radius : cof. azimuth :: cot. lat. : to tan. of a 4th arc $M = 57^\circ 54'$. And as sin. lat. : sin. decl. :: cof. M : to cof. of a 5th arc $N = 24^\circ 54'$. Then $M + N$, or $57^\circ 54' + 24^\circ 54' = 82^\circ 48' = ZA$, is the comp. of least alt. And $M - N$, or $57^\circ 54' - 24^\circ 54' = 33^\circ 00' = ZB$, is the comp. of greatest alt. Therefore, when the Sun has $7^\circ 12'$, or $57^\circ 00'$ of alt. he is on the giv. azim. Again. As cof. dec. : sin. azim. :: cof. least alt. : sin. hour fr. noon $88^\circ 02'$. And as cof. dec. : sin. azim. :: cof. greater alt. : sin. hour fr. noon $33^\circ 16'$. But $88^\circ 02' = 5^h. 52^m$; and $33^\circ 16' = 2^h. 13^m$, the respect. times fr. noon. Consequently the Sun will be seen on the ENE. azimuth at $6^h. 08^m$, and again at $9^h. 47^m$, both in the morning: Also, in the afternoon he will be on the WNW. azimuth at $2^h. 13^m$., and at $5^h. 52^m$. 162. Now to find at what time, and at what altitude, the greatest azimuth will happen at that place on the said day.

As the azimuth circle in this case is to touch the parallel nm , therefore the greatest distance of the azimuth from the equator will be $23^\circ 29'$; and as their poles must be at the same distance (IV. 33), therefore a small circle rs described about the pole s , at the distance of $23^\circ 29'$ (IV. 66), its intersection p with the horizon, is the pole of the circle zcn ; then describe an hour circle pcs through p .

In the spheric triangle zpc right angled at c .

Given the co-lat. $pz = 76^\circ 30'$, and the co-decl. $pc = 66^\circ 31'$. (IV. 34)

Required the greatest azim. $\angle pzc = 70^\circ 36'$, the dist. $zc = 54^\circ 08'$, and the hour from noon $= 56^\circ 27' = 3$ h. 46 m.

So that the azimuth is altered only $3^\circ 6'$ in 1 h. 33 m.; and consequently the variation of the compass in the torrid zone may be observed more accurately than elsewhere, could the altitudes when near the zenith be well taken.

163.

PROBLEM XX. Pl. XIV. Fig. 1

In the latitude of $20^\circ 00' N$. stands a horizontal dial, whose gnomon is perpendicular to the plane of the horizon: It is required to know at what hour in the afternoon on the longest day, the shadow of that gnomon shall stand still; and how many degrees shall the shadow run back?

CONSTRUCTION. Let the circle $zrnh$ be the meridian of the place, hr the horizon, whose poles are z , N ; $rp = 20^\circ$, the latitude, P and s the north and south poles: About P describe the tropic of Cancer aa , cutting the horizon in L ; about s , a small circle being described at the dist. of $23^\circ 29'$, the complement of the dist. of aa from P , its intersection p with the horizon, is the pole of the azimuth circle which will touch the parallel aa in O , the place of the Sun when he has the greatest azimuth that day: Through z , O , N , describe a vertical circle cutting the horizon in k ; and through O and L describe the hour circles $P \odot s$, PLs . Then will the $\angle zp \odot$ be equivalent to the hour when the shadow will stand still; and kL , the difference between the measures of the azimuth and amplitude, will shew how much the shadow will run back.

COMPUTATION. In the right angled spheric triangle PRL .

Given	$\angle R = 90^\circ 00'$	Given	$\angle \odot = 90^\circ 00'$
lat.	$= RP = 20^\circ 00'$	co-lat.	$= zP = 70^\circ 00'$
co-decl.	$= PL = 66^\circ 31'$	co-dec.	$= P \odot = 66^\circ 31'$

Requir. ampl. $= RL = 64^\circ 55'$

Then $33^\circ 06' = 2$ h. 12 m. 24 f: And $77^\circ 26' - 64^\circ 55' = 12^\circ 31'$.

So that the shadow will stand still at 2 h. 12 m. 24 f., and will run back $12^\circ 31' = 0$ h. 50 m. 4 f.

164.

PROBLEM XXI. Pl. XIV. Fig. 2.

A comet, whose declination was $47^\circ 00' N$., was observed to be distant from a star to the eastward $49^\circ 00'$; the star's declination was $36^\circ 00' N$. and its right ascension $45^\circ 00'$: What was the latitude and longitude of that comet?

CONSTRUCTION.

On the plane of the solstitial colure, where P and E are the poles of the equator and ecliptic, put the star at A by its right ascension and declination: About P and A describe small circles, at the distances of the comet from those points, their intersection c gives the place of the comet; describe great circles through AC , PC , EC , then EC will be the co-latitude, and the $\angle PEC$ the co-longitude; and their measures may be obtained from articles 70 and 72 of Book IV.

COMPUTATION.

COMPUTATION. In the triangle APC.

Given the $\star$'s co-decl. $PA = 54^{\circ}00'$
comet's co-decl. $PC = 43^{\circ}00'$
their distance $AC = 49^{\circ}00'$

In the triangle BPC.

Given comet's co-decl. $PC = 43^{\circ}00'$
obliq. of ecliptic $PE = 23^{\circ}29'$
comet's rt. asc. $\angle EPC = 69^{\circ}12'$

Req. com. rt. asc. $\star \angle APC = 65^{\circ}48'$
Then $65^{\circ}48' - 45^{\circ}00' = 20^{\circ}48'$
And $90^{\circ}00' - 20^{\circ}48' = 69^{\circ}12' = CPE.$

Req. comet's co-lat. $EC = 39^{\circ}53'$
and longit. $\angle PEC = 83^{\circ}47'$
Which being in the 4th quadrant,
gives $23^{\circ}47'$ in $\kappa.$

165.

PROBLEM XXII. Pl. XIV. Fig. 3.

At London, on the 10th of December, 1765, at what time of the night will the stars Aldebaran and Regel be on the same azimuth circle?

Aldeb. decl. $= 15^{\circ}59' N.$; right asc. $= 4^h. 22^m.$ Regel's decl. $= 8^{\circ}28' S.$; right asc. $= 5^h. 01^m.$

Their difference of right ascensions is 39 min., or $9^{\circ}45'.$

CONSTRUCTION. On the plane of the equinoctial put Aldebaran $\star$ A, and Regel at B, by their right ascensions and declinations, then a great circle through B and A will be the azimuth they are on at the time sought; and the parallel of London's lat. described about P, will cut the azimuth circle BA in zz the zenith, through which draw the meridians Pz, Pz.

Here the nearest intersection to the stars is taken for the zenith, for as the stars are both above the horizon, the greatest zenith distance is less than 90 degrees.

COMPUTATION. In the $\triangle APB.$

Given B's co-decl. $PB = 98^{\circ}28'$
A's co-decl. $PA = 74^{\circ}01'$
diff. rt. asc. $\angle APB = 9^{\circ}45'$

In the triangle APz.

Given A's co-decl. $PA = 74^{\circ}01'$
the co-lat. $Pz = 38^{\circ}28'$
the suppl. of BAP or $\angle PAz = 22^{\circ}14'$

Required the $\angle BAP = 157^{\circ}46'$

Req. $\angle APz = 144^{\circ}10'$ or $= 23^{\circ}00'.$

Now the star Aldebaran comes to the meridian at 11 h. 32 m. in the evening; which lessened by 1 h. 32 m. ($23^{\circ}00'$) gives 9 h. 40 m. for the time in the evening when those stars will be on the same azimuth.

166.

PROBLEM XXIII. Pl. XIV. Fig. 4.

At what time in the evening will the stars Betelgeuse and Pollux have one common altitude above the horizon of London, on the 10th of December, 1765?

Betelgeuse right ascen. $= 5^h. 42^m.$; decl. $= 7^{\circ}20' N.$ Pollux right ascen. $= 7^h. 28^m.$; decl. $= 28^{\circ}36' N.$

Their difference of right ascension is 1 h. 46 m. $= 26^{\circ}30'.$

CONSTRUCTION. On a primitive circle, where any point P represents the pole of the equinoctial, put the star Pollux at B, and Betelgeuse at A, by their declension and difference of right ascensions; through A, B describe a great circle; through C, the middle of AB, describe a great circle at right angles to AB, and cutting PA in D; then a small circle described about P, at the distance of $38^{\circ}28'$, the co-lat., will cut the circle CD in z the zenith.

For the stars A and B having a common altitude, are equally distant from z.

COMPT.

COMPUTATION. In the triangle APB.

Given A's co-decl. $PA = 82^{\circ} 40'$ } Required the $\angle BAP = 46^{\circ} 18'$
 B's co-decl. $PB = 61^{\circ} 24'$ } $AB = 32^{\circ} 49'$
 diff. rt. asc. $\angle APB = 26^{\circ} 30'$ } Its half $AC = 16^{\circ} 24'$

In the triangle ACD.

Given the $\angle C = 90^{\circ} 00'$ } Required the $\angle D = 46^{\circ} 05'$
 $\angle A = 46^{\circ} 18'$ } $AD = 23^{\circ} 04'$
 $AC = 16^{\circ} 24'$ } Theref. $PA - AD = PD = 59^{\circ} 36'$

In the triangle PDZ.

Given the co-lat. $PZ = 38^{\circ} 28'$ } Required $\angle ZPD = 66^{\circ} 04'$
 $PD = 59^{\circ} 36'$ } Or it is $= 58^{\circ} 30'$
 $\angle PDZ = 46^{\circ} 05'$

Now the star comes to the meridian at 12 h. 32 m. in the morning (133); from which take 3 h. 54 m. ($= 58^{\circ} 30'$), as the stars are to the east of the meridian) and it leaves 8 h. 38 m. in the evening for the time when those stars have the same altitude.

167.

PROBLEM XXIV.

Pl. XIV. Fig. 5:

Wanting to know the latitude and longitude of a comet C, its distance from two known stars A and B were observed, and are as follow:

A's lat. $= 49^{\circ} 12' N.$ Lon. $= 16^{\circ} 39' E$; distance from C $= 49^{\circ} 05'$
 B's lat. $= 30^{\circ} 05' N.$ Lon. $= 2^{\circ} 48' E$; distance from C $= 45^{\circ} 57'$

Hence the place of the comet C is required.

CONSTRUCTION. On the plane of the solstitial colure, where E is the pole of the ecliptic, put the stars A and B by their latitudes and longitudes, and describe a great circle through A and B; then small circles described about A and B as poles, at the respective distances of the comet, their intersection C will give its place; describe great circles through A, C; B, C; E, C; and E C will be the co-latitude, and from $\angle AEC$ will be obtained the longitude.

COMPUTATION. In the $\triangle ABE$.

Given A's co-lat. $AE = 40^{\circ} 48'$ } In the triangle ABC.
 B's co-lat. $BE = 59^{\circ} 55'$ } Given the distance $AB = 59^{\circ} 01'$
 diff. longit. $\angle AEB = 76^{\circ} 09'$ } distance $AC = 49^{\circ} 05'$
 distance $BC = 45^{\circ} 57'$

Required

$AB = 59^{\circ} 01'$ } Required the $\angle BAC = 56^{\circ} 27'$
 and $\angle EAB = 78^{\circ} 31'$ } Then $\angle EAB + \angle BAC = \angle EAC = 134^{\circ} 58'$

In the triangle CEA.

Given A's co-lat. $AE = 40^{\circ} 48'$ } Required the co-lat. $EC = 81^{\circ} 33'$
 distance $AC = 49^{\circ} 05'$ } diff. lon. $AEC = 32^{\circ} 43'$
 and the $\angle EAC = 134^{\circ} 58'$ } Hence lat. is $8^{\circ} 27' N.$ lon. $19^{\circ} 22' E$

168.

PROBLEM XXV.

Pl. XIV. Fig. 6.

The distance of the star C being observed from two stars, A, B, whose latitude and distance are known; thence to find the lat. and long. of C.

A's lat. $= 5^{\circ} 30'$; dist. from C $= 39^{\circ} 40'$; B's lat. $= 9^{\circ} 57'$; dist. from C $= 10^{\circ} 7\frac{1}{2}'$: And the distance of AB is $44^{\circ} 43'$.

Con.

CONSTRUCTION. On a circle of longitude, where E is the pole of the ecliptic, put the star B by its lat.; about the points B and E describe circles at the distances of A from those points, their intersection gives the place of A ; also circles described from A and B , at the distances of C respectively from them, their intersection is the place of C : Then describe the great circles EA , EC ; AC , AB ; BC ; and EC , will be the co-lat., and $\angle BEC$ the longitude from B .

COMPUTATION. In the $\triangle AEB$.

Given A 's co-lat. $AE = 84^{\circ} 30'$
 B 's co-lat. $BE = 80^{\circ} 03'$
 the distance $AB = 44^{\circ} 43'$

In the triangle ABC .

Given the distance $AB = 44^{\circ} 43'$
 distance $AC = 39^{\circ} 40'$
 distance $BC = 10^{\circ} 07\frac{1}{2}'$

Required the $\angle ABE = 92^{\circ} 14'$

Required the $\angle ABC = 55^{\circ} 22'$

In the triangle BEC .

Given B 's co-lat. $BE = 80^{\circ} 03'$ } Required C 's co-lat. $CE = 72^{\circ} 01'$
 the distance $BC = 10^{\circ} 07\frac{1}{2}'$ } C 's lon. fr. B , $\angle BEC = 6^{\circ} 22'$
 ($\angle ABE - \angle ABC = \angle CBE = 36^{\circ} 52'$)

169.

PROBLEM XXVI.

Pl. XIV. Fig. 7.

From the altitudes of two known fixed stars, and the altitude of a planet when in the same azimuth with one of these stars; to find the place of the planet.

EXAMPLE. Observed the Moon and Cor Leonis in the same azimuth, when the Moon's zenith distance was $36^{\circ} 37'$.

Cor Leonis's zenith dist. $= 45^{\circ} 00'$; decl. $13^{\circ} 09'N$.; rt. af. $9^h 53^m$.

Cor Hydra's zenith dist. $= 49^{\circ} 16'$; decl. $7^{\circ} 37'S$.; rt. af. $9^h 16^m$.

CONSTRUCTION. On the plane of the equinoctial, whose pole is P , draw the colures, and in the solstitial, take E for the pole of the ecliptic; put the given stars at B and A by their declinations and right ascensions: About E and A as poles, with their respective zenith distances, describe circles cutting in z the zenith; through z and B describe an azimuth circle, and making zD equal to the D 's zenith distance, gives her place: Then describe the great circles zB , zA , zD , ED ; and the arc ED will be the co-latitude, and $\angle PEB$ the longitude from ϖ .

COMPUTATION. In the $\triangle ABP$.

Given A 's co-decl. $PA = 97^{\circ} 37'$
 B 's co-decl. $PB = 76^{\circ} 51'$
 diff. rt. af. $\angle BPA = 9^{\circ} 15'$

Required the $\angle ABP = 155^{\circ} 38'$
 the side $AB = 22^{\circ} 43'$

Required the $\angle AEZ = 89^{\circ} 56'$
 Then $\angle ABP - \angle ABZ = \angle ZBP = 65^{\circ} 42'$.

In the triangle BDE .

Given B 's co-decl. $BP = 76^{\circ} 51'$
 ($BZ - ED =$) side $BD = 8^{\circ} 23'$
 $\angle PBD = 65^{\circ} 42'$

In the triangle PDE .

Given obl. of eclip. $PE = 23^{\circ} 29'$
 D 's co-decl. $PD = 73^{\circ} 32'$
 D 's co-rt. af. $\angle EPD = 129^{\circ} 22'$

Requ D 's co-decl. $PD = 73^{\circ} 32'$
 D 's rt. af. fr. B , $\angle BPD = 7^{\circ} 57'$
 Then $90^{\circ} + \angle BPD + \angle BPD = 129^{\circ} 22'$

Requir. D 's co-lat. $DE = 89^{\circ} 00'$
 D 's lon. fr. ϖ , $\angle PED = 47^{\circ} 52'$

R

SECTION

SECTION VI.

Of various methods to find the Latitude.

The usual way at sea to find the latitude, is from the Sun's meridional altitude and declination; the manner of doing this will be particularly shewn in Book IX: But as it frequently happens at sea, that the meridian altitude cannot be taken, therefore the mariner should be furnished with other means to come at this most useful knowledge of his latitude: To help him in this point, and as a farther exercise in the Astronomy of the sphere, the following problems are collected together.

170.

PROBLEM XXVII.

Pl. V.

Given the Sun's declination and his amplitude;

Required the latitude of the place.

EXAM. *Being in a place where the compass had no variation, on a day when the Sun's declination was $15^{\circ} 12' N$, I observed him to rise $62^{\circ} 30'$ from the north towards the east: Required the latitude of that place?*

CONSTRUCTION.

Having described the primitive circle, drawn the horizon HR, and (IV. 71) taken $RO = 62^{\circ} 30'$; then about O as a pole describe (IV. 66) a small circle, at the distance of $74^{\circ} 48' = \text{co-decl.}$, cutting the primitive in P, the place of the north pole: Draw the axis PS, the equator EQ, and the circle POS, cutting the equator in A.

COMPUTATION.

In the spheric triangle ROA right angled at A.

Given the co-amp. $RO = 27^{\circ} 30'$ | Then $\text{f. } \varphi O : \text{Rad} :: \text{f. } AO : \text{f. } \angle A \varphi O$.

the declin. $AO = 15 \ 12$ | Hence the latitude will be 55°

Required the co-latitude ARO. $24' N$.

171.

PROBLEM XXVIII.

Pl. V.

Given the Sun's declination, and his ascensional difference;

Required the latitude of the place.

EXAM. *When the Sun had $26^{\circ} 01'$ of declination S, he was observed to set at 4 h. 30 m.: Required the latitude of the place?*

As the ascensional difference is the time the Sun rises or sets before or after 6 o'clock; therefore 6h. — 4h. 30 m. $= 1 \text{ h. } 30 \text{ m.} = 22^{\circ} 30' =$ ascensional difference.

CONSTRUCTION. In the primitive circle representing the meridian of the place, draw the equator EQ, the axis PS, the parallel of declination ro , $20^{\circ} 01' S$: Make $\varphi B = 22^{\circ} 30'$, the ascensional difference; describe the circle of right ascension PBS, cutting ro in O; then a diameter HR through O will be the horizon, and RP the lat. sought.

COMPUTATION. In the spheric triangle φBO , right angled at B.

Given the asc. diff. $\varphi B = 22^{\circ} 30'$ | Then $\text{Rad} : \cot. OB :: \text{f. } \varphi B : \cot. \angle O \varphi B$

the declin. $OB = 20 \ 01$

Or $\text{Rad} : \cot. dec. :: \text{fin. rt. af.} : \tan. \text{lat.}$

Required the co-lat. $\angle O \varphi B$. Hence the lat. will be $46^{\circ} 25' N$, being contrary to the decl. when the asc. diff. falls between noon and fix.

172. PRO-

172.

PROBLEM XXIX.

Pl. V.

Given the Sun's declination, and altitude at six o'clock;
Required the latitude of the place.

EXAM. *Being at sea on a day the Sun's declination was $20^{\circ} 04' N$. at six o'clock in the evening, the Sun's altitude was $18^{\circ} 45'$: What was the latitude of the place of observation?*

CONSTRUCTION. Having described the meridian, drawn the horizon HR, the prime vertical ZN, and the parallel st of $18^{\circ} 45'$ altitude; from the center φ , with the half tangent of the declination $= 20^{\circ} 04'$, cut the parallel st in o : Through o draw the axis PS, and the azimuth circle ZON (II. 72), and the measure of RP will give the latitude sought.

COMPUTATION. In the spheric triangle φAO , right angled at A.

Given the decl. $\varphi o = 20^{\circ} 04' N$. Then $\sin. \varphi o : \text{Rad} :: \sin. AO : \sin. \angle O \varphi A$.
the alt. $AO = 18^{\circ} 45'$ | Or $\sin. \text{decl.} : \text{Rad} :: \sin. \text{alt.} : \sin. \text{lat.}$
Which is $69^{\circ} 32' N$, as the decl. is N.

Required the lat. $= \angle O \varphi A$.

173.

PROBLEM XXX.

Pl. V.

Given the Sun's declination, and altitude when he is due east or west;
Required the latitude of the place.

EXAM. *In a place where the compass had no variation, the Sun was observed to be due east when his declination was $16^{\circ} 38' N$., and his altitude $20^{\circ} 12'$: What is the latitude of that place?*

CONSTRUCTION. In the meridian HZRN, draw the horizon HR, the prime vertical ZN, and make $\varphi o = \text{half tan. of the alt. } 20^{\circ} 12'$: About o as a pole, at the distance of $73^{\circ} 22'$, the co-decl., describe (IV. 66) a small circle, cutting the meridian in P the elevated pole; draw the axis PS, equator EQ, and through P, O, S, describe an hour circle POS; then the measure of PR shews the latitude.

COMPUTATION. In the spheric triangle φAO , right angled at A.

Given the alt. $\varphi o = 20^{\circ} 12'$ | Then $\sin. \varphi o : \text{Rad} :: \sin. AO : \sin. \angle A \varphi O$.
the decl. $AO = 16^{\circ} 38' N$. | Or $\sin. \text{alt.} : \text{Rad} :: \sin. \text{decl.} : \sin. \text{lat.}$

Required the latit. $= \angle A \varphi O$, | Which is $56^{\circ} 00' N$., as the decl. is N.

But had the declination been S., the other intersection of the parallel circle and meridian must have been taken for the elevated pole, and the latitude would be south.

174.

PROBLEM XXXI.

Pl. V

Given the Sun's altitude and hour of the day on either equinox;
Required the latitude of the place.

EXAM. *On the day the Sun entered the vernal equinox, his alt. was found $22^{\circ} 56'$ at 9 o'clock in the morning: In what lat. was that observation made?*

CONSTRUCTION. Describe the meridian, draw the horizon, the prime vertical, and (IV. 68) the parallel st of $22^{\circ} 56'$ altitude; from the center φ , with the half tan. of $45^{\circ} = 3 h.$, the ascen. diff., cut st in o , and describe the vertical circle ZON, cutting the horizon in N.

COMPUTATION. In the spheric triangle φBO , right angled at B.

Given the time after 6 $\varphi o = 45^{\circ} 00'$ | As $\sin. \varphi o : \text{Rad} :: \sin. BO : \sin. \angle B \varphi o$
the altitude $BO = 22^{\circ} 56'$ | Or $\sin. a 6 : \text{Rad} :: \sin. \text{alt.} : \cos. l.$

Required the co-latitude $B \varphi O$. | Which is $56^{\circ} 34'$.

R 2

175. PRO-

Given the Sun's altitude, declination and azimuth;
Required the latitude of the place

EXAM. *Being at sea in a place where the compass had no variation, in the afternoon when the Sun was $42^{\circ} 30'$ high, his bearing was S. $57^{\circ} 45' W.$ and his declination $22^{\circ} 30' N.$: What is the latitude of that place?*

CONSTRUCTION. Draw the meridian, the horizon HR , the prime vertical ZN , and the parallel st at $42^{\circ} 30'$ above the horizon (IV. 68): The tangent of $57^{\circ} 45'$ set from A towards R gives the center of the azimuth circle zon , cutting the parallel of altitude st in o : About o as a pole (IV. 66), at the distance of $67^{\circ} 30'$, equal to the co-declination, describe a small circle, cutting the meridian in P , the place of the pole; then the measure of RP gives the latitude sought.

COMPUTATION. In the oblique angled spheric triangle zop .

Given the zenith dist. $zo = 47^{\circ} 30'$ } Required the co-latitude PZ .
the polar dist. $po = 67^{\circ} 30'$
the azimuth $\angle pzo = 122^{\circ} 15'$

As Rad	=	R	10,00000	As fin. alt.	=	$42^{\circ} 30'$	0,17032
To cof. azim.	=	$122^{\circ} 15'$	9,72723	To fin. decl.	=	$22^{\circ} 30'$	9,58284
So cot. alt.	=	$42^{\circ} 30'$	10,03795	So cof. 4th.	=	$30^{\circ} 13'$	9,93658

To tan. 4th = $30^{\circ} 13'$ 9,76518 To cof. 5th = $60^{\circ} 42'$ 9,68974

Then the difference between the 5th and 4th arcs, that is $30^{\circ} 13'$ taken from $60^{\circ} 42'$, the remainder $30^{\circ} 29'$ is the co-lat. Therefore $59^{\circ} 31' N.$ is the latitude sought.

Given the Sun's declination, his altitude and hour of the day;
Required the latitude of the place.

EXAM. *Being at sea, the Sun's altitude was observed to be $37^{\circ} 20'$ at 9 h. 45 m. in the morning, his declination at that time being $22^{\circ} 30' N.$: What is the latitude of the place of observation?*

CONSTRUCTION. In the meridian $PESQ$, draw the equator EQ , axis Ps , and parallel of declin. nm , $22^{\circ} 30'$ dist. from the equator ($3d$ 135). Set off from the center A towards Q the tang. of $33^{\circ} 45' = 2h. 15m.$, the distance between the time of observation and noon, which gives the center of the hour circle Pos , cutting the parallel nm in o : About the point o as a pole, describe (IV. 66) at the dist. of $52^{\circ} 40'$, the zen. dist., a small circle, cutting the meridian in z the zenith; through zo describe an azimuth circle zon ; then the measure of zE will give the lat. sought.

COMPUTATION. In the oblique angled triangle zop .

Given the zenith distance $zo = 52^{\circ} 40'$ } Required the co-lat. zP .
the polar distance $po = 67^{\circ} 30'$
the hour from noon $\angle zpo = 33^{\circ} 45'$

As Rad : cof. hour $A. M.$: : cot. decl. : tan. 4th arc $= 63^{\circ} 31'$.

As fin. decl. : fin. alt. : : cof. 4th arc : cof. 5th arc $= 45^{\circ} 02'$.

Their difference is the co-latitude $18^{\circ} 29'$. Therefore the lat. is $71^{\circ} 31' N.$

Given the altitudes of two known fixed stars, having the same azimuth; Required the latitude of the place.

EXAM. *Being at sea in an unknown latitude, I observed the star Schedar in Cassiopeia, and Almaach in Andromeda, to have the same azimuth, when the altitude of Schedar was $37^{\circ} 15'$, and that of Almaach was $17^{\circ} 45'$: What is the latitude of that place?*

CONSTRUCTION *upon the plane of the equator.* Let the primitive circle represent the equator, whose pole is P, and any point α the place where the right ascension begins, from whence lay off $\alpha a = 27^{\circ} 15'$ for Almaach's right ascension, and $\alpha b = 6^{\circ} 45'$ for Schedar's; draw the circles of right ascension pa , pb : Describe (3d 135) Almaach's and Schedar's parallel's of declination, cutting pa , pb , in A, B; A being Almaach, and B Schedar: A great circle passing through A and B (IV. 61) will be the azimuth they are on. About B at the distance of $52^{\circ} 45'$, Schedar's zenith distance, describe (IV. 66) a small circle, cutting the said azimuth circle in Z, the zenith of the place; draw PZ, which measured on the half tangents gives the co-latitude of the place of observation.

COMPUTATION.

1st. In the oblique angled spheric triangle ABP.

Given Almaach's co-dec. $PA = 48^{\circ} 52'$ Required the angle of position ABP.
Schedar's co dec. $PB = 34^{\circ} 49'$
their diff. of alt. $AB = 19^{\circ} 30'$ For the solution, see IV. 67.

$$\begin{array}{r} PB = 34^{\circ} 49' \\ BA = 19^{\circ} 30' \\ \hline PA = 48^{\circ} 52' \\ PB - BA = 15^{\circ} 19' \end{array}$$

$$\begin{array}{r} 64 \quad 11 \quad 32^{\circ} 05 \frac{1}{2}' = \frac{1}{2} \text{ sum.} \\ 33 \quad 33 \quad 16 \quad 46 \frac{1}{2}' = \frac{1}{2} \text{ diff.} \\ \hline \end{array}$$

$$\begin{array}{r} \text{Co-ar. fin. PB} = 34^{\circ} 49' \quad 0,24340 \\ \text{Co-ar. fin. BA} = 19^{\circ} 30' \quad 0,47650 \\ \text{Sin. } \frac{1}{2} \text{ sum} = 32^{\circ} 05 \frac{1}{2}' \quad 9,72532 \\ \text{Sin. } \frac{1}{2} \text{ diff.} = 16^{\circ} 46 \frac{1}{2}' \quad 9,46031 \\ \hline \text{Sum of four logs} \quad 19,90553 \\ \frac{1}{2} \text{ sum gives } 63^{\circ} 45 \frac{1}{2}' \quad 9,95276 \\ \text{Which doub. gives } 127^{\circ} 31' = \angle ABP. \end{array}$$

2d. In the oblique spheric triangle PBZ.

Given Schedar's co dec. $PB = 34^{\circ} 49'$ Required the co-lat. PZ.
Schedar's co-alt. $BZ = 52^{\circ} 45'$ For the solution, see IV. 165.
angle of position PBZ $= 52^{\circ} 29'$ Here $\angle PBZ = \text{sup. } \angle PBA$.

$$\begin{array}{r} \text{As Rad.} = R \quad 10,00000 \\ \text{To. cof } \angle \text{posit.} = 52^{\circ} 29' \quad 9,78461 \\ \text{So cot. Sch. decl.} = 55^{\circ} 11' \quad 9,84227 \\ \hline \text{To tan. 4th arc} = 22^{\circ} 57' \quad 9,62688 \\ \text{Which taken from } 52^{\circ} 45' = BZ, \\ \hline \text{Leaves 5th arc} \quad 29^{\circ} 48' \end{array}$$

$$\begin{array}{r} \text{As cof. 4th arc} = 22^{\circ} 57' \quad 0,03181 \\ \text{To cof. 5th arc} = 29^{\circ} 48' \quad 9,93840 \\ \text{So fin. Sch. dec.} = 55^{\circ} 11' \quad 9,91433 \\ \hline \text{To fin. latit.} = 50^{\circ} 41' \quad 9,88854 \end{array}$$

R 3

178. PRO-

178.

PROBLEM XXXV.

Pl. V.

Given the difference of time between the rising of two known stars; Required the latitude of the place.

EXAM. *Being at sea in an unknown place, the star Aldebaran was observed to rise 3 h. 15 m. latter than the bright star in Aries: Required the latitude of that place?*

Bright star in φ decl. $22^{\circ} 17' N.$; right ascen. 1 h. 53 m. Aldebaran's decl. $15^{\circ} 59' N.$; right ascension 4 h. 22 m.

CONSTRUCTION.

Let the primitive circle represent the equator, describe (3d 137) the parallels of declination of the two stars, that of Aries being $22^{\circ} 17' N.$ and of Aldebaran $15^{\circ} 59' N.$: Draw pa for the circle of right ascension passing through the star in φ , which suppose in A : From a lay off ab , $\equiv 37^{\circ} 15'$ = diff. of right ascensions; and $ac \equiv 48^{\circ} 45' \equiv 3$ h. 15 m., the diff. of time between their rising; draw bp , cp , cutting the parallels of declination in B , C : Through the points B , C , describe (IV. 61) the great circle HOR ; draw PO at right angles to HR , then the measure of PO will give the latitude sought.

COMPUTATION.

In the oblique angled spheric triangle PBC .

Given B 's co-decl. $PB \equiv 74^{\circ} 01'$ | Rad: $\cos. \angle BPC :: t PB : t. M \equiv 73^{\circ} 42'$.

A 's co-decl. $PC \equiv 67^{\circ} 43'$ | From M take Pc leaves $N \equiv 5^{\circ} 59'$.

$\angle APC - \angle APB = \angle BPC \equiv 11^{\circ} 30'$ | As $\sin. N : \sin. M :: \tan. \angle CPB : t. \angle C \equiv$

Required the angle PCB . | Hence $\angle PCO \equiv 61^{\circ} 54'$. [$118^{\circ} 06'$].

In the spheric triangle PCO , right angled at O .

Given C 's co-decl. $PC \equiv 67^{\circ} 43'$ | As Rad: $\sin. PC :: \sin. \angle PCO : \sin. PO$

and the $\angle PCO \equiv 61^{\circ} 54'$ | [$\equiv 54^{\circ} 43'$].

Required the latitude PO .

Therefore the latitude is $54^{\circ} 43'$.

179.

PROBLEM XXXVI.

Pl. V.

Two known fixed stars being observed to have the same altitude;

Required the latitude of the place of observation.

EXAM. *In the evening, the stars Capella and Procyon were observed at the same time to have each $38^{\circ} 00'$ of altitude: Required the latitude of the place where that observation was taken?*

Capella's decl. $\equiv 45^{\circ} 43' N.$ right ascen. 4 h. 59 m. Procyon's decl. $\equiv 5^{\circ} 49' N.$ right ascen $\equiv 7$ h. 24 m.

CONSTRUCTION.

On the plane of the equator, represented by the primitive circle, describe (3d 137) the parallels of the given declinations; take $ab \equiv 36^{\circ} 15'$, the diff. of the given right ascensions; draw pa , pb , then A represents Procyon, and B Capella.

About A and B as poles, describe (IV. 66) arcs of small circles, at the distance of 52° , the co-altitude, and their intersection gives Z the zenith of the place: Through the points A , B ; Z , A ; Z , B ; describe great circles (IV. 61), and draw ZP , whose measure will give the co-latitude of the place of observation.

C O M P U]

COMPUTATION.

In the oblique angled spheric triangle APB.

Given A's co-decl. $AP = 84^{\circ} 11'$ Rad: $\cos \angle P :: t. BP : t. M = 38^{\circ} 11'$.
 B's co-decl. $BP = 44^{\circ} 17'$ And M tak. fr. PA leaves $N = 46^{\circ} 00'$.
 diff. rt. asc. $\angle ABP = 36^{\circ} 15'$ Sin. $N : \sin. M :: \tan. \angle P : \tan. \angle BAP$
 Required the stars distance AB. $[= 3213']$

And the angle BAP.

Cof $M : \cos. N :: \cos. BP : \cos. BA = 50^{\circ} 45'$.

In the oblique angled spheric triangle AZB.

Given A's co-alt. $ZA = 52^{\circ} 00'$ The angle BAZ will be found =
 B's co-alt. $ZB = 52^{\circ} 00'$ $68^{\circ} 15'$.
 stars distance $AB = 50^{\circ} 45'$ Then $\angle BAZ - \angle BAP = \angle PAZ =$
 Required the angle ZAB. $36^{\circ} 02'$.

In the spheric triangle AZP.

Given A's co-alt. $ZA = 52^{\circ} 00'$ Rad: $\cos. \angle A :: t. ZA : t. M = 45^{\circ} 59'$.
 A's co-lat. $AP = 84^{\circ} 11'$ M taken from PA leaves $N = 38^{\circ} 12'$.
 the angle $ZAP = 36^{\circ} 02'$ Cof. $M : \cos. N :: \cos. ZA : \cos. PZ = 45^{\circ} 52'$.
 Required the co-lat. ZP . Therefore the latitude is $44^{\circ} 08' N$.

180

PROBLEM XXXVII.

Pl. V.

Given the altitudes of two known fixed stars;

Required the latitude of the place.

EXAM. The altitude of the Hydra's heart was observed to be $40^{\circ} 44'$, and of the Lion's heart $45^{\circ} 00'$: What is the latitude of the place of observation?

Hydra's heart, decl. $= 7^{\circ} 37' S$.; right ascen. $= 9$ h. 16 m. Lion's heart, decl. $= 13^{\circ} 09' N$.; right ascen. 9 h. 53 m.

CONSTRUCTION.

If this problem is constructed on the plane of the equator, it will be in every respect like the last; only the small circles, described about A and B are to be unequally distant from their respective poles A, B.

COMPUTATION.

Here, as in the last, there will be three spheric triangles to work in; namely, the triangles APB, ZAB, and ZPB.

In the triangle APB, where $AP = 97^{\circ} 37'$, $BP = 76^{\circ} 51'$, $\angle APB = 9^{\circ} 15'$.

As Rad: $\cos. \angle APB :: \tan. BP : \tan. M = 76^{\circ} 41'$. Then $AP - M = N = 20^{\circ} 56'$.

As $\sin. N : \sin. M :: \tan. \angle APB : \tan. \angle BAP = 23^{\circ} 55'$.

And as $\cos. M : \cos. N :: \cos. BP : \cos. BA = 22^{\circ} 42'$.

In the triangle BAZ, where $AZ = 49^{\circ} 16'$, $BZ = 45^{\circ} 00'$, $AB = 22^{\circ} 42'$. The angle BAZ will be found equal to $68^{\circ} 56'$.

Then $\angle BAZ - \angle BAP = \angle PAZ = 45^{\circ} 01'$.

In the triangle APZ, where $AP = 97^{\circ} 37'$, $AZ = 49^{\circ} 16'$, $\angle PAZ = 45^{\circ} 01'$. As Rad: $\cos. \angle PAZ :: \tan. AZ : \tan. M = 39^{\circ} 23'$. Then $AP - M = N = 58^{\circ} 14'$.

And as $\cos. M : \cos. N :: \cos. AZ : \cos. PZ = 63^{\circ} 23'$.

Hence the latitude sought is $26^{\circ} 37' N$.

R 4

181. PRO-

181.

PROBLEM XXXVIII.

Pl. V.

Given the Sun's declination, two altitudes, and the time between the observations;

Required the latitude of the place.

EXAM. On a day when the Sun's declination was $20^{\circ} 00' N.$, in the forenoon the Sun's altitude was observed to be $18^{\circ} 30'$, and three hours after, his altitude was $44^{\circ} 00'$: What was the latitude of the place?

CONSTRUCTION.

Let the primitive circle represent that hour circle on which the Sun was at the first observation, EQ being the equator, then Aa, the parallel of 20° of declination, gives A the Sun's place at first; and as CQ is the tangent of 45° , Q will be the center of the hour circle PBS three hours distant from the former, its intersection B with the parallel of declination, is the Sun's place at the second observation: About A as a pole, at the distance of $71^{\circ} 30'$, the first zenith distance, describe (IV. 66) a small circle; about B as a pole, at the distance of $46^{\circ} 00'$, the second zenith distance, describe (IV. 66) another small circle, cutting the former in z the zenith: Through z, A; z, B; A, B; P; z; describe (IV. 61) great circles; then Pz is the co-latitude required,

COMPUTATION.

Here are three triangles to work in; namely, ABP, ABZ, BPZ.

In the isosceles spheric triangle APB.

Given $AP = 70^{\circ} 00'$ | Suppose a perpendicular Pb is drawn.

$BP = 70^{\circ} 00'$ | $\text{Rad} : \tan. Bpb :: \text{cof. PB} : \cot \angle PBA = 81^{\circ} 56'.$

$\angle APB = 45^{\circ} 00'$ | $\text{Rad} : \sin. PB :: \sin. \angle Bpb : \sin. Bb = 21^{\circ} 04\frac{1}{2}'.$

Req. $\angle ABP$ and AB. | Then Bb doubled gives $AB = 42^{\circ} 09'.$

In the oblique angled spheric triangle ABZ.

Given $AZ = 71^{\circ} 30'$ | Then working with the three sides, the angle

$BZ = 46^{\circ} 00'$ | ABZ will be found $= 114^{\circ} 11'.$

$AB = 42^{\circ} 09'$ | And $\angle ABZ - \angle PBA = PBZ = 32^{\circ} 15'.$

Required $\angle ABZ.$

In the oblique angled spheric triangle PBZ.

Given $PB = 70^{\circ} 00'$ | As $\text{Rad} : \text{cof. } \angle PBZ :: \tan. BZ : \tan. M = 41^{\circ} 13'.$

$BZ = 46^{\circ} 00'$ | And $PB - M = N = 28^{\circ} 47'.$

$\angle PBZ = 32^{\circ} 15'$ | As $\text{cof. M} : \text{cof. N} :: \text{cof. BZ} : \text{cof. PZ} = 35^{\circ} 58'.$

Req. the co-lat. Pz. | Therefore the latitude is $54^{\circ} 02' N.$

182. If the Sun's altitude can be taken both before and after noon, when he has equal heights, then the time between these two observations being bisected, will give the time when the Sun was on the meridian: Now the co-declination, the co-altitude, and the time from noon at either observation being known, the latitude may be readily computed in one oblique angled triangle, wherein are known two sides, and an angle opposite to one of them to find the other side, which is the co-latitude; for which see the problem, art. 176.

183. PRO-

183.

PROBLEM XXXIX.

PL V.

Given the Sun's declination, two altitudes, and the difference of the magnetic azimuths;

Required the latitude of the place.

EXAM. On the 21st of May, the Sun's declination being $20^{\circ} 16' N$, in the morning when the Sun was in the ESE. point of the compass, his altitude was $43^{\circ} 30'$; and when he bore S. $20^{\circ} 30' E$. his altitude was $58^{\circ} 30'$: What is the latitude of the place of observation?

CONSTRUCTION.

Let the primitive circle represent the azimuth circle the Sun was on at the greatest altitude $58^{\circ} 30'$, A being the Sun's place at that time, HR the horizon, and Z the zenith; draw aa a parallel of $43^{\circ} 30'$ of altitude, and (IV. 75) describe a vertical circle, making an angle with AZ, of $47^{\circ} 00'$, the difference of the observed azimuths; where this cuts the parallel of altitude aa , gives B the Sun's place at the first observation: Then small circles being described about A and B, as poles, at the distances of $69^{\circ} 44'$, the co-declination (IV. 66), their intersection will give P the place of the pole: Through A, B; P, A; P, B; and Z, P, describe great circles (IV. 61) then pz is the co-latitude sought.

COMPUTATION.

1st. In the spheric triangle AZB: Given AZ = A's co-alt. = $31^{\circ} 30'$

BZ = B's co-alt. = $46^{\circ} 30'$

AZB = diff. of az. = $47^{\circ} 00'$

Required $\angle ABZ$ = $45^{\circ} 41'$
AB = $32^{\circ} 17'$

2^d. In the spheric triangle APB: Given AP = A's co-decl. = $69^{\circ} 44'$

BP = B's co-decl. = $69^{\circ} 44'$

AB = distance = $32^{\circ} 17'$

Required $\angle ABP$ = $83^{\circ} 52'$

3^d. In the spheric triangle ZBP: Given BZ = B's co-alt. = $46^{\circ} 30'$

PB = P's co-decl. = $69^{\circ} 44'$

$\angle ZBP$ = $38^{\circ} 11'$

Requir. zp = co-lat. = $39^{\circ} 21'$
Therefore the latitude of the place is $50^{\circ} 39' N$.

184. PRO-

184.

PROBLEM XI.

Pl. V.

Two known stars being observed on the same azimuth, and two other known stars being observed on another azimuth, and the time between the observations being known; to find the latitude of the place.

EXAM. *The stars Aldebaran in Taurus, and Regel in Orion, were observed on the same azimuth; and 2 h. 35 min. after, the stars Castor in Gemini and the Hydra's heart were also observed on another azimuth: What was the latitude of the place of observation?*

CONSTRUCTION.

On the plane of the equator put the stars Aldebaran and Regel at A, B, also the stars Castor and Hydra at c, d, by the means of their right ascensions and declinations (2d and 3d of 137): Let the stars c, d, be removed forwards $2^h 35^m$, or $38^\circ 45'$ to c, d; through A, B, and D, c, describe (IV. 61) great circles intersecting in z, the zenith of the place; draw the great circles AC and PZ; then the measure of PZ gives the co-latitude required.

COMPUTATION.

Aldebaran's declin.	$= 15^\circ 59' \text{ N.}$	right ascen.	$= 4^h 22^m = 65^\circ 30'.$
Regel's	$= 8 \ 28 \ \text{S.}$		$= 5 \ 01 \ = 75 \ 15.$
Castor's	$= 32 \ 24 \ \text{N.}$		$= 7 \ 17 \ = 109 \ 15.$
Hydra's heart	$= 7 \ 37 \ \text{S.}$		$= 9 \ 16 \ = 139 \ 00.$

1st. In the triangle PAB, where $PB = 988 \ 28'$, $PA = 74^\circ 01'$, $\angle APB = 9^\circ 15'$.

Then the $\angle PAB$ will be $157^\circ 46'$, and the $\angle PAZ$, the suppl. $= 22^\circ 14'$.

2d. In the triangle PCD, where $PD = 97^\circ 37'$, $PC = 57^\circ 36'$, $\angle CPD = 29^\circ 45'$.

Then the $\angle PCD$ will be $139^\circ 21'$, and the $\angle PCZ$, the suppl. $= 40^\circ 39'$.

3d. In the triangle APC, where $AP = 74^\circ 01'$, $CP = 57^\circ 36'$, $\angle APC = 5^\circ 00'$, which is the difference between 2 h. 35 m. and the difference of the right ascensions of A and C.

Then the $\angle PAC$ will be $14^\circ 06'$, $\angle PCA = 163^\circ 23'$, and $AC = 16^\circ 57'$.
Now $\angle PAC + \angle PAZ = \angle CAZ = 36^\circ 20'$; and $\angle PCA - \angle PCZ = \angle ACZ = 122^\circ 44'$.

4th. In the triangle ACZ, where $AC = 16^\circ 57'$, $\angle CAZ = 36^\circ 20'$, $\angle ACZ = 122^\circ 44'$.

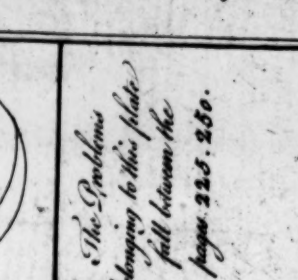
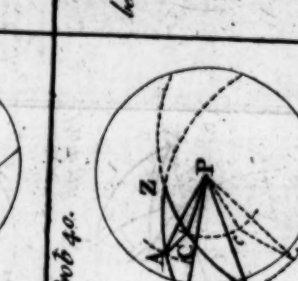
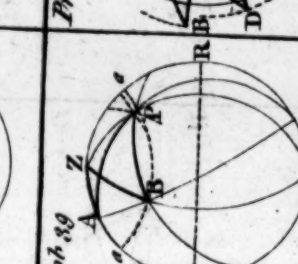
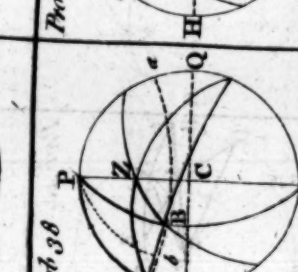
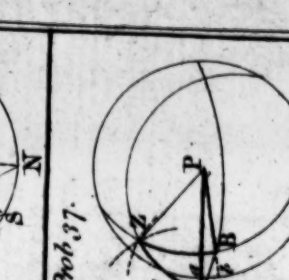
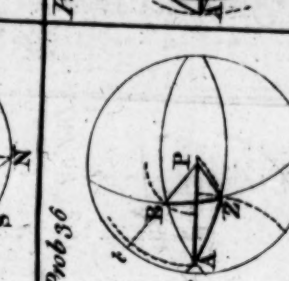
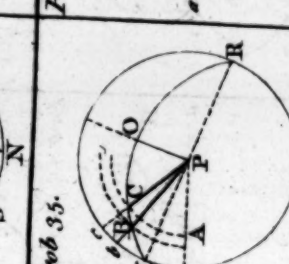
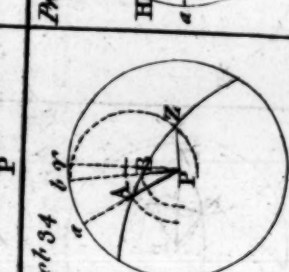
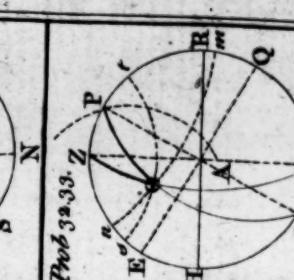
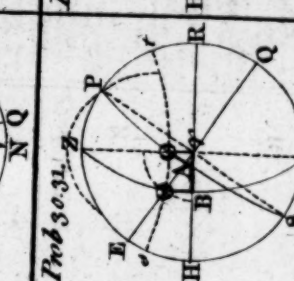
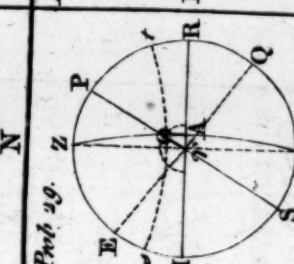
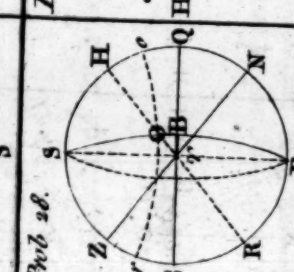
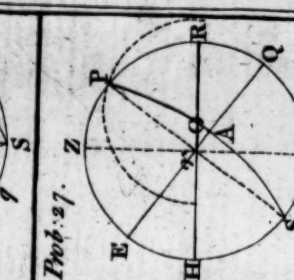
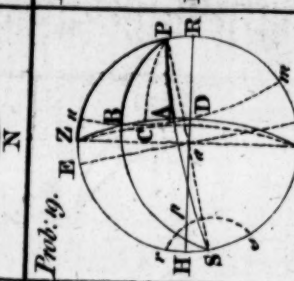
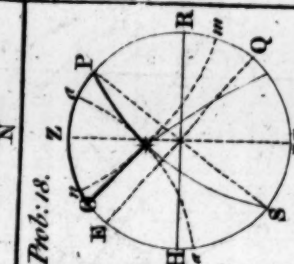
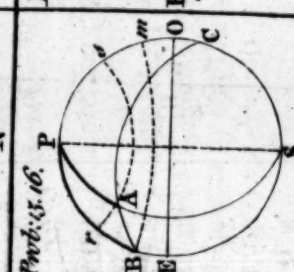
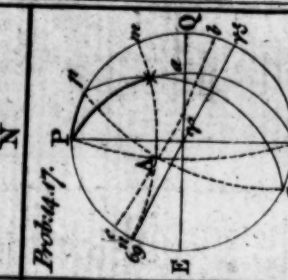
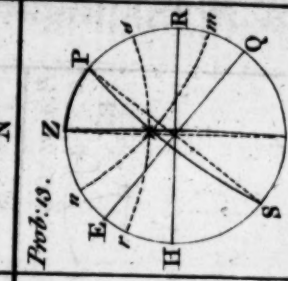
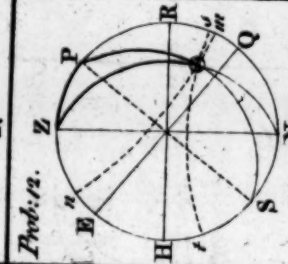
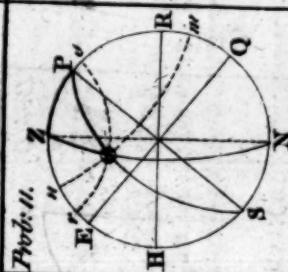
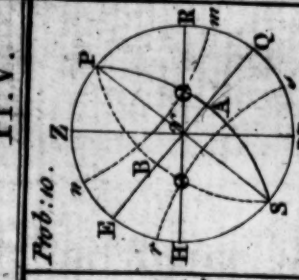
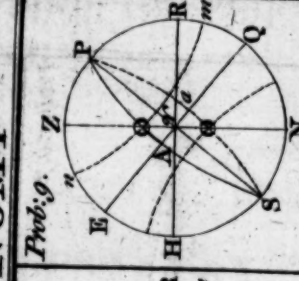
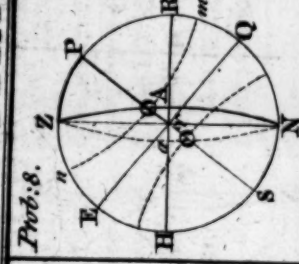
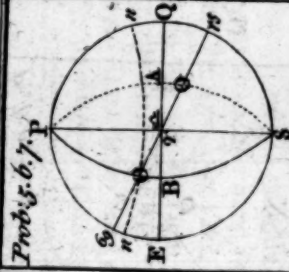
Then CZ will be found equal to $24^\circ 58'$.

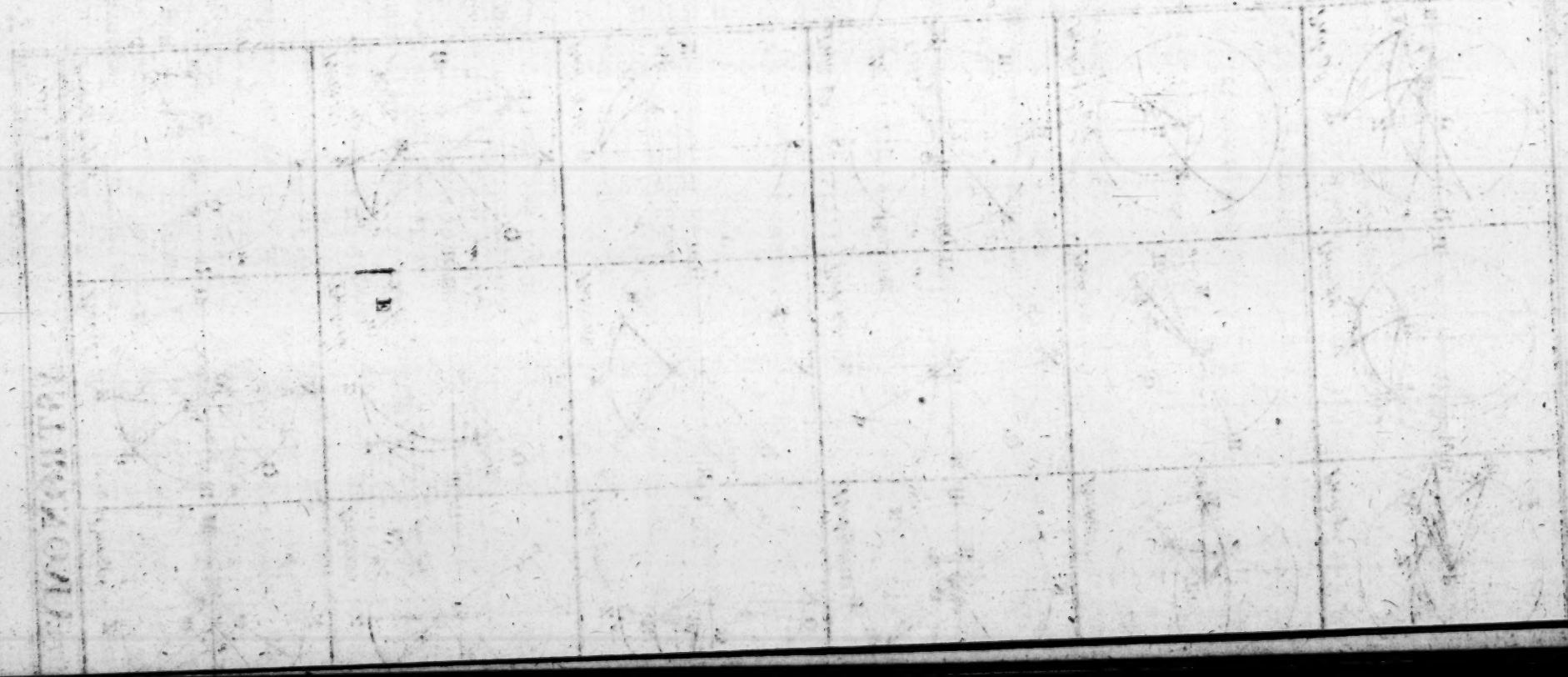
5th. In the triangle CPZ, where $CZ = 24^\circ 58'$, $CP = 57^\circ 36'$, $\angle PCZ = 40^\circ 39'$.

Then PZ will be found equal to $40^\circ 52'$.

And the latitude of the place of observation is $49^\circ 08' \text{ N.}$

There





PROBLEM I

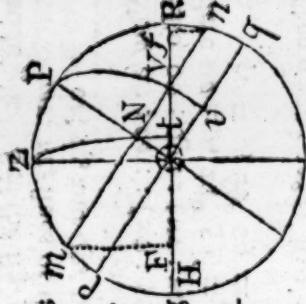
N A T A ch fu un

187.

PROBLEM XLIII.

Being at sea in the western ocean, some time in July the Sun was observed to rise at 4 h. 24 m. 36 s. A. M.; and in the same place, his altitude at noon was 62° 00': Required the latitude of that place, and day of the month?

Let m, v, n , be places of the Sun, at noon; m at rising, and at midnight.
 mf, vf , fines of mer. alt. & midnight depr.
 ov the fine of the ascensional difference.
 vq the versed fine of the time of setting from midnight.
 qv = radius + fine of the ascensional difference.



Now $qv : vq :: (mv : vn ::) mf : vf$.

Then $vf = \frac{vq \times mf}{qv}$; or $Lf, vf = L, qv + Lf, mf + Lv, L$

Or, $L, qv + Lf$, merid. alt. + $2Lf$, $\frac{1}{2}$ time @ midnight + $L, z = Lf$, mid-night depression.

Here $ov = (s, 1 h. 35 m. 24 s. = s, 23^{\circ} 51' =) 0.40435$; and $qv = 1.4043$; Time from midnight = 4 h. 24 m. 36 s = $66^{\circ} 09'$; its half = $33^{\circ} 4\frac{1}{2}'$.

$qv = 1.4043$ its L 9,85254
 $mf = 1.62^{\circ}$ its Lf 9,94593
 $vq = \left. \begin{array}{l} 11, 33^{\circ} 4\frac{1}{2}' \\ 2 \end{array} \right\}$ $2Lf$ 9,47397
 its L 0,30103

$vf = 21^{\circ} 59' 40''$ its Lf 9,57347

Merid. alt. = $62^{\circ} 0' 0''$

Midnt. depr. = $21^{\circ} 59' 40''$

Sum = $83^{\circ} 59' 40''$; its $\frac{1}{2} = 41^{\circ} 59' 50''$

Diff. = $40^{\circ} 0' 20''$; its $\frac{1}{2} = 20^{\circ} 0' 10''$

Latitude = $48^{\circ} 0' 10''$ N.

Declination = $20^{\circ} 0' 10''$ N. on July 23.

188.

PROBLEM XLIV.

At a place in the northern hemisphere, some time in the month of May, the Sun was observed to have $14^{\circ} 43\frac{1}{2}'$ altitude at 6 h. P. M. and to set at 7 h. 35 m. 24 s.: Required the latitude of that place, and day of the month?

Let m , N , v , n , be places of the Sun, at noon, 6 o'clock, setting and midnight. See the fig. to Problem 43.

mf , nf , sines of the altitudes at m and N .

ov , vq , and qv , the same as in the last problem.

Now $ov : vq :: (nv : vn ::) nt : nf$. Then $L, nf = L, ov + L, nt + L, vq$.

Also $ov : qv :: (nv : vm ::) nt : mf$. And $L, mf = L, ov + L, nt + L, qv$.

Here $ov = 1, 23^{\circ} 51' = 0,40435$; and $qv = 1,4043$.

$qv = v, 66^{\circ} 09' = 25, 33^{\circ} 4\frac{1}{2}'$.

Then $ov = 1, 23^{\circ} 51'$ its $L's$, 0,39325 | And $ov = 1, 23^{\circ} 51'$ its $L's$, 0,39325

$nt = 1, 14^{\circ} 43\frac{1}{2}'$ $L's$, 9,40514 | $nt = 1, 14^{\circ} 43\frac{1}{2}'$ its $L's$, 9,40514

$qv = \left\{ \begin{array}{l} 1, 33^{\circ} 4\frac{1}{2}' \\ 2 \end{array} \right. 2L's, 9,47397$ | $qv = 1, 4043$ its L , 10,14746

$mf = 1, 22^{\circ} 0'$ | $mf = 1, 62$ $\bar{O}$ $L's$, 9,94585

$9,57339$

Then $\frac{62^{\circ} + 22^{\circ}}{2} = 42^{\circ}$ the co-latitude.

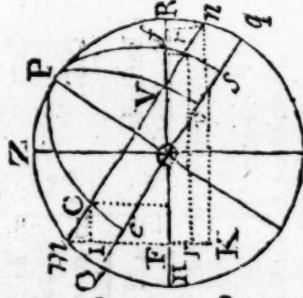
And $\frac{62 - 22}{2} = 20^{\circ}$ the declination answering to May 20.

189.

PROBLEM XLV.

At a place in the western ocean, some time in July, the Sun's altitude was observed to be 36° at 3 h. 51 m. 49 s. P. M.; and was seen to set at 7 h. 35 m. 24 s. P. M.: Required the latitude of the place and day of the month?

Let m , c , v , n , be the Sun's places, at noon, 3 h. 51 m. 49 s. at setting, and at midnight. mf , nf , the sines of the altitudes at m , c ; nf the sine of the mid-night depression. oc , ov , the sines of the times from 6 h., and ov , their sum.



Now

Now $cv : Qv :: (cv : vm ::)$ IF : mf . Or $Ls, mf = Ls, cv + Ls, IF + Ls, Qv$.
 And $cv : qv :: (cv : vn ::)$ IF : nf . Or $Ls, nf = Ls, cv + Ls, IF + Ls, qv$.
 Here $oc = s, 2h. 8m. 11s. = s, 32^{\circ} 2' 45'' = 0, 53059$ } $cv = 0, 93494$;
 $ov = s, 1h. 35m. 24s. = s, 23^{\circ} 51' 00'' = 0, 40435$ } $Qv = 0, 93494$;
 $Qv = s, 1, 4043; qv = v, 66^{\circ} 9' = 2s, 33^{\circ} 4' 30''$; or $L, qv = 2Ls, 33^{\circ} 4\frac{1}{2}' + L2$.

Then $L, cv = 0, 93494$ } $0, 02922$. And $L, cv = 0, 93494$ } $0, 02922$
 $Ls, IF = 36^{\circ} 00'$ } $9, 76922$ } $Ls, IF = 36^{\circ} 00'$ } $9, 76922$
 $L, Qv = 1, 4043$ } $10, 14746$ } $L, qv \left\{ \begin{array}{l} s, 33^{\circ} 4\frac{1}{2}' \\ 2 \end{array} \right.$ } $9, 47397$
 $L, mf = 62^{\circ} 00'$ } $9, 94590$. } $Ls, nf 22^{\circ} 00'$ } $0, 30103$
 $\underline{\hspace{1cm}}$ $\underline{\hspace{1cm}}$ $\underline{\hspace{1cm}}$ $\underline{\hspace{1cm}}$
 $9, 57344$

Then $\frac{62^{\circ} + 22^{\circ}}{2} = 42^{\circ}$ the co-latitude.

And $\frac{62 - 22}{2} = 20$ the declination answering to July 23.

190.

PROBLEM XLVI.

Some time in the month of May, at a place in the western ocean, the day broke at 1 h. 45 m. 36 s. A. M.; and at 8 h. 8 m. 11 s. A. M. the Sun's altitude was observed to be 36° : Required the latitude and day of the month.

Let m, c, r, n , be the Sun's places, at noon, at 8 h. 8 m. 11 s., at the beginning of twilight, and at midnight.

mf, IF , the sines of the altitudes at m C.

FT, FK , the sines of the depressions at r, n .

oc, os , the sines of the times from 6 h., and c s their sum.

Now $cs : Qc :: (cr : mr ::)$ IT : mt . And $mt - FT = mf$.

Also $cs : qc :: (cr : cn ::)$ IT : IK. And $IK - IF = nf$.

Here $IF = s, 36^{\circ} 00' = 0, 58778$ } $IT = 0, 89679$,
 $FT = s, 18^{\circ} 00' = 0, 30901$ }

$oc = s, 2h. 8m. 11s. = s, 32^{\circ} 2' 45'' = 0, 53059$; and $qc = 1, 5306$

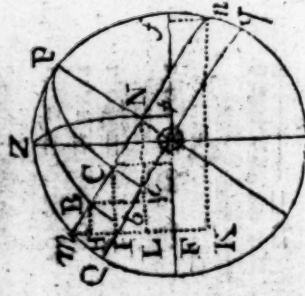
$os = s, 4h. 14m. 24s. = s, 63^{\circ} 36' 00'' = 0, 89571$; and $Qc = 1, 8957$

Then cs	$9, 84579$	cs	$9, 84579$	$mt - FT = mf = 0, 8829$	the $s, 62^{\circ} 00'$
IT	$9, 95269$	IT	$9, 95269$	$IK - IF = nf = 0, 37458$	the $s, 22^{\circ} 00'$
Qc	$10, 27777$	qc	$10, 18486$	Hence the lat. is $48^{\circ} N$.	
mt	$10, 07625$	IK	$9, 98334$	Decl. $22^{\circ} 00' N$, answering to May 20.	
mt	$1, 1919$	IK	$0, 96236$		

191.

PROBLEM XLVII.

In the month of May, at some place in the western ocean, the Sun's altitude at 6^h A. M. was $14^{\circ} 43\frac{1}{2}'$, and at 8 h. 8 m. 11 s. its altitude was 36° : Required the latitude of the place and day of the month?



Let m, c, n, n , be the Sun's places at noon, at 8 h. 8 m. 11 s., at 6 h., and at midnight. mf, if, lf , the fines of the alts. at m, c, n . nf the fine of the midnight depression. oc the fine of 2 h. 8 m. 11 s.

Now $oc : oq :: (nc : nm ::) il : ml$. Then $ml + lf = mf$.
And $oc : cq :: (nc : cn ::) il : ik$. Then $ik - if = nf$.

Hence $L, ml = L', oc + Rad + L, il$. And $L, ik = L', oc + L, cq + L, il$.

Here $oc = 52^{\circ} 2' 45''$ its L , 8 m. 11 s. $= 52^{\circ} 2' 45'' = 0,53059$; and $cq = 1,5306$.

$if = 5,3600'$ its L , and $lf = 5,14^{\circ} 43\frac{1}{2}' = 0,25418$; so $il = 0,3336$
 $oc = 32^{\circ} 2' 45''$ its L , 0,27524
 $il = 0,3336$ its L , 9,52323
Rad 10,00000
 $cq = 1,5306$ its L , 10,18486

$ml = 0,62874$

9,79847

$ik = 0,96234$

9,98333.

Then $mf = 0,88292$ the fine of $62^{\circ} 00'$ the meridian altitude.
And $nf = 0,37456$ the fine of $22^{\circ} 00'$ the midnight depression.
Hence the latitude is 48° N. decl. 20° N. on May 20.

(185)

192.

PROBLEM XLVIII.

At a place in the western ocean, in the month of July, the Sun's altitude was found to be 46° at 2^h 49^m 9^s P. M.; and to be 36° high at 3^h 51^m 49^s P. M.: Required the latitude of the place and day of the month?

Let m, b, c, n , be the Sun's places at noon, at 2 h. 49 m. 9 s., at 3 h. 51 m. 49 s. and at midnight; and mf, hf, if , the fines of the alts. at m, b, c, ob, oc , the co-fines of the times from noon; or the fines of the times to 6 o'clock.

Now $bc : qc :: (bc : mc ::) hi : mi$. Then $mi + if = mf$.

And $bc : bq :: (bc : bn ::) hi : hk$. Then $hk - hf = ik = nf$.

Here $ob = 5,3$ h. 10 m. 51 s. $= 5,47^{\circ} 42' 45'' = 0,73978$ } Hence $bc = 0,20919$
 $oc = 5,2$ h. 8 m. 11 s. $= 5,32^{\circ} 45' = 0,53059$ }

$hf = 5,46^{\circ} = 0,71933$; $if = 5,36^{\circ} = 0,58778$; $hi = 0,13155$; $bq = 1,7398$.
Also $qc = (\text{ver. fine of } 57^{\circ} 57' 15'' =) 24,5 57^{\circ} 57' 15''$.

Then

Also $AB : de :: mn : mk$; and $km - fm = fk$, the sine of rn .
Hence the latitude and declination are found.

Here $AB = (\frac{1}{2}ma - \frac{1}{2}mb = \frac{mb + ma}{2} \times 2s, \frac{mb - ma}{2} = \frac{1}{2}M \times 2s, N$. (185)

And $de = (s, 2d \text{ alt.} - s, 1st \text{ alt.} = \frac{2d + 1st}{2} \times 2s, \frac{2d - 1st}{2} = \frac{1}{2}W \times 2s, V$. (IV. 181)

(IV. 182)

Now $L, \frac{de}{AB} \times mn = L, mk = L's, M + L's, N + L's, W + L's, V + L, 2$.

And $L, \frac{de}{AB} \times MB = L, me = L, mk + 2Ls, \frac{1}{2} \angle meb$.

In this example, the $\angle meb = 3h. 51.49 = 57^\circ 57\frac{1}{2}'$; its $\frac{1}{2} = 28^\circ 58\frac{1}{4}'$.
 $\angle mea = 2h. 49. 9 = 42^\circ 17\frac{1}{2}'$.

$57^\circ 57\frac{1}{2}'$ 42 17 $\frac{1}{2}$	46° 36	$L's, M$ $L's, N$ $L's, W.$ $L's, V$ $L, 2$	$50^\circ 7\frac{1}{8}'$ $7 49\frac{7}{8}$ $41 0$ $5 0$ $—$	$0, 11407$ $0, 86564$ $9, 87778$ $8, 94030$ $0, 30103$
$100 14\frac{3}{4}$ 15 39 $\frac{3}{4}$	$50^\circ 7\frac{3}{8}' = M$ $82 41 0 = W.$ $—$ $—$	L, K, M $2Ls, \frac{1}{2}mb$ L, me	$1, 2581$ $28^\circ 58\frac{1}{8}'$ $0, 29527$	$10, 09972$ $9, 37050$ $9, 47022$
$s, 36^\circ = fe = 0, 58778$ $me = 0, 29527$	$7 49\frac{7}{8} = N$ $110 5 0 = V$			
$fm = 0, 88305 = 62^\circ 0' 26''$ $km = 1, 25810$				
$fk = 0, 37505 = 22^\circ 2' 00''$				

$$fk = 0, 37505 = 22^\circ 2' 00''$$

$$84 \quad 2 \quad 26$$

$$39 \quad 58 \quad 26$$

$42^\circ 1' 13''$ the co-latitude.

19 59 13 the decl. on May 20th.

198. The following is another solution, on different principles.

In the triangles BPZ , APZ , are given, (see the foregoing figure)
 BZ , AZ ; BPZ , APZ , hour angles; and $PB = PA$;

To find PB , PZ ; for whose sum and diff. put M and N .

Now (IV. 239) $\left\{ \begin{array}{l} s, PB \times s, PZ \times s, BPZ + s, PB \times s, PZ = s, BZ. \\ s, PA \times s, PZ \times s, APZ + s, PA \times s, PZ = s, AZ. \end{array} \right.$

Or (IV. 181. 174) $\left\{ \begin{array}{l} \frac{1}{2}s, N - \frac{1}{2}s, M \times s, BPZ + \frac{1}{2}s, N + \frac{1}{2}s, M = s, BZ. \\ \frac{1}{2}s, N - \frac{1}{2}s, M \times s, APZ + \frac{1}{2}s, N + \frac{1}{2}s, M = s, AZ. \end{array} \right.$

Therefore $(\frac{1}{2}s, N - \frac{1}{2}s, M = \frac{s, BZ}{s, BPZ})$
 $\frac{s, BPZ}{s, APZ}$

And $(\frac{1}{2}s, N + \frac{1}{2}s, M = \frac{s, BZ - \frac{1}{2}s, N - \frac{1}{2}s, M \times s, BPZ = s, AZ - \frac{1}{2}s, N - \frac{1}{2}s, M}{s, APZ})$

Hence $\frac{s, BZ \times s, APZ - \frac{1}{2}s, N + \frac{1}{2}s, M \times s, APZ = s, AZ \times s, BPZ - \frac{1}{2}s, N + \frac{1}{2}s, M}{s, BPZ}$.

Therefore

Therefore, $BZ \times \dot{s}, APZ - \dot{s}, AZ \times \dot{s}, BPZ = \dot{s}, APZ - \dot{s}, BPZ \times \frac{1}{2}\dot{s}, N + \frac{1}{2}\dot{s}M$.

Again

$$\dot{s}, AZ - \dot{s}, BZ = \dot{s}, APZ - \dot{s}, BPZ \times \frac{1}{2}\dot{s}, N - \frac{1}{2}\dot{s}M.$$

Therefore $\frac{1}{2}\dot{s}, N + \frac{1}{2}\dot{s}M = \frac{\dot{s}, BPZ \times \dot{s}, AZ - \dot{s}, APZ \times \dot{s}, BZ}{\dot{s}, APZ - \dot{s}, BPZ}$.

And $\frac{1}{2}\dot{s}, N - \frac{1}{2}\dot{s}M = \frac{\dot{s}, AZ - \dot{s}, BZ}{\dot{s}, APZ - \dot{s}, BPZ}$

$$\dot{s}, M = \left(\frac{\dot{s}, AZ + \dot{s}, BPZ \times \dot{s}, AZ - \dot{s}, BZ - \dot{s}, APZ \times \dot{s}, BZ}{\dot{s}, APZ - \dot{s}, BPZ} \right)$$

$$\frac{I + \dot{s}, BPZ \times \dot{s}, AZ - I + \dot{s}, APZ \times \dot{s}, BZ}{\dot{s}, APZ - \dot{s}, BPZ}$$

$$\dot{s}, N = \left(\frac{\dot{s}, BPZ \times \dot{s}, AZ - \dot{s}, AZ - \dot{s}, APZ \times \dot{s}, BZ + \dot{s}, BZ}{\dot{s}, APZ - \dot{s}, BPZ} \right)$$

$$\frac{I - \dot{s}, BPZ \times \dot{s}, AZ - I - \dot{s}, APZ \times \dot{s}, BZ}{\dot{s}, APZ - \dot{s}, BPZ}$$

$$\text{Or } \dot{s}, M = \left(\frac{\dot{s}, BPZ \times \dot{s}, AZ - \dot{s}, APZ \times \dot{s}, BZ}{\dot{s}, APZ - \dot{s}, BPZ} \right)$$

$$\frac{2\dot{s}, \frac{1}{2}\dot{s}, BPZ \times \dot{s}, AZ - 2\dot{s}, \frac{1}{2}\dot{s}, APZ \times \dot{s}, BZ}{\dot{s}, APZ - \dot{s}, BPZ}$$

$$\text{And } \dot{s}, N = \left(\frac{\dot{s}, BPZ \times \dot{s}, AZ - \dot{s}, ABZ \times \dot{s}, BZ}{\dot{s}, APZ - \dot{s}, BPZ} \right)$$

$$\frac{2\dot{s}, \frac{1}{2}\dot{s}, BPZ \times \dot{s}, AZ - 2\dot{s}, \frac{1}{2}\dot{s}, APZ \times \dot{s}, BZ}{\dot{s}, APZ - \dot{s}, BPZ}$$

Here $FPZ = 57^{\circ} 57\frac{1}{4}'$, its $\frac{1}{2} = 28^{\circ} 58\frac{1}{8}'$; $APZ = 42^{\circ} 17\frac{1}{2}'$, its $\frac{1}{2} = 21^{\circ} 8\frac{5}{8}'$.
 $BZ = 54^{\circ}$; $AZ = 44^{\circ}$; $\dot{s}, BPZ = 0,53060$; $\dot{s}, APZ = 0,73978$; their dif-
 $= 0,20918$.

$2L\dot{s},$ 28° 58 $\frac{1}{8}'$	$2L\dot{s},$ 28° 58 $\frac{1}{8}'$	$\left\{ \begin{array}{l} 0,30103 \\ 9,88384 \end{array} \right\}$	$2L\dot{s},$ 21° 8 $\frac{5}{8}'$	$\left\{ \begin{array}{l} 0,30103 \\ 9,37050 \end{array} \right\}$
$L\dot{s},$ 44	$L\dot{s},$ 44	9,85693	$L\dot{s},$ 54	9,85693
L, A 1,10104	L, a 0,33764			9,52846
$2L\dot{s},$ 21° 8 $\frac{5}{8}'$	$2L\dot{s},$ 21° 8 $\frac{5}{8}'$	$\left\{ \begin{array}{l} 0,30103 \\ 9,93946 \end{array} \right\}$		$\left\{ \begin{array}{l} 0,30103 \\ 9,11418 \end{array} \right\}$
$L\dot{s},$ 54	$L\dot{s},$ 54	9,76922		9,76922
L, B 1,07824	L, b 0,15291			9,18443
$L, A - B$ 0,07842	$L, a - b$ 0,18472	8,89443		9,26651
$L\dot{s}, APZ - \dot{s}, BPZ$ 0,20918	$L\dot{s}, APZ - \dot{s}, BPZ$ 0,20918	9,32052		9,32052
$L\dot{s}, M$ 112° 1'	$L\dot{s}, N$ 27° 59'	9,57391		9,94599

Hence the latitude is $47^{\circ} 59' N$; declination 20 , which answers to the 20th of May.

199. And hence is readily derived the investigation of that method, published in the year 1759, and then used by some for finding the true latitude at sea, by knowing the latitude by account, (or dead reckoning) the Sun's declination, two altitudes of the Sun, and the time between the observations. Thus. See the last figure.

Let M and N represent the half sum and half diff. of the times from noon. w and v , the half sum and half diff. of the two altitudes.

AB the diff. of the co-fines of the times from noon, to the radius Em . AD the diff. of the fines of the altitudes.

$\angle BAD$ represents the latitude; Em the cof. of the declination.

Now $(197) s, M \times 2s, N = AB$ reduced to the rad. $\frac{1}{2} Qq$; and $s, w \times 2s, v = AD$. But in the triangle ABD .

$$s, \text{ lat.} : R :: AD : AB = \frac{I}{s, \text{ lat.}} \times AD, \text{ to rad. } Em.$$

And $s, \text{ decl.} : R :: AB : AB \times \frac{I}{s, \text{ decl.}} = \frac{I}{s, \text{ decl.}} \times \frac{I}{s, \text{ lat.}} \times AD = AB$ reduced to the radius $\frac{1}{2} Qq$.

$$\text{Then } s, M \times 2s, N = \frac{I}{s, \text{ decl.}} \times \frac{I}{s, \text{ lat.}} \times s, w \times 2s, v.$$

$$\text{And } s, M = \frac{I}{s, \text{ decl.}} \times \frac{I}{s, \text{ lat.}} \times \frac{I}{s, N} \times s, w \times s, v.$$

Then $M + N = \angle meb$, the time from noon at the least altitude.

And $M - N = \angle mea$, the time from noon at the greatest altitude.

Hence the versed fines of the arcs mb or ma , are known to rad. $\frac{1}{2} Qq$.

Now $R : Em :: v, ma : ma = Em \times v, ma$; or $mb = Em \times v, mb$.

And $R : s, mad :: ma : md (:: mb : me.)$

Then $Fm = Fd + dm$, or to $Fe + em$, is the sine of the mer. alt.

Hence the two operations.

1st. $Ls, \text{ decl.} + Ls, \text{ lat.} + Ls, N + Ls, w + Ls, v = Ls, M$.

Hence the times are known; viz. arcs ma , mb .

2d. $Ls, \text{ decl.} + Ls, \text{ lat.} + L, 2 + 2Ls, \frac{1}{2} ma = L, md$.

Then by the merid. alt. and declination the latitude may be found.

If this latitude and that assumed are the same, then the latitude by account, or dead reckoning, may be taken as the true latitude.

But if they differ, it is plain that the $\angle mad$, the co-latitude, is less or greater than was assumed.

200.

PROBLEM LIV.

Given three descending (or ascending) altitudes of the Sun, taken on the same day at unequal known intervals of time; thence to find those times, the latitude of the place of observation, and the Sun's declination. Thus, suppose in July, 1763, the altitudes were $54^\circ 30'$, 46° , and 36° ; and the intervals of time 60 m. 55 s. and 62 m. 40 s.; required the rest?

Let

Let A, B, C , be the places of the Sun when observed; m, n , the places at noon and midnight; and let mf, df, ef, fe, FK , represent the fines of the distances of those places from the horizon HR .

Also let man be the parallel of declination, described on mn ; where in a, b, c , correspond to A, B, C . Then will the angles mEc, mEb, mEa, H represent the times from noon when the observations were taken.

Now as cb is one interval and ab the other; then $\frac{1}{2} \angle aEc$, and its complement $\angle acE$, are known.

Draw cg parallel to mn ; and $\angle mEc = (Ecg) = \angle acE = \angle acg$.

Hence if $\angle acg$ was known, the times from noon would be known.

The lines df, de , the diff. of the fines of the altitudes, are known.

And the lines ca, cb, ba , the chords of the intervals, are known.

Then in the $\triangle cbo \left\{ \begin{array}{l} R : (bco) :: s, \frac{1}{2} \text{ arc } ab :: (cb) = 2s, \frac{1}{2} \text{ arc } cb : bo. \\ R : (cbo) :: s, \frac{1}{2} \text{ arc } ab :: (cb) = 2s, \frac{1}{2} \text{ arc } cb : co. \end{array} \right.$

Now the lines fd, ac, ac , are cut in e, B, r , in like ratio to their wholes.

And the line ac cutting the parallels aa, bb , make sim. $\triangle, acg, rbo$.

Then $df : de :: (ca : cb ::) ca : cr$; and $co - cr = ro$.

And $bo : ro :: R : r, ro = acg$; and $acE - acg = gce = cEm$.

Hence the times from noon are known, and also mo , the versed sine of cEm .

Again $R : s, acg :: ac : cg = AC$.

And $AC : mc :: fd : dm$. Then $fd + dm = fm$, the sine of Hm .

Also $AC : mn :: fd : mk$. Then $Km - fm = FK$, the sine of Rn .

This Problem has, at times, for near a century past, exercised the talents of many ingenious persons, as well in Russia, Germany, Holland and France, as in England; perhaps, on account of its apparent use at sea: And among the different solutions, there seems no one more intelligible, particularly to beginners, nor shorter in its numeral operations, than that above; however, for the sake of the more inquisitive, another solution, which has been commonly given, is here subjoined.

201. In the last figure, the three triangles CPZ, BPZ, APZ , are those concerned; wherein the same two sides are common in each.

$s, CPZ \times s, PC \times s, PZ + s, PC \times s, PZ = s, ZC$
 $s, BPZ \times s, PC \times s, PZ + s, PC \times s, PZ = s, ZB$
 $s, APZ \times s, PC \times s, PZ + s, PC \times s, PZ = s, ZA$ } by IV. 239.

Then $s, CPZ - s, BPZ \times s, PC \times s, PZ = (s, ZC - s, ZB =) d$.

And $s, CPZ - s, APZ \times s, PC \times s, PZ = (s, ZC - s, ZA =) D$.

Hence $D \times s, CPZ - s, BPZ = d \times s, CPZ - s, APZ$.

Then $D \times s, CPZ - d \times s, CPZ = D \times s, BPZ - d \times s, APZ$.

S 4

Let

(II. 48)

(II. 147)

Let $\left\{ \begin{array}{l} \text{APZ} - \text{CPZ} = n, \text{ then } \text{APZ} = \text{CPZ} + n. \text{ And } s, \text{APZ} = s, \text{CPZ} \times s, n - s, \text{CPZ} \times s, n. \\ \text{BPZ} - \text{CPZ} = n. \end{array} \right. \text{BPZ} = \text{CPZ} + n. \quad s, \text{BPZ} = s, \text{CPZ} \times s, n - s, \text{CPZ} \times s, n.$

Then $D \times s, \text{CPZ} - d \times s, \text{CPZ} = \left\{ \begin{array}{l} D \times s, n \times s, \text{CPZ} - D \times s, n \times s, \text{CPZ}. \\ -d \times s, n \times s, \text{CPZ} + d \times s, n \times s, \text{CPZ}. \end{array} \right. \quad (\text{IV. 216})$

Hence $D - D \times s, n - d + d \times s, n \times s, \text{CPZ} = d \times s, n - D \times s, n \times s, \text{CPZ}.$

Then $\frac{1 - s, n \times D \cos 1 - s, n \times d}{d \times s, n \cos D \times s, n} = \frac{s, \text{CPZ}}{s, \text{CPZ}} = t, \text{CPZ}. \quad (\text{II. 156})$

Or $\frac{s, \text{BPZ} \times D \cos s, \text{APC} \times d}{s, \text{BPC} \times D \cos s, \text{APC} \times d} = t, \text{CPZ},$ the hour from noon at C.

Here $Fd = s, 54^\circ 30' = 0,81411$
 $Ff = s, 46^\circ 00' = 0,71934$ } Then $df = 0,22643$; $de = 0,09477$
 $Ff = s, 36^\circ 00' = 0,58778$

The arc cb , or $\angle CPB$, $= 1$ h. 0 m. 55 s. $= 15^\circ 13\frac{1}{2}'$; its $\frac{1}{2} = 7^\circ 36\frac{1}{2}'$.

arc ab , or $\angle BEA$, $= 1$ h. 2 m. 40 s. $= 15^\circ 40'$; its $\frac{1}{2} = 7^\circ 50'$.

arc ca , or $\angle CPA$, $= 2$ h. 3 m. 35 s. $= 30^\circ 53\frac{1}{2}'$; its $\frac{1}{2} = 15^\circ 26\frac{1}{2}'$.

To find the hour, by method 1β .

To find the hour, by method $2d$.

$1s, \frac{1}{2}ba$	$7^\circ 50'$	$9,13447$	
$1s, \frac{1}{2}cb$	$7^\circ 36\frac{1}{2}'$	$9,12224$	$s,cb = 26268$; $s' = 96488$; $v = 0,03512$.
$1s, \frac{1}{2}bo$	$0,01826$	$8,25671$	$s,ca = 51347$; $s' = 83809$; $v = 1,4191$.
$1s, \frac{1}{2}ba$	$7^\circ 50'$	$9,99593$	$do = 0,09477$; $df = 0,22643$.
$1s, \frac{1}{2}cb$	$7^\circ 36\frac{1}{2}'$	$9,12224$	Then $v,ca \times de = 0,01348$.
$1s, \frac{1}{2}co$	$0,13127$	$9,11817$	$v,cb \times df = 0,007951$.
$1d, df$	$0,22633$	$0,64526$	Their diff. $= 0,005497$, the dividend.
$1d, ed$	$0,09477$	$8,97653$	Also $s,ca \times de = 0,048660$.
$1d, \frac{1}{2}ca$	$15^\circ 26\frac{1}{2}'$	$9,42547$	$s,cb \times df = 0,059477$.
$1s, \frac{1}{2}cr$	$0,11150$	$9,04726$	Their diff. $= 0,010817$, the divisor.
			Then $\frac{0,005497}{0,010817} = 0,5082 = 1,26^\circ 56\frac{1}{2}'$.

Then $\frac{1}{2}ro = (\frac{1}{2}co - \frac{1}{2}cr =) 0,01977$

Now $\frac{1}{2}bo : \frac{1}{2}ro :: R : t, 47^\circ 35' 20''$.

Hence if obser. was at 1 h. 47 m. 51 f.

at 2 48 46.

at 3 51 26.

202. If the altitudes were taken at equal intervals of time; the two first proportions are useless: For ab and bc being equal, the point a falls in the middle of the chord ca ; and bo the versed sine is known.

Then $df : de :: (CA : CB ::) ea : cr$; and $co - cr = ro$.

And $bo : ro :: \text{Rad} : t, rbo = acg$; and $ace - acg = gec = cem$.

Hence the times from noon are known; and also mc the versed sine of cem .

Again, in $\triangle acg$. $R : s, acg :: ac : eg = AC$.

And $AC : mc :: fd : dm$. Then $Fd + dm = fm$.

Also $AC : mn :: fd : mk$. Then $km - fm = FK$.

Here

Here $dv = 0,81412$; $ev = 0,71772$; $fv = 0,58778$; $df = 0,22634$; $de = 0,09640$.

Also, $arc\,cb = 1\text{h.}47\frac{1}{2}' = 15^{\circ}26\frac{1}{2}'$; $co = 0,26636$; $eo = 0,96387$; $bo = 0,03613$.

Then $df:de::ea:cr = 0,22688$; and $eo-cr = ro = 0,03948$.

And $bo:ro::Rad:tr\,bo = 47^{\circ}32'$; then $90^{\circ} - 15^{\circ}26\frac{1}{2}' - 47^{\circ}52' = 27^{\circ}14'$.

Then $27^{\circ}14' = 1\text{h.}48\text{m.}4\frac{1}{2}\text{s.}$ from noon at 3d. alt.

27 $14' + 15^{\circ}26\frac{1}{2}' = 42^{\circ}28' = 2^{\circ}49'52''$ noon at 2d.

42 $28' + 15^{\circ}26\frac{1}{2}' = 57^{\circ}54\frac{1}{2}' = 3^{\circ}51'39\frac{1}{2}''$ noon at 1st.

Again $R:s,acg::ac:AC = 0,35967$.

And $AC:mc::fd:dm = 0,06867$; then $mf = 0,88279 = 5,61^{\circ}59'$.

Also $AC:mn::fd:mk = 1,25860$; then $FR = 0,37581 = 5,22^{\circ}5'$.

Hence the lat. is $47^{\circ}58'N$; decl. $19^{\circ}57'$, on May 19.

203. But the operations in this case may be much contracted. Thus,

Since $(df:de::ea:cr = \frac{de \times ea}{df})$; thence $ro = (co - \frac{de \times co}{df} =)$

$$\frac{df - 2de \times co}{df}.$$

And $(bo:ro::r:t,rb\,o = (\frac{ro}{bo} = \frac{D \times co}{df \times 2s, \frac{1}{2}cb} =) \frac{D \times \frac{1}{2}co}{df \times s, \frac{1}{2}cb})$

where $D = df - 2de$.

But $R \times \frac{1}{2}v,cb = s, \frac{1}{2}cb = \frac{1}{2}s,cb \times t, \frac{1}{2}cb$.

Then $tr\,bo = \frac{D}{df \times t, \frac{1}{2}cb}$; or $L,rb\,o = L', \frac{1}{2}cb + L', df + L, D$.

(IV. 193. 195)

Also $(R:s,acg::ac:AC = (s,acg \times ac =) s,acg \times 2s,cb$.

And $(AC:mn::fd:mk = (\frac{mn \times fd}{AC} = \frac{2 \times fd}{s,acg \times 2s,cb} =) \frac{fd}{s,acg \times s,cb}$.

And $(AC:mc::fd:md = (\frac{mc \times fd}{AC} = \frac{fd \times 2s, \frac{1}{2}mec}{s,acg \times 2s,cb} =) \frac{fd}{s,acg \times s,cb}$

$\times s, \frac{1}{2}mec$.

Hence $L's,acg + L's,cb + L,fd$

$= L, mk$.

And $L's,acg + L's,cb + L,fd + 2Ls, \frac{1}{2}mec = L, md$.

Here $df = 0,22634$; $2de = 0,19280$; and $D = (df - 2de =) 0,03354$.

Also $\frac{1}{2}cb = 7^{\circ}43'26''$; and $ac\,E = 74^{\circ}33\frac{1}{2}'$.

Then $L', \frac{1}{2}$	$7^{\circ}43'26''$	0,86765	$L's,acg$	$47^{\circ}32'$	0,17059
L', df	$= 0,22634$	0,64524	$L's,cb$	$15^{\circ}26\frac{1}{2}'$	0,57452
L, D	$= 0,03354$	8,52556	L, fd	$0,22634$	9,35476
$L, acg = 47^{\circ}32'$		10,03845	L, mk	$1,25860$	10,0987
$ac\,E = 74^{\circ}33\frac{1}{2}'$			$2Ls, \frac{1}{2}mec$	$13^{\circ}30\frac{1}{2}'$	8,73689
$mec = 27^{\circ}1\frac{1}{2}'$; its $\frac{1}{2} = 13^{\circ}30\frac{1}{2}'$			L, md	$0,06867$	8,83676

PRO.

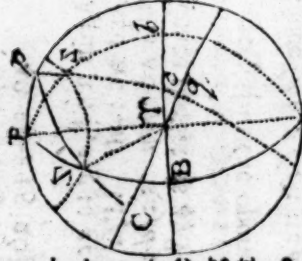
PROBLEM LV.

204. Given the obliquity of the ecliptic, the latitude of a place, and the apparent time at that place. To find the longitude of the nonagesime degree.

CONSTRUCTION.

The given time applied to the sun's right ascension, gives the right ascension of the unknown meridian, or mid-heaven.

Then let the primitive circle represent the solstitial colure, where P, p , represent the poles of the equator φB , and ecliptic φC , the center φ being their intersection. In the equator apply the right ascension of the unknown meridian from φ to B , and the circle PzB being described, is that meridian; the intersection of which by a parallel of latitude described about P , gives z , the place of the zenith; through z describe a circle of longitude pzc , and the point c is the nonagesime degree, and φc is longitude.



COMPUTATION.

In the triangle $p p z$. Given $p p$ the obliquity of the ecliptic.

$p z$ the co-latitude.

$$\begin{array}{rcl} \angle z p p & \text{the right as. of the mid-heaven.} & \\ \text{Required } z p & \text{---} & \text{(IV. 253)} \\ \angle p p z & & \text{(IV. 144)} \end{array}$$

DETERMINATION.

When the right ascension of the mid-heaven falls in the first quadrant, its quantity in degrees, increased by 90, gives the angle $z p p$, and the acute angle $p p z$ is the complement of the longitude of the nonagesime degree.

When in the second quadrant, the said right ascension in degrees taken from 270, leaves the angle $z p p$; and the angle acute $p p z$, increased by 90 degrees, is the longitude of the nonagesime degree.

When the said right ascension falls in the third quadrant; its degrees, taken from 270, leaves the angle $z p p$; and the obtuse angle $p p z$, increased by 90 degrees, is the longitude sought.

When in the fourth quadrant, the said right ascension in degrees, lessened by 270, leaves the angle $z p p$; and the supplement of the obtuse angle, $p p z$, added to three right angles, or 270 degrees, gives the longitude of the nonagesime degree.

EXAM-

EXAMPLE. At Greenwich, in latitude $51^{\circ} 28' N$. the obliquity of the ecliptic being $23^{\circ} 28'$: What is the longitude of the nonagesime degree on the 15th of May 1772, at 1 b. 25 m. P. M. at 3 b. 41 m. P. M. at 13 b. 25 m. P. M. and at 15 b. 41 m. P. M.?

The several right ascensions of the mid-heaven are thus found:

	h	m	h	m	h	m
1772 May 14th, at noon, the right asc. =	3	27	3	27	3	27
Time =	1	25	3	41	13	25
Right asc. of mid-heaven, in time =	4	52	7	8	16	52
Right asc. of mid-heaven, in degrees =	73	0	107	0	253	0
	90	0	270	0	270	0
Angle $z p p$ =	163	0	163	0	17	0

The two following operations are wrought by IV. 253.

$\frac{1}{2} \angle z p p = 81^{\circ} 30'$	$\left\{ \begin{array}{l} 9,99520 \\ 9,99520 \end{array} \right.$	Or, $8^{\circ} 30'$	$\left\{ \begin{array}{l} 9,16970 \\ 9,16970 \end{array} \right.$
$\frac{1}{2} p z = 38^{\circ} 31\frac{1}{2}'$	9,79436	—	9,79436
$\frac{1}{2} p p = 23^{\circ} 28'$	9,60012	—	9,60012
Sum of the four logs	39,38488	—	37,73388
Half sum, call arc x	19,69244		18,86694
$\frac{1}{2} p z - \frac{1}{2} p p = 7^{\circ} 31\frac{1}{2}'$	— 9,11730	Subtract	— 9,11730
$\frac{1}{2}$ arc, call it v	10,57514		9,74964
$\frac{1}{2}$ arc v	9,98528		9,69010
Log, arc x —log, arc v	9,70716		9,17684
Which gives $\frac{1}{2} z p = 30^{\circ} 38'$			
Hence $z p = 61^{\circ} 16'$			
		Here $\frac{1}{2} z p = 8^{\circ} 38'$	
		Hence $z p = 17^{\circ} 17'$	

Then $\frac{1}{2} z p = 61^{\circ} 16'$	0,05707	Or, $17^{\circ} 17'$	—	0,52710
To $\frac{1}{2} z p = 38^{\circ} 31\frac{1}{2}'$	9,79436	38	$31\frac{1}{2}$	9,79436
So $\frac{1}{2} \angle z p p = 163^{\circ} 0'$	9,46593	17	0	9,46593
To $\frac{1}{2} \angle z p p = 11^{\circ} 59'$	0,31736	142	12	9,78739

1st quadrant 78 01

Hence the longitude sought is $\left\{ \begin{array}{l} 2d \\ 3d \\ 4th \end{array} \right.$

101 59
232 12
307 48

SECTION VII.

Of Practical Astronomy.

205. DESCRIPTION and Use of ASTRONOMICAL INSTRUMENTS.

By PRACTICAL ASTRONOMY, is meant the knowledge of observing the celestial bodies with respect to their position, and time of the year; and of deducing, from those observations, certain conclusions useful in calculating the time when any proposed position of those bodies shall happen.

For this purpose, the Astronomer, or Observer, should have an observatory properly furnished.

An OBSERVATORY is a room, or place, conveniently situated, contrived, and furnished with proper astronomical instruments for observing the motions of the heavenly bodies: it should have an uninterrupted view, from the zenith, down to (or even below) the horizon, at least towards its cardinal points; and for this purpose, that part of the roof which lies in the direction of the meridian, in particular, should have moveable covers, which may be easily removed and put on again; whereby an instrument may be directed to any point of the heavens between the horizon and zenith, as well to the northward as southward.

The furniture should consist of some, if not of all, of the following instruments.

1st. A PENDULUM CLOCK, for shewing equal time.

2d. An Achromatic REFRACTING TELESCOPE, or a REFLECTING ONE, of at least two feet in length; for observing of particular phenomena.

3d. A MICROMETER for the measuring of small angular distances.

4th. An ASTRONOMICAL QUADRANT for the taking of the meridian altitudes of the celestial bodies.

5th. A TRANSIT INSTRUMENT, for observing of objects as they pass over the meridian.

6th. An ASTRONOMICAL SECTOR, to observe angular distances of several degrees, and the differences of right ascension and declination.

7th. An EQUAL ALTITUDE INSTRUMENT, for finding when an object has the same altitude on both sides of the meridian.

It is not intended to give in this work any other than a general account of these instruments, most of which have met with considerable improvements (if not contrived) by the late Mr. George Graham, F. R. S., one of the most eminent artists in mechanical contrivances that this, or any other, nation has produced: those readers who are curious to see a minute description of such, and other, instruments, together with their use fully exemplified, may consult the second volume of Dr. Smith's complete treatise of Optics, Stone's treatise of Mathematical Instruments, the Philosophical Transactions, and the works of many writers who have treated on such subjects.

206. *Of the Pendulum Clock.*

A clock which shews time in hours, minutes, and seconds, should be chosen; with which the observer, by hearing the beats of the pendulum may count them by his ear, while his eye is employed on the motion of the celestial object he is observing.

Just before the object arrives at the position desired, the Observer should look on the clock and remark the time; suppose it $9^h 15^m 25^s$; then saying 25, 26, 27, 28, &c. responsive to the beats of the pendulum, till he sees, through the instrument using, the object arrived at the position expected, which suppose to happen when he says 38; he then writes down $9^h 15^m 38^s$ for the time of observation, annexing the year and day of the month.

If two persons are concerned in taking the observation, one may read the time audibly, while the other observes through the instrument, the Observer repeating the last second read, when the desired position happens.

207. *Of the Telescope.*

THE REFRACTING TELESCOPE is an instrument almost every person is acquainted with, especially the marine gentlemen; and therefore it need here be only remarked, that an *astronomical telescope* has only two convex glasses; viz. the eye-glass used next to the eye; and at the other end the object-glass, which has much the longer focal distance: such an instrument, although it inverts all objects, is yet as useful for viewing those in the heavens, as if it shewed them erect; the Observer knowing that the motions are in an opposite direction to those he sees through this telescope: But the Achromatic Refracting Telescope, which is a new invention of Dollond's, has its object glass compounded of three glasses, and combined with two eye-glasses placed near each other. This instrument, which shews objects in their true position, need not exceed three feet in length.

THE REFLECTING TELESCOPE, as is generally well known, shews objects in their true positions; and as it is much shorter than the old reflecter, is therefore, by some, in greater esteem for use.

A *telescope*, used in astronomical observations, should have fixed in the focus of its object-glass a metal frame, carrying fine silver wires stretched at right angles to one another; one of them is to be vertical, and the other horizontal; the intersection of those wires to be exactly in the middle of the focus of the object-glass; a line passing through this intersection and the center of the object-glass, is called the line of sight, or line of *collimation*.

208.

Of the Micrometer.

A MICROMETER is an instrument used to measure small angular distances, by being placed in the focus of a telescope: This is effected by turning a screw, which moves a fine wire in a position parallel to itself, and also parallel to a fixed wire; both being in a plane at right angles to the line of collimation: the distance of these parallel wires is measured by the number of turns the screw has taken to cause their recess; which number of turns is shewn on a graduated circular plate (like that of a clock) by an index,

index, or hand, which revolves by the turning of the screw: now the divisions on the plate, answering to a known angle or arc intercepted between the parallel wires, being known by experiment, any other distance, to which the wires can recede, may be known by proportion; and so a table of angles answering to every division on the circular plate may be formed, by which the observed angles will be readily known.

Thus in observing the diameter of a planet; when the wires are removed so far asunder, as to become parallel tangents at the same time to opposite points of the planet, the measure of the recess of the wires will shew the diameter of the planet in minutes and seconds.

There is another micrometer published by the late very ingenious Mr. Dollond*, an account of which was given to the Royal Society by Mr. James Short, F. R. S. and published in the Philosophical Transactions for the year 1753, which is thus.

Let a good circular object-glass be neatly cut into two semicircles; and each semicircle fitted in a metal frame, so that their diameters sliding on one another (by the means of a screw,) may have their centers so brought together as to appear like one glass, and so form one image; or by their centers receding, may form two images of the same object: it being a property of such glasses, for any segment, to exhibit a perfect image of an object, although not so bright as the whole glass would give it.

Now proper scales being fitted to this instrument, to shew how far the centers recede, relative to the focal length of the glass, will also shew how far the two parts of the same object are asunder, relative to its distance from the object-glass; and consequently give the angle under which the distance of the parts of that object are seen.

209.

Of the Astronomical Quadrant.

An ASTRONOMICAL QUADRANT, is an instrument in the form of a quarter of a circle, contained under two radii at right angles to one another, and an arch equal to one fourth part of the circumference of the circle, and consists of the following parts.

1st *Its frame.* This is usually composed of iron or brass bars, set at right angles to one another in as strong and neat a manner as a workman can contrive, to preserve the face of the instrument in the same plane, and be as little affected by heat and cold as is possible.

2d. *Its center.* The center, which is a very fine point, should be contained in a separate piece of work screwed to the bars; and so contrived, that if the index, or telescope, by frequent motion in a length of time, should become irregular in its rotation, by the parts wearing, a new collar and socket may be fitted to the first center, and the instrument restored to its original accuracy.

* The first notion of such a micrometer was given by *Roemer*, a Dane, in the year 1675: *Mr. Savery*, an Englishman, also thought of such a contrivance, which he communicated to the Royal Society in the year 1743: *Mr. Bouguer*, a Frenchman, also proposed it in the year 1748; and *Mr. Dollond*, an Englishman, published it in the year 1753: but the public are obliged to *Mr. Short* for putting the theory into execution, otherwise it might still have continued only as an ingenious thought.

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Book V.

ASTRONOMY

3d. *Its limb.* This is a brass arch of about two inches broad, well fixed to the said frame work, and generally continued a little farther at each end than the extent of 90 degrees; the two perpendicular radii may be also covered with plates of brass screwed to them; and the whole face of the instrument is, with the greatest care, to be worked smooth, and brought into the same plane.

4th. *Its divisions.* The arcs of 60, 30, and 90 degrees, and also the intermediate degrees, together with such subdivisions as the size of the degrees will conveniently contain, are laid down by accurate methods well known to the good workmen.

5th. *Its index or telescope.* This, which is usually a brass tube containing the proper glasses and cross wires, is fixed near the object end to a brass plate, a little above a circular hole, or socket, in the plate: this socket goes round a collar concentric to the center, and fixed to the center-piece: so that, although the axis of the telescope does not, as a radius, pass through the center, yet it always keeps at the same distance from it in every position: to the eye-end of the tube is screwed a flat plate, which slides along the limb with the telescope; this plate, called the VERNIER, contains certain divisions, which used with those on the limb, give the angle to minutes or seconds of a degree, according to the size of the instrument: the beginning of the divisions called the index, on the VERNIER, is as far distant from the axis, or line of collimation, as the center is; and therefore the position of an object is given as truly, as if the line of collimation coincided with a radius.

6th. *Its pedestal.* This part, which should, by its construction, be very steady, may be either moveable or fixed: the moveable pedestal is commonly a strong pillar standing on a tripod, or three-footed stand; with holes through each foot, either to screw them to a floor, or to pin them to the ground: the fixed pedestal may be either a strong timber frame, or the wall of the observatory, or a stone shaft built from the ground through the middle of the floor of the observatory. On the top of the pillar, of either sort of pedestal, may be fixed a piece of machinery called the *arm*, which is attached by screws to the middle of the plane of the quadrant, on the under side. The arm is contrived to give to the instrument, either an horizontal, vertical, or oblique motion; which motions should be steady and free from jerks, or shakes: but when a wall, or stone shaft, is used as the supporter, the quadrant is then fixed to the wall, or shaft (without its arm attached,) and is called a MURAL ARCH; its plane is adjusted to that of the meridian, and is the best method of fixing the quadrant for taking the meridian altitude of the stars, or planets.

7th. *Its plummet.* This is a sufficient ball, or weight, hanging to one end of a very fine silver wire, the upper end being fixed in the radius continued above the center. Now when the face of the quadrant is to be set in the plane of an azimuth circle, one of its radii is brought into a vertical position by the help of the plummet, the wire being made to bisect the center-point and the division of 90° on the arch; and to distinguish these bisections with accuracy, they are to be examined with a small prospect, or magnifying-glass: the ball should hang freely in a vessel of water to check its vibrations.

210.

Of the transit Instrument.

This instrument is a telescope of about $2\frac{1}{2}$ feet long, fixed at right angles to an horizontal axis of about 15 inches long; which axis must be so supported, that the line of collimation may move in the plane of the meridian.

The axis, to the middle of which the telescope is fixed, should gradually taper toward its ends, and terminate in cylinders well turned and smoothed: and a proper balance is to be put on the tube, so that it may stand at any elevation when its axis rests on the supporters.

Two upright posts of wood or stone, firmly fixed at a proper distance, are to sustain the supporters to this instrument: these supporters are two thick brass plates, having well smoothed angular notches in their upper ends to receive the cylindrical arms of the axis: each of the notched plates are contrived to be moveable by a screw, which slides them upon the surfaces of two other plates immovably fixed to the two upright posts; one plate moving in a vertical, and the other in an horizontal, direction, adjust the telescope to the planes of the horizon and meridian: to the plane of the horizon, by a spirit level hung in a parallel position to the axis, and to the plane of the meridian in the following manner.

Observe the times by the clock when a circumpolar star, seen through this instrument, transits both above and below the pole: and if the times of describing the eastern and western parts of its circuit are equal, the telescope is then in the plane of the meridian; otherwise, the notched plates must be gently moved till the time of the star's revolution is bisected by both the upper and lower transits, taking care at the same time of the horizontality of the axis.

When the telescope is thus adjusted, a mark must be set at a considerable distance (the greater the better) in the horizontal direction of the intersection of the cross-wires, and in a place where it can be illuminated in the night-time by a lanthorn hanging near it; which mark being on a fixed object, will serve at all times afterwards to examine the position of the telescope, by first adjusting the transverse axis by the level.

211. *To adjust the Clock by the Sun's transit over the Meridian.*

Note the times by the clock, when the preceding and following edges of the sun's limb touch the cross wires: the difference between the middle time and 12 hours, shews how much the mean, or clock time, is faster or slower than the apparent, or solar, time for that day; to which the equation of time being applied, will shew the time of mean noon for that day, whereby the clock may be adjusted.

212.

Of the Astronomical Sector.

This is an instrument contrived for finding the difference in right ascension and declination between two objects, whose distance is too great

great to be observed through a fixed telescope, and consequently to apply a micrometer to; and consists of the following particulars.

1st. A brass plate, called a sector, formed like a T, having the shank (as a radius) of about $2\frac{1}{2}$ feet long, and 2 inches broad, and the cross piece (as an arch) of about 6 inches long and $1\frac{1}{2}$ inch broad; upon which with a radius of 30 inches, is described an arch of 10 degrees, each being subdivided into as small parts as are convenient.

2d. Round a small cylinder, containing the center of this arch, and fixed in the shank, moves a plate of brass, to which is fixed a telescope, having its line of collimation parallel to the plane of the sector, and passing over the center of the arch and the index of a *Vernier's* dividing plate, fixed to the eye end of the telescope, to slide along the arch; which motion is performed by a long screw at the back of the arch, communicating with the Vernier through a slit cut in the brass, parallel to the divided arc.

3d. A circular brass plate of 5 inches diameter, round whose center moves a brass cross, having the opposite ends of one bar turned up perpendicularly about 3 inches, to serve as supporters to the sector, and screwed to the back of its radius; so that the plane of the sector is parallel to the plane of the circular plate, and can revolve round the center of that plate in this parallel position.

4th. An axis of 18 inches long is screwed flat to the back of the circular plate along one of its diameters, so that the axis is parallel to the plane of the sector: the whole instrument is supported on a proper pedestal, so that the said axis shall be parallel to the earth's axis, and proper contrivances annexed to fix it in any position.

Now the instrument thus supported, can revolve round its axis parallel to the earth's axis, with a motion like that of the stars, the plane of the sector being always parallel to the plane of some hour circle, and consequently every point of the telescope describing a parallel of declination: and if the sector be turned round the joint of the circular plate, its graduated arch may be brought parallel to an hour circle; and consequently any two stars, whose difference of declination does not exceed the degrees in that arch, will pass over it.

213. *To observe their passage.* Direct the telescope to the preceding star, and fix the plane of the sector a little to the westward of it; move the telescope by the screw, and observe, at the transit of each over the cross wires, the time shewn by the clock, and also the division shewn by the index; then is the difference of the arches the difference of declination; and that of the times shews the difference of right ascension of those stars.

214. Of the Equal-Altitude Instrument.

AN EQUAL-ALTITUDE INSTRUMENT is that used to observe a celestial object, when it has the same altitude on both the east and west sides of the meridian, or in the morning and afternoon; and consists of a telescope of about 30 inches long (with two vertical, and 3 or 5 horizontal, wires in its focus,) supported on the end of an iron bar, or axis, of 30 inches long, and about an inch in diameter; the axis being sustained in

in a vertical position by passing through a hole in one end of a brass box, whose other, or lower, end sustains the lower point of the axis. The box, which is about 21 inches long, with ends about 4 inches square, has only two sides, which sides are fixed at right angles: From one of these sides jut out four flat arms, with a hole in each, whereby the box is, by screws, fixed in a vertical position to an upright post. On the lower end of the box lies a brass plate which slides in grooves, and can, by means of a screw, be gently moved forwards or backwards; in this plate is a fine punched hole to receive the smooth conical point, into which the lower end of the axis is formed. On the upper end of the box are two plates which slide also in grooves; and, by the means of screws, can be gently moved sideways, till their angular notches embrace the axis; which in this part, is made perfectly cylindrical, and very smooth.

To the upper part of the axis is fixed, by its radius, a brass sextant (or arch of 60° , to a radius of seven or eight inches,) with the arch downwards, so that the center is just above the top of the axis: also a spirit level is fixed at right angles across the axis, just under the arch, so as to be clear of the upper end of the box.

To the under part of the telescope is fixed a brass semicircle, of the same radius with the sextant, both arches having a common center-pin. In the semicircle is a groove cut through the plate parallel to its limb, to receive two screw-pins, which go into the sextantal arch near its ends; by these screw-pins the two arches may be pressed close, and the telescope fixed in any desired elevation; which might be nearly ascertained, by graduating the semicircle, and putting a Vernier's scale on the sextant.

To use the Instrument. Fix the box to the post, put the axis into the box, letting the conical point drop into the punched hole, screw on the level, and annex the telescope, observing to insert the center and arch pins; then, by the help of the screw plates at the bottom and top ends of the box, correct the vertical position of the axis, so that the same end of the air-bubble in the level may stand at the same point throughout the whole revolution of the axis, which will thereby be known to be then truly vertical, so that the telescope will describe a parallel of altitude; direct the tube to the sun, or star, and fix it at the desired elevation, by pressing the two arches together with the two screw pins.

Some instruments have been contrived to answer both kinds of observation; viz. either a transit, or equal altitudes.

215. To adjust the Clock by equal altitudes of the Sun.

Having rectified the instrument by the level, and being provided with a piece of transparently coloured or smoked glass to preserve the eye; then about 6 or 5 or 4 or 3 hours before noon, direct the telescope to the sun, and fix it by the arch, so that the whole body of the sun shall be above the upper wire (the ascent of the sun appearing through the telescope as a descent:) mark the times shewn by the clock, when the preceding edge of the sun touches each of the wires; and also when the following edge

edge touches those wires, writing down those times; the instrument being turned horizontally on its axis to follow the Sun, and keep his center in the middle of the telescope, between the vertical wires. About the same time after noon (taking care to be early enough), turn the instrument on its vertical axis, the telescope remaining fixed at the same elevation, as used in the morning, and rectifying its horizontality by the level, observe the Sun in its descent, which through the telescope apparently ascends, and write down the times, when the preceding edge touches each wire, and also when they are touched by the following edge, keeping the Sun in the middle of the telescope; and the sets of observations are made.

There are as many sets of observations, as there are horizontal wires: for the fore and afternoon contacts of the same edge of the Sun with the same wire, make one set; and the same edge which precedes in the forenoon, follows in the afternoon; and that which follows in the forenoon, precedes in the afternoon; therefore the

1st, $\left\{ \begin{array}{l} \text{A. M. preceding, and the} \\ \text{2d, \&c.} \end{array} \right\}$ last, $\left\{ \begin{array}{l} \text{last but one,} \\ \text{last but 2, \&c.} \end{array} \right\}$ P. M. following,
make a set.

1st, $\left\{ \begin{array}{l} \text{A. M. following, and the} \\ \text{2d, \&c.} \end{array} \right\}$ last, $\left\{ \begin{array}{l} \text{last but one,} \\ \text{last but 2, \&c.} \end{array} \right\}$ P. M. preceding,
make a set.

Then to each set, or pair of observations, find the middle time, which added to the time of the morning observation, gives the time shewn by the clock when the Sun was on the meridian, if the observations were made within two or three days of a solstice, when the Sun's declination would not sensibly alter between the fore and afternoon observations: but on other days, this time must be corrected, by applying to it an equation, shewing the alteration in time, arising from the alteration in declination between the fore and afternoon observations.

The time by the clock of the solar, or apparent, noon being thus obtained, the time of the mean noon may be had, by applying the proper equation of time.

When the time of noon is sought from two or more pairs of observations; if they give different times, it is best to take the medium between them, which is found by dividing the sum of all the times by their number.

216.

PROBLEM LV.

Given the latitude, the declination of the Sun, and interval of time between the Sun's having equal altitudes before and after noon, to find the distance from noon of the middle point of time between the observations.

1st. Find the change made in the Sun's declination during the interval between the observations; which will nearly bear the same proportion

T 2

portion to the change made between the noon of the day, whereon the observations are made, and the noon of the day immediately preceding or following, as the interval of time between the observations to 24 hours.

2dly. Add the co-tangent of the latitude to the co-sine of half the interval of time reduced to degrees and minutes of the equator, the sum, rejecting the radius, is the tangent of an arc to be taken less than a quadrant, when the interval of time is less than twelve hours, and greater than a quadrant, should the interval of time exceed twelve hours.

3dly. Add together the arithmetical complements of the sine of this arc, and of the sine of the Sun's distance from the pole at noon, the logarithmic sine of the difference of these arcs, the logarithmic co-tangent of half the interval of time in degrees, and the logarithm of half the change in the Sun's declination during the interval between the observations; the sum, rejecting twice the radius, is the logarithm of an arc, which divided by 15 gives in seconds of time the distance of the middle point between the observations from noon.

4thly. When the Sun's distance from the elevated pole increases, this middle point of time precedes noon, otherwise it falls beyond.

DEMONSTRATION.

Let P be the pole of the equator bd , Z the zenith, A the morning place, c the afternoon place, AND a parallel of declination, ABC a parallel of altitude.

Then the afternoon hour angle ZPC differs from the morning hour angle ZPA , by the hour angle BPC , the points A and B having the same declination and distance from the zenith; and the arc PO bisecting the $\angle BPC$, the $\angle ZPO$ will be half the interval of time, which being increased or diminished by half the $\angle BPC$, will give the position of the meridian to A or C ; also PO will be the Sun's distance from the pole at noon.

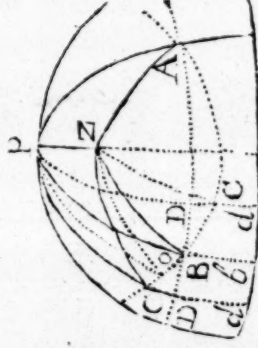
Now here the difference between PB , PO , PC being but small, the $\angle BPO$ will be to the difference between PO and PB , or PC , that is, half $PB \propto PC$, nearly as t, POZ to s, PO .*

Again, in the triangle OPZ , an arc M being taken, that as rad. : s, ZPO :: t, PZ : t, M the arc M to be less than a quadrant, when the $\angle ZPO$ is acute, but greater, should the $\angle ZPO$ be obtuse; then (IV. 123)

$$\frac{s, PO \propto M}{s, PO \propto M \times ZPO} = \frac{t, POZ}{s, M \times t, POZ} \text{ and consequently}$$

$$\text{But } \angle BPO : \frac{PB \propto PC}{2} :: t, POZ : s, PO :: s, M \times t, POZ : s, M \times s, PO;$$

therefore $\angle BPO : \frac{PB \propto PC}{2} :: s, PO \propto M \times t, ZPO : s, M \times s, PO$, conformably to the rule above laid down.



* See Case/. *Æstimat. Error. in Mixt. Math. Theorem. 23.*

217.

EXAMPLE.

In the latitude 50° N. on the 27th of October, 1761, the Sun was observed to have equal altitudes at 9 h. 11 m. 50 f. A. M. and at 2 h. 22 m. 22 f. P. M. by a clock adjusted nearly to the true measure of time, to find what correction may be wanted to set this clock to the true hour of the day, the Sun's distance from the pole on the 27th day at noon being $102^{\circ} 58' 52''$, and on the 28th $103^{\circ} 19' 0'' = 1^{\circ} 20' 11''$ *S. Dielm. 13. q. o.*

Here the interval of time is 5 h. 10 m. 32 f. its half is 2 h. 35 m. 16 f. in degrees and minutes of the equator $38^{\circ} 49'$, and the difference in declination in one day is $20' 8''$.

Then $24 \text{ h.} : 5 \text{ h.} :: 10 \text{ m.} 32 \text{ f.} : 20' 8'' : 4' 20'' = 260''$, the alteration in declination, the half of which is $2' 10'' = 130''$.

Then

Latitude 50°		log. $t, 9, 92381$	$33^{\circ} 10' 3''$	$L, s,$	$0, 26:85$
$\frac{1}{2}$ time $= 38^{\circ} 49'$		log. $s, 9, 89162$	$102 \ 58 \ 52$	$L, s,$	$0, 01:3$
			$69 \ 48 \ 20$	log. $s,$	$9, 97245$
$33^{\circ} 10' 32''$	log. $t,$	$9, 8:543$	$38 \ 49 \ 0$	log. $t,$	$10, 09447$
			$130''$	log.	$2, 11394$
		corr. $= 284\frac{1}{2}''$		log.	$2, 45394$
		or 19 sec. of time.			

Hence, for setting the clock to the true hour of the day, add the half interval of time 2 h. 35 m. 16 f. to 9 h. 11 m. 50 f., the sum 11 h. 47 m. 6 f. is the middle time between the observations, as noted by the clock:

And 11 h. 47 m. 6 f. + 19 f. = 11 h. 47 m. 25 f. will be the time pointed out by the clock, when the Sun passes the meridian, and shews the clock to be 12 m. 35 f. behind the Sun.

Though the clock should not keep time with perfect exactness, yet if the deviation is but small, the correction computed will not much differ from the truth; and the clock being again examined within a few days, will shew whether it keeps time truly, or moves too fast or too slow, whereby its going may be corrected accordingly.

218. However, the method here directed supposes the ship to be stationary: But the Abbé de la Caille proposes a method of correcting a watch at sea, even while the ship is in motion, by taking with a quadrant two equal altitudes of the Sun, one before and the other after noon.

His method is this. With the common altitude observed, together with the latitudes at the time of each observation, and the Sun's correct declination to those times, the times from noon are to be computed at both observations, which times being applied to the times of observation, give the respective times of noon: Then the mean of the two noons being taken, will give the time shewn by the watch when it was the true mid-day.

Or. Half the difference of the computed noons being applied to either of them, will also give the true time of noon.

And although the latitudes used in the computations be something erroneous, yet the altitudes being equal, the error in each of the computed distances from noon will be nearly the same, if the change in latitude and longitude between the observations be duly attended to.

T 3

219. Of

219. *Of the Vernier dividing plate.*

When the relative unit of any line is to be divided into many small equal parts, those parts may be too many to be conveniently introduced, or if introduced, they may be too close to one another to be readily estimated; and on these accounts there have been a variety of methods contrived for estimating the aliquot parts of the small divisions, into which the relative unit of a line may be commodiously divided: among those methods, that is most justly preferred which was published by PETER VERNIER (a gentleman of Franche Comté) at Brussels, in the year 1631; and which, by some strange fatality, is most unjustly, although commonly, called by the name of NONIUS: for *Nonius's* method is not only very different from that of *Vernier's*, but much less convenient.

Vernier's method is derived from the following principle.

If two equal right lines, or circular arcs A, B, are so divided, that the number of equal divisions in B is one less than the number of equal divisions of A; then will the excess of one division of B above one division of A be compounded of the ratios of one of A to A, and of one of B to B.

For let A contain 11 parts; then one of A to A, is as 1 to 11; or $\frac{1}{11}$.

B contain 10 parts; then one of B to B, is as 1 to 10; or $\frac{1}{10}$.

$$\text{Now } \frac{1}{10} - \frac{1}{11} = \left(\frac{1 \times 11}{10 \times 11} - \frac{1 \times 10}{11 \times 10} \right) \text{ (II. 148) } = \frac{11 - 10}{10 \times 11} = \frac{1}{10 \times 11} = \frac{1}{110}.$$

Or. If B contains n parts, and A is of $n+1$ parts;

Then $\frac{1}{n}$ is one part of B, and $\frac{1}{n+1}$ is one part of A.

$$\text{And } \frac{1}{n} - \frac{1}{n+1} = \left(\frac{1 \times \overline{n+1}}{n \times \overline{n+1}} - \frac{1 \times n}{\overline{n+1} \times n} \right) \text{ (II. 148) } = \frac{n+1-n}{n \times \overline{n+1}} = \frac{1}{n \times \overline{n+1}} = \frac{1}{n} \times \frac{1}{\overline{n+1}}.$$

Or th's. Let A and B be unequal right lines, or circular arcs; and let any part of A, considered as the relative unit, be divided into n parts; and a part of B, equal to $m+1$ parts of A, be divided into m parts: then will $\frac{1}{m}$ th of B = $\frac{1}{n}$ th of A = $\frac{1}{m}$ th of B $\times \frac{1}{n}$ th of A.

But n parts of A : 1 unit of A :: $m+1$ parts of A : $\frac{m+1}{n}$ units of A.

But m parts of B = $(m+1)$ parts of A = $\frac{m+1}{n}$ units of A.

Then m parts of B : $\frac{m+1}{n}$ units of A :: 1 parts of B : $\frac{m+1}{m \times n}$ units of A.

Therefore

$$\text{Therefore } \frac{m+1}{m \times n} - \frac{1}{n} = \left(\frac{m+1}{m \times n} - \frac{1 \times m}{n \times m} = \frac{m+1-m}{m \times n} = \frac{1}{n \times m} \right)$$

$$\frac{1}{m} \times \frac{1}{n}.$$

The most commodious divisions, and their aliquot parts, into which the degrees on the circular limb of an instrument may be supposed to be divided, depend on the radius of that instrument.

Let R be the radius of a circle in inches; and a degree to be divided into n parts, each being $\frac{1}{p}$ -th of an inch.

Now the circumference of a circle in parts of its diameter $2R$ inches, is $3,1415926 \times 2R$ inches.

(II. 197)

Then $360^\circ : 3,1415926 \times 2R :: 1^\circ : \frac{3,1415926}{360} \times 2R$ inches.

Or, $0,01745329 \times R$ is the length of one degree, in inches.

Or, $0,01745329 \times R \times p$ is the length of 1° , in p th parts of an inch.

But as every degree contains n times such parts,

Therefore $n = 0,01745329 \times R \times p$.

The most commodious perceptible division is $\frac{1}{8}$ or $\frac{1}{10}$ of an inch.

EXAM. Suppose an instrument of 30 inches radius: into how many convenient parts may each degree be divided? how many of those parts are to go to the breadth of the Vernier, and to what parts of a degree may an observation be made by that instrument?

Now $0,01745 \times R = 0,5236$ inches, the length of each degree.

And if p be supposed about $\frac{1}{8}$ of an inch for one division.

Then $0,5236 \times p = 4,188$, shews the number of such parts in a degree.

But as this number must be an integer, let it be 4, each being $15'$.

And let the breadth of the Vernier contain 31 of those parts, or $7\frac{1}{2}$, and be divided into 30 parts.

Here $n = \frac{1}{4}$; $m = \frac{1}{30}$; then $\frac{1}{4} \times \frac{1}{30} = \frac{1}{120}$ of a degree, or $30''$,

Which is the least part of a degree that instrument can shew.

If $n = \frac{1}{5}$, and $m = \frac{1}{36}$; then $\frac{1}{5} \times \frac{1}{36} = \frac{60}{5 \times 36}$ of a minute, or $20''$.

220. The following table, taken as examples in the instruments commonly made from 3 inches to 8 feet radius, shews the divisions of the limb to nearest tenths of inches, so as to be an aliquot of 60's, and what parts of a degree may be estimated by the Vernier, it being divided into such equal parts, and containing such degrees, as their columns shew.

T 4 Rad.

Rad. inches	Parts in Breadth		Parts		Rad. inches	Parts in Breadth		Parts	
	a deg.	Vernier.	of Ver.	observed.		a deg.	Vernier.	of Ver.	observed
3	1	15	$15\frac{10}{12}$	4' 0"	30	5	30	$7\frac{10}{12}$	0' 20"
6	1	20	$20\frac{1}{4}$	3 0	36	6	30	$5\frac{1}{4}$	0 20
9	2	20	$10\frac{1}{4}$	1 30	42	8	30	$3\frac{1}{2}$	0 15
12	2	24	$12\frac{1}{2}$	1 15	48	9	40	$4\frac{5}{8}$	0 10
15	3	20	$6\frac{1}{2}$	1 0	60	10	36	$3\frac{7}{16}$	0 10
18	3	30	$10\frac{1}{4}$	0 40	72	12	30	$2\frac{7}{12}$	0 10
21	4	30	$7\frac{1}{4}$	0 30	84	15	40	$2\frac{2}{3}$	0 6
24	4	36	9	0 25	96	15	60	4	0 4

By altering the number of divisions, either in the degrees or in the Vernier, or in both, an angle can be observed to a different degree of accuracy. Thus, to a radius of 30 inches, if a degree be divided into 12 parts, each being five minutes, and the breadth of the Vernier be 21 such parts, or $1\frac{10}{12}$, and divided into 20 parts, then $\frac{1}{12} \times \frac{1}{20} = \frac{1}{240} = 15''$: or taking the breadth of the Vernier of $2\frac{1}{12}$, and divided into 30 parts; then $\frac{1}{12} \times \frac{1}{30} = \frac{1}{360}$, or $10''$: Or $\frac{1}{12} \times \frac{1}{50} = \frac{1}{600} = 6''$; where the breadth of the Vernier is $4\frac{1}{4}$.

SECTION VIII.

Practical Astronomy.

The ELEMENTS of the EARTH'S MOTION.

221. By the theory of the Sun, or Earth, is meant the knowledge of all the requisites, or elements, necessary for determining his place in the ecliptic at any proposed time.

222. MEAN MOTION, or MEAN ANGULAR VELOCITY, is a motion made uniformly in the circumference of a circle, the center of motion being the center of that circle.

The mean motion of a planet is the degree and parts shewing its distance from the first point of Aries, reckoned in the order of the signs.

223. ANOMALY, or TRUE ANOMALY, is an angle made by two lines drawn from the center of motion, one to the Aphelion, or Apogee, and the other to the place of the revolving body, or planet: Or ANOMALY, is the angular distance of a planet from its Aphelion, the angular point being the center of motion.

224. MEAN ANOMALY, is that made by an uniform circular motion about the center, and is the same as mean motion, beginning at the Aphelion.

225. Ex.

225. **EXCENTRIC ANOMALY**, is an angular distance from the aphelion, determined in a circle on the transverse axis, by a normal to that axis, passing through the planet's place in its elliptical orbit.

226. The **EQUATION OF THE CENTER**, sometimes called the *prosthaphæresis*, is the difference between the mean and true anomalies.

227. The *motion of the equinoxes*, is the same as the *precession of the equinoxes*, which is backwards, or contrary to the order of the signs; whereby the stars appear to have advanced forwards, from the equinoctial point Aries: this motion, is about 50 seconds of a degree in a year.

228. The *motion of the apsides*, is a slow motion of the Earth's orbit around the sun, in the order of the signs: discovered by the apogee point changing its place among the zodiacal constellations: this motion is found, by comparing distant observations together, to be about 16 seconds of a degree in a year, in respect to the fixed stars; and about 66 seconds ($= 50' + 16''$) with respect to the equinoxes.

229. A **TROPICAL or SOLAR YEAR**, is the time elapsed between two successive passages of the Sun through the same Equinoctial or Solstitial points of the ecliptic.

230. A **SYDERIAL YEAR**, is the time the Sun takes between his departure from any fixed star to his next return to that star.

231. An **ANOMALISTIC YEAR**, is the interval of time between two succeeding passages of the Sun through the same apside.

232. By the annexed figure, the foregoing articles may be easily comprehended.

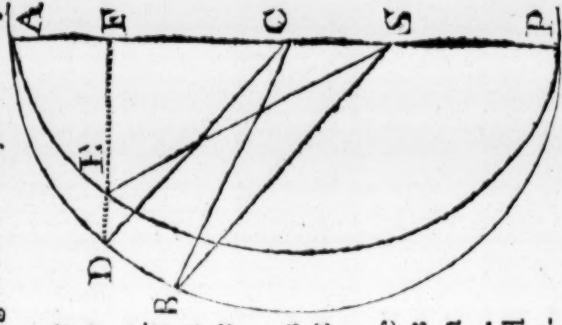
On the line of the apsides AP describe a circle ADP, called the excentric; and an ellipsis AEP for the Earth's orbit, having the eccentricity CS.

Let s be the place of the Sun, c the center of the orbit, A the aphelion, P the perihelion; SA the aphelion, or apogeeon, distance; SP the perigeon distance.

Let E be a true place of the earth in its orbit; D a corresponding place in the excentric, in FE continued, normal to AP.

Let $\angle ACB$ represent the mean anomaly; the $\angle ACD$ is the excentric anomaly; and the $\angle ASB$ is the true anomaly; the difference between $\angle ACB$ and $\angle ASE$ is the equation of the center.

When the Earth is in the apsides, then B and E fall together in A and P, and here is no equation of the center, the mean and true anomalies being equal; but the greatest equation of the center must be, when the Earth is at its mean distance from the Sun.



233. Observations shew, that in this age the Earth passes the apogee on the 30th of June, when its daily motion is $57' 12''$: and passes the perigee on the 30th of December, when the daily motion is $61' 12''$: and is at the mean distance about the 28th of March and 30th of September, when its daily motion is $59' 8''$.

237. REMARK. By the second method may the declinations of the fixed stars, or of any other celestial phenomena, be found: observing that their declination is of the same name, viz. north or south, with the latitude of the place, when its complement is less than the altitude; otherwise, of a contrary name with the latitude.

238.
PROBLEM LVIII.

To find the time of an Equinox.

SOLUTION. In a place whose latitude is known, let the Sun's meridian altitude be taken on the day of the equinox, and on the day preceding, and that following it. Then the difference between those altitudes and the co-latitude will be the Sun's declinations at the times of observations. (236)

If either of the altitudes is equal to the co-latitude, that observation was made at the time of the equinox.

But if the co-latitude is unequal to either of the altitudes, proceed thus. Let DG represent the equator; AC the ecliptic, E the equinoctial point; the points A, B, C , the places of the Sun at the times of observation; the arcs AD, BE, CG , the corresponding declinations.



Now using either the two first, or two last observations, suppose the latter, in the  A B right-angled spherical triangles CEG , BEF , wherein are known the obliquity of the ecliptic, and the declinations: find EC , EB : then BC , the sum or difference of EC , EB , is the ecliptic arc described in 24 hours. Then say,

As BC to BE, so 24 hours for BC, to the time for BE.
And this time shews the distance of the equinox from the time of the middle observation.

239. PROBLEM LIX.

To find the length of the tropical, periodical and anomalistical revolutions of the Earth.

SOLUTION. Let two observations be chosen, among the most authentic of those on record, of the time when the Sun had like positions, viz.

- 1st. In regard to his longitude, or place in the ecliptic.
2d. In regard to the right ascension of some noted star.
2d. In respect to the line of the apsidæ.

The greater the interval (suppose 80 or 100 years) between each two observations, the more accurate will be the result: then that interval being divided by the number of revolutions made during that time, will give the time of one periodical revolution.

According to Mayer's tables the numbers are these

A tropical year	is made in	365 ^d	5 ^h	48 ^m	42 ^s
A periodical, or syderial revolution		365	6	9	7.
An anomalistic revolution		365	6	15	29 ¹ / ₂
					240. RE.

240. REMARKS. 1. The tropical year being shorter than the syderial by 20 m. 25 f., shews that the Sun has returned to the same point of the ecliptic before he has made one complete revolution with regard to the stars; and consequently every point of the ecliptic must have moved in antecessentia during that tropical period, and so have produced what is called the precession of the equinoxes.

Now 365 d. 6 h. 9 m. 7 f. : 360° :: 20 m. 25 s. : $50''$, 3, or nearly $50''$, for the precession in one year.

If there was no precession, the tropical and syderial years would be equal.

241. 2d. A syderial revolution being performed sooner by 6 m. 22 f. than the anomalistic, shews that the line of the apsidæ has a motion in consequentia : now 365 d. 15 h. 29 m. : 360° :: 6 m. 22 f. : $15''$, 7, the yearly quantity by which the Sun's apogee is advanced in respect to the stars : and as the equinoxes move in antecessentia, and the apsidæ in consequentia, their sum $66''$ ($= 50, 3 + 15, 7$) shews the motion of the apsidæ from the equinoxes.

242. 3d. From the comparison of many observations, it appears that the length of the solar year, deduced from two very distant observations made at the time when the Sun was in the same point of the ecliptic near its apogee, differs by many seconds from the length of the year deduced by like observations, when the Sun was in another part of the ecliptic, near its perigee : those made near the apogee giving the revolutions less, and those made near the perigee making them greater, than the revolutions deduced from observations taken at the Sun's mean distance : this also shews, that the line of the apsidæ has a motion in consequentia ; and that the length of a tropical revolution should be determined from very distant observations, made at the times when the Sun is at its mean distance from the Earth ; or that the mean revolution should be taken between those deduced from observations made on the Sun's place, when he is in both the apogee and perigee.

243. PROBLEM LX.

To find the right ascension of some noted fixed star.

Having a good clock well regulated to mean or equal time, a large astronomical quadrant fixed in the plane of the meridian, and an equal altitude or transit instrument : then, on some day a little before or after the vernal equinox, when the daily alteration of the Sun's declination is about 18 or 20 minutes, observe the Sun's meridian altitude ; and by equal altitudes, find the times when both the Sun and star come to the meridian ; the difference of these times, is their difference of right ascension.

Again. At some time a little after or before the autumnal equinox, before the Sun has passed the said declination, observe his meridian altitude ; and by equal altitudes find the times of the Sun and same star's coming to the meridian, the difference of those times is also the difference of their right ascensions.

If the vernal and autumnal meridian altitudes are the same, then those observations were made when the Sun was on the same parallel of

of declination: now the sum, or diff. of the two observed differences of right ascension, shews the equatorial arc described by the Sun between those times; which arc bisected, shews the distance of the nearest solstice from the Sun at the time when the observations have equal altitudes; and that distance corrected and taken from 90° , shews the Sun's right ascension at the vernal observation, or its complement to 360 degrees.

From hence, and the first difference of right ascension between the Sun and star, the star's right ascension will be obtained.

224. If the two meridian altitudes of the Sun are not the same, their difference shews the difference of the mid-day declinations, when those observations were taken: now from some tables of right ascension and declination, take the Sun's daily alteration in declination and right ascension on the day the lesser altitude was taken; then say, *As the daily change of decl. is to that of right ascen. ; so is the diff. of the altitudes to the correction in right ascension.*

This correction added to the vernal, or subtracted from the autumnal difference of right ascension, according to which is least, fits that difference of right ascension to the same declination with the other; and then the difference between those two differences of right ascension, fitted to the same declination, shews the equatorial arc, as before recited.

245. At the Royal Observatory at Greenwich, in the year 1770, observations were made on the Sun and the star α Aquilæ.

March 15, Sun's mer. zen. dist. cleared of refraction and parallax, was $53^{\circ} 28' 29''$; and their diff. of rt. asc. was $60^{\circ} 30' 7,8''$.
 Sep. 28th. Sun's mer. zen. dist. cleared of refraction and parallax, was $53^{\circ} 36' 26''$; and their diff. of rt. asc. was $109^{\circ} 59' 22,8''$.
 Then $7' 57''$ is the diff. of zen. dists. or the alteration in declination.
 Also $23' 40''$ and ($3^m 39^s$ or) $54' 45''$ are the dists. of decl. and rt. asc. between the 15th and 16th days of March 1770.

Now $23' 40'' : 7' 57'' :: 54' 45'' : 18' 23,5''$ the increase of the difference of right ascension after the noon of the 15th of March.

Then $60^{\circ} 30' 07,8'' - 18' 23,5'' = 60^{\circ} 11' 44,3''$ which is the first diff. rt. asc. when the Sun had the same decl. as at the second observation.

Here, the times of the two observations fall nearest the winter solstice.

Then $60^{\circ} 11' 44,3'' + 109^{\circ} 59' 22,8'' = 170^{\circ} 11' 7''$; its half $85^{\circ} 5' 33,5''$ is the distance of the winter solstice from the Sun.

Hence $270^{\circ} + 85^{\circ} 5' 33,5'' + 18' 23,5'' = 355^{\circ} 23' 57''$ is the Sun's rt. asc. on March 15.

Also $355^{\circ} 23' 57'' - 60^{\circ} 30' 7,8'' = 294^{\circ} 53' 49,2''$ is the rt. asc. of α Aquilæ.

246. The right ascension of one star being known, the right ascensions of all the rest are found, by noting the times shewn by the clock, when those stars come to the meridian: for the differences of those times, from the transit of the chosen star, are the differences of right ascension; by which, the right ascension of all the observed stars will be known; taking care to augment, or diminish the right ascension of the chosen star by those differences, according as the chosen star is preceded, or followed by the other observed stars.

247.

PROBLEM LXL.

To find the Sun's place.

SOLUTION. Let the times be observed both when the Sun, and a star whose right ascension is known, pass the meridian, and hence the Sun's right ascension is known.

With that right ascension, and the obliquity of the ecliptic, compute (142) the longitude, and thus his place in the ecliptic will be known.

248.

PROBLEM LXII.

To find the greatest Equation of the Center.

SOLUTION. At the times when the Sun is near his mean distances, let his longitude be found; their difference will shew the true motion for that interval of time.

Find also the Sun's mean motion for that interval of time.

Then half the difference between the true and mean motions will shew the greatest equation of the center.

Observations made at the Royal Observatory at Greenwich, shews that 1769 October 1st. at $23^h 49^m 12^s$ mean time, $\odot$ long was $6^\circ 9' 32'' 0,6''$
1770 March 29th. at $0^h 4^m 50^s$ mean time, $\odot$ long. was $0^h 8^m 50^s 27,5''$

The diff of time $178d. 0^h 15^m 38^s$; True diff. long. $5^h 29^m 18^s 27''$
The tropical year $= 365d. 5^h. 48^m. 42^s. f. = 365,2421527$
The observed interval $= 178^d. 0^h 15^m 38^s = 178,01085648$.
Then $365,2421527 : 178,01085648 :: 360^\circ : 175,455948$ mean motion.
So $175^\circ 27' 21''$ of mean motion, answers to $179^\circ 18' 27''$ true motion.
Their diff. $= 3^\circ 51' 6''$; its half $1^\circ 55' 33''$ is the greatest equation of the center according to these observations.

249.

PROBLEM LXIII.

To find the Eccentricity of the Earth's orbit.

SOLUTION. Say, As the diameter of a circle in degrees,

To the diameter in equal parts;

So the greatest equation of the center in degrees,

To the eccentricity in equal parts.

The greatest equation of the center $1^\circ 55' 33'' = 1^\circ,9268166$ &c.

The diam. of a circle being 1, its circumf. is $3,1415926$. (II. 197)

Then $3,1415926 : 1 :: 360^\circ : 114^\circ,591599$ equal to the diameter.

And $114,591599 : 1,00$ &c. $:: 1,9268166 : 0,0168061$ the eccentricity.

Hence $1,016806 (= 1,000000 + 0,016806) =$ aphelion distance.

And $0,983194 (= 1,000000 - 0,016806) =$ perihelion distance.

250. PROBLEM LXIV.

To find the time and place of the Sun's Apogee.

SOLUTION. On each day of two successive apsidæ, let the Sun's place and the time be observed.

Then if the interval of those times and places are equal to the halves of 365d. 6h. 15m. 29s. and $360^{\circ} 1' 6''$; those observations were made when the Sun was in the apsidæ.

For such intervals of time and place belong to no other points of the Earth's orbit.

But if those observed intervals of time and place differ from the said halves, take the difference between the interval of place and $180^{\circ} 0' 33''$.

Then to the apogee daily motion (233), the said diff. and 24h. find the proportional time; which proportional time and difference, being applied to the time, and place, of the apogee observation, gives a time and place when it is $180^{\circ} 0' 33''$ distant from the observed perigee place: now if the interval of these times is equal to 182d. 15h. 7m. $44\frac{1}{2}$ the times and places of the apsidæ are known.

But if the interval of time differs from 182d. 15h. 7m. $44\frac{1}{2}$ f, say, *As the diff. between the perigee and the apogee daily motions, to the daily motion in apogee; so the diff. of the interval of time, to a second correction of the apogee time.*

This correction applied to the before corrected apogee time, will give the true time of the Sun's apogee.

Also, to the last correction of time, find the proportional motion of the Sun in apogee; and apply it to the last corrected place of the apogee, and the true place of the apogee will be obtained.

By observations made at the Royal observatory at Greenwich in the year 1769.

July 1st. at $0^h 3^m 20^s$ mean time $\odot$ long. $= 3^{\circ} 9' 46''$ 38,5

December 29th. at $0 2 49$ mean time $\odot$ long. $= 9 8 10$ 58,1

Interval 180d. 23 59 29. Interval of place $= 5 28 24$ 19,6

The Sun's motion in half an anomalistic year $\frac{6 0 0}{33}$

The Sun's place at 1st observation is too forward by $1 36 13.4$

Then $57' 12'' : 1^{\circ} 36' 13.4 : 24h : 40h$ 22m. 24f. to be taken from the time of July 1st, to make the distance of the times answer to the half $360^{\circ} 1' 6''$; and it leaves June 29d. 7h. 40m. 56f.; at which time, the sun was in $3^{\circ} 8' 10' 25.1''$, which is distant from the December observation by $180^{\circ} 0' 33''$: But here, the interval of time is 182d. 16h. 21m. 53f.; which is greater than 182d. 15h. 7m. $44\frac{1}{2}$ f. the half anomalistic revolution, by 1h. 14m. $8\frac{1}{2}$ f.; therefore the Sun has some time to run before he comes to the apogee.

Now $4' 0'' : 57' 12'' : 1h. 14m. 8\frac{1}{2}$ f. : 17h. 13m. 14f. correction of time.

And $24h. : 17h. 13m. 14f. : 57' 12'' : 42' 68''$. correction of place.

Then June 29d. 7h. 4m. 56f. + 17h. 13m. 14f. gives

June 30d. oh. 21m. 10f. for the time of the apogee.

And $3^{\circ} 8' 20'' : 25,1'' + 42' 6,8''$ gives $3^{\circ} 8' 52' 33''$ for the place of the apogee.

251. PRO-

251.

PROBLEM LXV.

At any given time, to find the Sun's mean anomaly.

SOLUTION. Let an epocha of the Sun's passage through its aphelion be accurately determined. Then say,

As the time of a tropical revolution, or solar year,

To the interval between the aphelion and given time;

So is 360 degrees, to the degrees shewing the mean anomaly.

OR. From the tables or mean motions, find the Sun's mean motion for the given time, and this will be the mean anomaly.

252. If the Sun's motion in the ecliptic was uniform, his true place for any time could be found by the tables of his mean motion; but the Sun's longitude found by those tables, called his mean longitude, must, on account of his irregular motion, be corrected.

As the Earth revolves in an elliptical orbit about the Sun, placed in one of its foci, its angular motion round the Sun will differ from the angular motion it would have, was the Sun in the center of the ellipsis.

Now the table of mean motions gives the angular motions from the center of the ellipsis, in a circle described on the line of the apsides, and reckoned from the first point of Aries; this motion, lessened by that of the apogee, gives the Sun's mean longitude, or mean anomaly, from the aphelion point.

But the motion of the Earth being in an elliptic orbit, its true anomaly will differ from its mean; this difference, called the equation of the center, is the correction wanted to reduce the mean motions to the true ones.

253. To find the equation of the center, or solving (what is called) the Keplerian problem, is the most difficult operation, particularly in orbits whose eccentricity bears a considerable proportion to the mean distance: how to do this has been shewn by Newton, Gregory, Keil, La Caille, and many others, by methods little differing from one another, and consists chiefly in finding an intermediate angle, called the excentric anomaly, as shewn in the following problem.

254.

PROBLEM LXVI.

The Sun's mean anomaly being known, and the dimensions of its orbit; to find the excentric anomaly.

SOLUTION

SOLUTION. Say,

As the aphelion distance,
To the perihelion distance;

So is the tan. $\frac{1}{2}$ the mean anomaly,
To the tan. of an arc.
Which arc added to half the mean anomaly, gives the excentric anomaly.

For let $\angle DBP$ be the excentric.

AEP the Earth's orbit.
c the center, s the Sun,
a the aphelion, p the perihelion, E the true place, D the corresponding place in the excentric, and B the mean place.

Now it is evident, that the less the eccentricity is, the nearer will the elliptic orbit approach the excentric circle; the nearer will the true and mean places, E and B, approach one another; and the less will be the difference between the mean, the excentric, and the true anomalies; also the nearer will the lines CD, SB, approach to parallelism, or coincidence: so that in orbits of small eccentricities, CD and SB may be taken as parallel lines, particularly in the Earth's orbit, where CS is only about $\frac{1}{80}$ of CP.

Therefore $\angle ASB = \angle ACD$ the excentric anomaly.

Then in the triangle BCS, where the sum of the sides $BC + CS = SA$; the diff. of the sides $BC - CS = SP$, and $\angle BCS (= \text{supplement of } \angle ACB)$ are known, the $\angle CSB$ may be found.

Thus $SA : SP :: \tan. \frac{1}{2} (\text{sum } \angle S, \angle CSB) : B = \angle ACB : \tan. \text{ of an arc.}$

Then $\frac{1}{2} \angle ACB + \text{that arc} = \angle CSB$ (III. 47) the excentric anomaly.

255.

PROBLEM LXVII.

The Sun's excentric anomaly, and the dimensions of its orbit, being known; to find the true anomaly.

SOLUTION. Say, As the square root of the aphelion distance,

To the square root of the perihelion distance;

So tangent of half the excentric anomaly,

To tangent of half the true anomaly.

For let a semicircle be described from E through the other focus s, cutting AP in s, I, and SE, produced, in G, H.

Then (II. 172) $SH : SI :: SI : SG = \left(\frac{SI \times SS}{SH} = \frac{SI \times SI}{SE + Es} = \right) \frac{2CF \times 2CS}{2CA}$ Or

Or $CF \times CS = (\frac{1}{2}SG =) \frac{1}{2}SE - \frac{1}{2}EF$; $CA (= \frac{1}{2}SE + \frac{1}{2}EF)$ being radius.

Therefore $SE = (I + CS \times CP \text{ (III. 47)} =) I + CS \times S, ACD$. (III. 9)

Again. In $\triangle SFE$. As $SE : R :: SF : S, ASE = (\frac{SP}{SE} =) \frac{SC + S, ACD}{I + SC \times S, ACD}$.

Then $I + S, ASE : I - S, ASE :: I + \frac{SC + S, ACD}{I + SC \times S, ACD} : I - \frac{SC + S, ACD}{I + SC \times S, ACD}$.

Or $\frac{I - S, ASE}{I + S, ASE} = \left(\frac{I + SC \times S, ACD - SC - S, ACD}{I + SC \times S, ACD + SC + S, AC} \right)$

$$= \frac{I - SC + SC \times S, ACD - S, ACD}{I + SC + SC \times S, ACD + S, ACD}$$

$$= \frac{SP + CS - I \times S, ACD}{SA + CS + I \times S, ACD} = \frac{SP - S, ACD \times SP}{SA + S, ACD \times SA}$$

$$= \frac{I - S, ACD \times SP}{I + S, ACD \times SA}$$

But $\frac{I - S, ASE}{I + S, ASE} = t, \frac{1}{2}ASE$; and $\frac{I - S, ACD \times SP}{I + S, ACD \times SA} = t, \frac{1}{2}ACD \times \frac{SP}{SA}$. (IV. 217)

Then $\frac{SP}{SA} \times t, \frac{1}{2}ACD = t, \frac{1}{2}ASE$. Or $\frac{SP}{SA} = \frac{t, \frac{1}{2}ASE}{t, \frac{1}{2}ACD}$.

Therefore $\left(\frac{\sqrt{SP}}{\sqrt{SA}} = \frac{t, \frac{1}{2}ASE}{t, \frac{1}{2}ACD} \right.$ Or $\left. \sqrt{SA} : \sqrt{SP} :: t, \frac{1}{2}ACD : t, \frac{1}{2}ASE \right.$

256. REMARK. The $\angle CQS = \angle ACB \cap \angle ASE$ (II. 95), is the equation of the center, to be applied to the mean anomaly; and is subtractive from the aphelion to the perihelion, or in the first six signs of anomaly; and additive from the perihelion to the aphelion, or in the last six signs of anomaly.

For the lines CB , SE ; cb , se , which coincide in SA , SP , will in every other position cross one another; in Q while revolving from A to P , and in q while revolving from P to A : in the first half revolution, the mean anomaly, or the external $\angle ACB$, exceeds the $\angle ASE$, the true anomaly, by the $\angle CQS$ (II. 96): in the latter half, the true anomaly, or external $\angle PSA$, exceeds the mean anomaly, or $\angle PCB$, by the $\angle sqc$.

SECTION IX.

Practical Astronomy.

Of the EQUATION OF TIME.

257. Time, which of itself flows uniformly, has its parts measured by the motion of some visible object; and the Sun being the most conspicuous moving object in the heavens, its motion has been chosen as the most proper measure of the parts of time, as well for the day, as for the year.

258. The astronomical day, at any place, begins when the Sun's center is on the meridian of that place; and is divided into 24 hours, reckoned in a numeral succession from 1 to 24: the first 12 are sometimes distinguished by the mark P. M., signifying *post meridiem*, or afternoon; and the latter 12 are marked A. M., signifying *ante meridiem*, or before noon: but astronomers generally reckon through the 24 hours, from noon to noon; and what is by the civil, or common way of reckoning, called morning hours, is by Astronomers reckoned in the succession from 12, or midnight, to 24 hours.

Thus 5 o'clock in the morning of April the 10th, is by astronomers called April the 9th, at 17 h.

259. *The Sun's daily motion in longitude*, is the arc of the ecliptic run through in that day; and his *daily motion in right ascension*, is the corresponding arc of the equator; and the mean daily motion in either circle is measured by $59' 8''$ nearly. For $365 \text{ d} : 1 \text{ d} :: 360^\circ : 59' 8''$.

260. An *ASTRONOMICAL* or *SOLAR DAY*, is the interval of time between two successive transits of the Sun's center over the same meridian; and is measured by the sum of the whole equator, and an arc thereof equal to the daily motion in right ascension.

For at the end of a diurnal rotation, which by observations is known to be uniform, the meridian is returned to the same star, or point of the ecliptic, it was against at the preceding noon: but the Sun, during this rotation, has removed from that star to another, which has a greater right ascension: therefore, before the meridian can be again opposite to the Sun, so much of another rotation must be described, as is equal to the daily motion in right ascension.

261. A *SYDERIAL DAY*, is the interval between two successive returns of the same meridian to the same fixed star, is less than the solar day, and is measured by 360° .

262. A *MEAN* or *EQUATORIAL DAY*, is the time elapsed between two successive transits of the Sun over the meridian, and is measured by $360^\circ 59' 8''$ nearly.

263. *MEAN* or *EQUAL TIME*, is that shewn by a *clock*, whose 24 hours measures the time the Sun takes to describe an equatorial arc equal to $360^\circ 59' 8''$ nearly.

264. The difference between the measures of a mean solar day and a syderial day, viz. $59' 8''$, reduced to time (132), gives 3 m. 56 f.; which shews, that a star which was on the meridian with the Sun on

U 2

one

one noon, will return to that meridian 3 m. 56 f. before the next noon; therefore a clock which measures mean or equal days by 24 hours, will give 23 h. 56 m. 4 f. for the length of a syderial day.

265. APPARENT, or TRUE TIME, is that shewn by a *sun-dial*; whole 24 hours, or day, is measured by the sum of 360° , and that day's motion in right ascension.

266. The solar days are unequal to one another, for observations shew that the sun's daily motion in right ascension is continually varying.

The true and mean solar days are never equal, but when the Sun's daily motion in right ascension is $59' 8''$; which happens about February 11th, May 14th, July 26th, and November 1st: at all other times, the lengths of the *true* and *mean* days differ: The accumulation of these differences produces the equation of time; and sometimes the apparent noon will precede the time of the mean noon, and sometimes fall after it; their difference amounting to above 16 minutes at the beginning of November.

267. The EQUATION of TIME, is the difference between the times shewn by a *clock* and a *sun-dial*; or between the *mean* and true noons; or between the Sun's right ascension and his mean motion taken in the equator.

This difference arises on two accounts. First, because of the obliquity of the ecliptic, the daily motions in longitude and right ascension are unequal. Secondly, because of the unequal motion of the Earth in an elliptic orbit.

In the first and third quadrants, or between the signs ♈ , ♊ , ♋ , the right ascension being less than the longitude (140), or the mean motion taken in the equator; the point of right ascension is to the west, and therefore the apparent noon precedes, or comes in consequentia to the meridian before the mean noon: but in the 2d and 4th quadrants, or between the signs ♌ , ♍ , ♎ , the right ascension being greater than the longitude, or mean motion taken in the equator, the mean noon is westward, and therefore precedes, or comes in consequentia to the meridian before the apparent noon.

From the aphelion to the perihelion, or in the first six signs of anomaly, the mean noon precedes the apparent; and in the last six signs of anomaly, the apparent noon precedes the true; their difference in either case is the equation of the center, which convert into time.

Now because the points of Aries, and of the Sun's apogee, the places where the two parts of the equation of time commence, do constantly recede from one another; therefore the whole equation of time made up of those two parts will serve only for a few years, and requires to be corrected from time to time.

268. To calculate the equation, or difference between the mean and apparent noons, for any proposed day.

Find the mean and true anomalies for that time (255); their difference, or the equation of the center, is one part.

The true anomaly gives the Sun's longitude; with which, and the obliquity of the ecliptic, compute the right ascension (139); the difference between the longitude and right ascension, gives the other part.

The sum, or diff. of the two parts, turned into time, gives the equation sought.

SECTION

SECTION X.

Practical Astronomy.

To make SOLAR TABLES.

269. I. *Tables of the mean motions of the Sun.*(301, 302,
303, 304.)

Divide 360 degrees by a solar revolution, the quotient shews the mean motion for one day $0^{\circ} 59' 08''$ &c.

Take the multiples of one day's motion, from 1 to 365, for every day in the year; and these properly disposed, according to the month days, will give the mean motions for every day of each month. (304)

The 24th part of one day's motion, will give that for one hour, and its multiples to 24 times, will shew the mean motions answering to each hour: from hence, those for the minutes of an hour, the seconds of a minute, &c. are easily obtained. (303)

The mean motion of a year of 365 days (viz. for the last of December) being doubled, tripled, and quadrupled, those for 1, 2, 3, and 4 years will be obtained, adding one day's mean motion to the 4th year, it being leap-year, and containing 366 days: the motion for leap-year being increased by those of 1, 2, 3, and leap-years, give those for 5, 6, 7, and 8 years: the mean motion for 8 years being increased by those for 1, 2, 3, and 4 years, give those for 9, 10, 11, and 12 years: and thus increasing the mean motion for the last leap-year by those of 1, 2, 3, and 4 years, the mean motions may be continued for any number of absolute years. (301)

270. In the following tables, the numbers used were,

365d. 5h. 48m. $54\frac{1}{2}$ f.

Length of the year

Yearly motion of the apogee

Place of the apogee, beginning the year 1760,

Greatest equation of the Earth's orbit.

$1^{\circ} 5' 0''$.
3' 8' 47" 25.
1 55 39.

271. Now 365 d. 5 h. 48 m. $54\frac{1}{2}$ f. = 365,2423002472 days.

Then 365,2423003472 d. : 360° :: 1 d. : 0,9856470613 degrees.

Hence the mean motion for 1 day = $0^{\circ} 0' 0'' 59' 8'' 19''' 45^{iv} 54^{v} 50^{vi}$

January 30th	5 days =	0 4 55 41	38 49 34 10
March 31st	30 days =	0 29 34 9	52 57 25 0
June 29th	90 days =	2 28 42 29	38 52 15 0
December 26th	180 days =	5 27 24 59	17 44 30 0
December 31st	360 days =	11 24 49 58	35 29 0 0
	365 days =	11 29 45 40	14 18 34 10

U 3

Now

Now 1 year's mean motion = $11^{\circ}29'45''40'''14'''18''34''10'''$.

2 years

= $11^{\circ}29'31''20''28''37''8''20'''$.

3 years

= $11^{\circ}29'17''0''42''55''42''30'''$.

4, or leap year

= $0^{\circ}0'1'49''17''0''11'30''=3y.+1y.+1d.$

5 years

= $11^{\circ}29'47''29''31''18''45''40''=4y.+1y.$

6 years

= $11^{\circ}29'33''9''45''37''19''50''=4y.+2y.$

7 years

= $11^{\circ}29'18''49''59''55''54''0''=4y.+3y.$

8 years B

= $0^{\circ}0'3'38''34''0''23''0''=4y.\times 2.$

20 years B

= $0^{\circ}0'9''6''25''0''57'30''=4y.\times 5.$

100 years B

= $0^{\circ}0'45'32''5''4'47'30''=20y.\times 5.$

1000 years B

= $0^{\circ}7'35'20''50''47'55''0''=100y.\times 10.$

Where B stands for biffextile, or leap-year.

272. But to find the mean motions for the years related to any particular epocha, the mean motion for some particular time in that epocha must be known. Thus.

Let the mean motion of the Sun be determined by observation (or otherwise) when the Sun is in some noted point of the ecliptic, sup-
pole near Aries: or let the time of his entering the sign Aries be well ascertained, where his mean motion will be nothing. Take the difference between the time of that ingrefs and the 31st of December at noon, in days, hours, minutes, and seconds (reckoning the end of the 31st of December to be the beginning of January at noon,) and find the mean motions for those days, hours, minutes, and seconds, and it will shew the motion, from Aries, for the 31st of December, or the mean motion at the beginning of the year proposed; or the radix for that year, relative to the proposed epocha.

The relative mean motions for one year being known, those for any number of succeeding years, relative to that epocha, may be had, by adding such of the before found absolute years, to the first relative year, as will make the number wanted: and the mean motions for any past year, relative to that epocha, will be found, by lessening the radical years by such a number of the absolute years, as will produce the relative years required.

And in this manner are tables constructed, whereby the mean motions of the Sun for any time, past, or to come, may be computed.

273. Suppose in the year 1760, the Sun entered Aries on the 20th of March, at 13 h. 42 m. $3\frac{1}{2}$ f. P. M.: required the Sun's mean motion for the beginning of the year 1760?

Now 1760 being leap-year, February has 29 days, and from the equinox to the commencement of the year is 80 d. 13 h. 42 m. $3\frac{1}{2}$ f. Then 1 d.: 0,9856470613 deg.: 80 d. 13 h. 42 m. $3\frac{1}{2}$ f.: 79,4144 &c. degrees.

Therefore at the beginning of the year 1760, the Sun's mean motion was $2^{\circ}19'24''52'''$ short of Aries; or his mean motion was $9^{\circ}10'35''8'''$; which is the radical mean motion for the year 1760.

274. II. *Of the mean motions of the Sun's apogee.* (301, 302)

The yearly motion of the apogee being determined (241): the motion for any number of absolute years, will be that multiple of one year's

year's motion, and so for any part of a year: the monthly motions will be 5 seconds for some months, and 6 for others, to make 65 in the 12 months.

Let the time of the Sun's passage through the aphelion be accurately determined by observation (250), and also its place in the ecliptic; then the distance of the place of the apogee from Aries will be known at that time: let this distance be lessened by the apogee's motion from the last day of the year preceding the proposed epocha to the time of the apogeeon passage, and the mean motion of the apogee will be known for the beginning of that year, taken as a radix.

Then that radical mean motion, increased by the multiples of the yearly motion, will give those for succeeding years; but being diminished by those multiples, will give them for past years.

The Sun's mean motion for any time, lessened by that of the apogee for that time, gives the Sun's mean anomaly.

275. III. Of the equation of the Sun's center. (305)

To every degree of the first fix signs of mean anomaly assumed, find the true anomaly (253, 254): the difference between the mean and true anomalies, will be the equations of the center to those degrees of mean anomaly; which serve also for the degrees of the last fix signs; as equal anomalies are at equal distances on both sides of either apside.

Set the equations of the center orderly to their signs and degrees of anomaly, the first fix being reckoned from the top of the table downwards, and signed at top with the title *subtract*; the last fix, for which the same equations serve, but taken in a contrary order, viz. from the bottom of the table reckoned upwards, are signed at bottom with the title *add*; and let the difference between every adjacent two equations, called tabular differences, be set in another column.

From these equations of the center, augmented or diminished by the proportional parts of their respective tabular differences for any given minutes and seconds, are deduced equations of the center to any given mean anomaly.

276. Astronomical tables are usually computed to answer to two given denominations only; as to signs and degrees; degrees and minutes; months and days; &c.: for if made to more names, such tables would swell into a bulk so great, as to be tedious to compute, expensive to print, and not much advantage gained in their use; but it generally happens in calculations, that numbers are wanted from tables to answer to given numbers of three, or more denominations as to signs, degrees, minutes, and seconds; months, days, hours, minutes, and seconds; &c.: and to obtain from the tables numbers answering to all the given names, the tabular numbers are to be increased or diminished by a proportional part of their difference.

Thus. To find the equation of the center to $4^{\circ} 21' 44'' 36''$?

Now the equation to this number will fall between those belonging $4^{\circ} 21'$ and $4^{\circ} 22'$; which equations are $1^{\circ} 13' 59''$ and $1^{\circ} 12' 24''$ (205) Their diff. is $1' 35'' = 95''$; the pro. pt. of which is to be taken for $44' 36''$.

U 4 And

And as the diff. 1° or $60'$: diff. $95'' :: 44', 6 : 70'', 6 = 1' 11''$ the proportional part

Now to $4^{\circ} 21' 0''$, $1^{\circ} 13' 59''$ is the equation of the center.

And to $0^{\circ} 044' 36''$, $1' 11''$ is the prop. part to be subtracted.

Then to $4^{\circ} 21' 44' 36''$, $1' 12' 48''$ is the equation of the center.

When the tabular numbers are increasing, the proportional part is to be added; but when decreasing, the proportional part is to be subtracted.

277. IV. Tables of the Sun's true place.

(308)

The Sun's true place at any proposed time is thus found.

Collect together the mean motions of the Sun, and also those of the apogee, for the given year, month, day (hour, minute, and second, if given); and their sum will be the mean motions of the Sun and its apogee.

The Sun's mean motion, lessened by that of the apogee, gives the mean anomaly; to which find the proper equation of the center, by proportioning for the minutes and seconds.

Then the Sun's mean motion, augmented or diminished by the equation of the center, as the title of its table directs, gives the Sun's true longitude, or place, for that given time.

The Sun's place thus found to every day for four successive years, viz. for leap-year, and 1, 2, 3 years after; and those places ranged under their proper-years, according to their respective months and days, constitute the tables of the Sun's place.

These tables only find the Sun's place at noon; but the place for any intermediate time is found, by applying to the noon-place the proportional part of the daily-difference at that time.

278. To find the Sun's longitude, suppose on May 4, 1776, at noon?

In the table of the Sun's longitude (308) for 1776, against May 4, stands $1^{\circ} 14' 30' 33''$, which shews that the Sun's longitude, reckoned from Aries, is $44^{\circ} 30' 33''$; or that his place is in $814^{\circ} 30' 33''$.

But to find the Sun's place at any other hour, suppose on May 4th, at 7 h. 24 m. 36 f. P. M. proceed thus.

The difference between the noon places of the 4th and 5th of May is $58' 0'' = 3480''$, answering to 24 hours in time.

Then 24 h. : 7 h. 24 m. 36 f. :: $3480' : 1074'' = 17' 54''$, the proportional part.

And $1^{\circ} 14' 30' 33'' + 17' 54'' = 1^{\circ} 14' 48' 27''$, the Sun's longitude at that time.

279. Or. The Sun's place may be found by the tables of mean motions.

1776

1776. ☉ M. Mot. (302) $9^{\circ} 10' 42'' 25''$ M. Ap. $3^{\circ} 9' 4' 45''$ (302)
 May 4 (304) 4 2 13 13 27 (307)

7 hours (303) 17 15 3 9 5 12 M. Apogee.

24 minutes (303) 59 Diff. 10 4 18 41 mean Anom.

26 seconds (303) 1 Now to $10^{\circ} 4'$ the equation of the center is $1^{\circ} 34' 45''$. (305)

☉ M. longitude 1 13 13 53 And the diff. is $70''$, decreasing.
 Eq. center + 1 34 34 $60' : 70'' :: 8' 41'' : 11''$, the proportional part.

Sun's true longitude 1 14 48 27 And $1^{\circ} 34' 45'' - 11' = 1^{\circ} 34' 34'' =$ equation of the center.

280. To find the Sun's longitude at any given time and place.

Seek in the table of the longitudes of places, at the end of Book VI. for the difference of longitude between London and the proposed place; and convert the diff. of longitude into time.

If the proposed place is to the eastward of London, take the diff. between the proposed time and diff. of longitude, and this will shew the corresponding time at London; after noon, if the proposed time is greater than the diff. of longitude; but before noon, if the proposed time is less.

If the proposed place is to the westward of London, the sum of the proposed time and diff. of longitude will be the corresponding time at London.

The Sun's place found to the corresponding time at London, will be the Sun's longitude sought for the proposed time and place.

Thus, In a place 6^{h} . to the east of London, when it is 8 h. P. M. at that place, it is 2 h. P. M. at London; and when it is 4 h. P. M. at that place, it is 2 h. before noon at London.

For when it is noon at London, it is 6 h. P. M. at the other place.

Also, in a place 6 h. to the west of London; when it is 8 h. P. M. at that place, it is 14 h. P. M. at London.

For when it is noon at the proposed place, it is 6 h. P. M. at London.

281. V. Tables of the Sun's declination, (309)

With each of the Sun's longitudes, already found, and the obliquity of the ecliptic, find the declination to each day of the four years. (139).

Or thus. To each degree of the three first signs of the ecliptic, taken as longitudes, find the declination (139): and of these declinations, regularly ranged to their sign and degree, take the difference of each adjacent two, which set against them in another column; and this auxiliary table is prepared, answering to each sign and degree of longitude (306): for to equal longitudes, taken on both sides of each equinox, belong equal declinations.

Now these auxiliary declinations augmented, or diminished (according as they are increasing or decreasing), by the proportional part of

of their difference, for the minutes and seconds in any given longitude, will give the declination for that longitude.

And this being done for every day in the four years, using the longitudes already computed, will give the declinations sought: which are to be ranged according to their year, month, and day. (309)

282. To find the Sun's declination. Suppose on May 4, 1772, at noon. In the table of the Sun's declination (309) for 1772, against May 4, stands $16^{\circ} 12' 33''$ for the Sun's declination, which is N, as being between the vernal and autumnal equinoxes.

283. But if the declination was wanted on May 4, 1772, at 7h. 24m. 36f. P. M. proceed thus.

The difference between the noons of May 4 and 5, is $17' 1''$, which answers to 24 h. Then $24 \text{ h.} : 17' 1'' :: 7 \text{ h. } 24 \text{ m. } 36 \text{ f.} : 5' 15''$, the proportional part.

And as the declination is increasing; then $16^{\circ} 12' 23'' + 5' 15''$ gives $16^{\circ} 17' 48''$ for the Sun's decl. at the proposed time.

284. But art. 311 is a table for finding the proportional part at sight, for fitting the noon declination to any other time. Thus.

Seek in the left-hand column for a daily difference, nearest to the given one; against which, in a column marked at top with hours, nearest those given, stands the proportional part sought.

Thus against $17' 0''$ of daily diff. and to 7 h. 24 m. time stands $5' 15''$, the proportional part sought.

Although this table goes no farther than 8h., yet it may be applied quite to 12 h. or 180 degrees.

— 285. EXAM. What will be the Sun's declination at London, on the 25th of Augst, 1775, at 10h. 35m. P. M.?

In 1775, the daily diff. between the noons of the 25th and the 26th of Augst is $20' 50''$ decreasing. (309)

Now 10 h. 35 m. is equal to 2 h. 35 m. + 8 h. 0 m.

To the diff. $20' 50''$, and to 2 h. 35 m., answers $2' 14''$. (311)

To the diff. $20' 50''$, and to 8 h. 0 m., answers $6' 57''$.

The sum $9' 11''$ is the proportional part, by which the decl. $10^{\circ} 45' 39''$ to Augst 25th, is to be diminished; so $10^{\circ} 36' 28''$ is the decl. sought.

Here $20' 50''$ is the middle between $20' 40''$ and $21' 0''$.

And 2 h. 35 m. is $\frac{3}{4}$, the interval between 2 h. 20 m. and 2 h. 40 m.

Now the middle of $2' 1''$, and $2' 2''$, is $2' 1''$; and the middle of $2' 18''$, and $2' 20''$, is $2' 19''$; the diff. of the middles, is $18''$; $\frac{3}{4}$ of which is $13''$; so $2' 14''$ answers to $20' 50''$ and 2 h. 35 m : also, under 8 h., the middle between the numbers to $20' 40''$, and $21' 0''$, is $6' 57''$.

— 286. From the table of declination, fitted to the meridian of London, or Greenwich, the declination may be found at any time, under any other meridian, at a given difference of longitude from London. Thus.

Required

286 Required the Sun's declination at noon, under a meridian 110° to the west of London, on the 24th of February, 1773?

(311) Now at 110° to the west of London, it is noon 7 h. 20 m. after it is noon at London; that is, when it is 7 h. 20 m. P. M. at London, it will be noon at the proposed place; so the declination found to that time at London (285) will be the declination sought.

In 1765, the diff. between the declinations of the 24th and 25th of February, is $22' 19''$ decreasing (309): and against $22' 20''$ of daily diff., and under 110° , or 7 h. 20 m., is $6' 49''$ in table, art. 311, which taken from $9^{\circ} 12' 54''$, leaves $9^{\circ} 6' 5''$, the declination sought.

EXAM. II. What is the Sun's declination on September 2d, 1773, at 20 h. 30 m., under a meridian 100° to the eastward of London?

Now under a meridian 100° to the eastward of London, it is noon 6 h. 40 m. before it is noon at London (311); or when it is noon at London, it is 6 h. 40 m. after noon at the proposed place; and when 20 h. 30 m. after noon at that place, it is 13 h. 50 m. after noon at London; so the declination found to that time (285), will be the declination sought.

In 1773, the daily diff. at September 2d, is $22' 5''$ (309), against which (in tab. art. 311), and under 8 h. and 5 h. 50 m., stand $7' 21''$ and $5' 21''$, their sum $12' 42''$ taken from the decl. to September 2, viz. $7^{\circ} 43' 55''$, leaves $7^{\circ} 31' 13''$, the declination sought.

Here 5 h. 50 m. falls in the middle between 5 h. 40 m. and 6 h. 0 m.; so $5' 21''$, the middle between $5' 12''$ and $5' 30''$, is taken.

287. VI. Tables of the Sun's right ascension. (310)

To the obliquity of the ecliptic, and each degree in the three first signs of longitude, find the right ascensions (139), and of each take the supplement.

Range the right ascensions according to their sign and degree for the three first signs; and for the three next signs, range the supplements, so that the 4th sign begins with the least supplement, and the 6th sign ends with the greatest: because the right ascensions in the 2d and 4th quadrants are the supplements of those in the first and third.

Let the differences of these right ascensions, viz. each adjacent two, through the six signs be taken, and set in other columns. (307)

Then this auxiliary table, used like that of declination, will give the right ascension to each day in the four years.

288. To find the Sun's right ascension; suppose on June 12 at noon, in the year 1774, at London.

In the table of the Sun's right ascension (310) for 1774, against June 12, stands 5 h. 22 m. 54 s., which is the right ascension sought, and shews how much later the Sun passed the meridian of London than the equinoctial point Aries.

EXAM.

EXAM. II. *Required the Sun's right ascension at London on the 2d of November, 1774, at 9 h. 30 m. P. M.?*

Between the 2d and 3d of November, 1774, the daily diff. is 3 m. 57 f., which answers to 24 h. Then 24 h. : 3 m. 57 f. :: 9 h. 30 m. : 1 m. 34 f., the proportional part.

Then the right ascension on the 2d at noon, 14 h. 31 m. 7 f. + 1 m. 34 f. gives 14 h. 32 m. 41 f. for the right ascension at the time required. By this table, the right ascension may also be found at any time in places that are to the eastward, or westward, of London, the difference of longitude of those places being known: by finding the time at London corresponding to the given time at the proposed place, and seeking the right ascension to that corresponding London time.

The table at art. 311. pages 222, 223, may be applied to the tables of the Sun's longitude and right ascension, as well as to those of the declination, for finding the proportional parts of the difference between the noons of adjoining days, which shall answer to any intermediate hours. Thus in the Exam. page 296. To find the pro. pts. of 58' to 7 h. 24 m. Now $\frac{1}{4}$ th of 58' is 14' 30"; which falls between 14' 20" and 14' 40". And the time 7 h. 24 m. falls between 7 h. 20 m. and 7 h. 40 m. The mean of the equations under 7 h. 20 m. and 7 h. 40 m. and against 14' 20" and 14' 40", are 4' 26" and 4' 38'; their diff. is 12". And 20 : 12 :: 4 : $2\frac{1}{2}$: and 4' 26" + $2\frac{1}{2}$ = 4' 28" $\frac{1}{2}$ the pro. pts. to $\frac{1}{4}$ of 58'. Then the proportional parts to 58', are 17' 54".

Again. In the Exam. page 300. To find the parts proportional to 3 m. 57 f. as 9 h. 30 m. is to 24 h.

Here 3 m. 57 f. being taken as 4 m.; and 4 h. 40 m as the half of 9 h. 30 m. The equation is 47", which doubled gives 1' 34" for the proportional parts required.

289. VII. *Of the right ascensions and declinations of the fixed Stars.* (312)

This table, which contains 120 of the principal fixed stars, viz. 60 having north declination, and 60 with south declination, are fitted to the year 1764; and are a part of a larger catalogue given by M. La Caille, which, he says, "are all derived from his own observations made, during ten years attention to this business, either at Paris, or at the Cape of Good Hope; that the positions are ascertained with all the accuracy that could be derived from the modern Astronomy; and that he had all proper helps, with regard to instruments, assistants, and convenience, and neither care or pains were wanting to perfect the work.

"The right ascensions were determined by a multitude of corresponding altitudes of each, taken with a quadrant of three feet radius, to have their passage over the meridian with the greatest exactness. Almost all the stars in the northern hemisphere have been compared with the bright star in the Harp; and those in the southern hemisphere, to Sirius; that is to say, that on each day that the time of the star's passing the meridian had been found by equal altitudes; that of the Harp and Sirius were found in like manner; the right ascensions of these

“ these two stars having been settled by a great many observations
 “ taken when those stars were in the properest situation for this
 “ purpose.”

“ The declinations have been deduced from a sufficient number of
 “ observations of their zenith distances, taken with an instrument of
 “ six feet radius, made with great care for this purpose.”

The table consists of 9 columns; that on the left hand contains the name of the constellation; the next shews in what part of the constellation the star is; in the 3d are the names by which certain stars are distinguished: the 4th column shews the Greek characters by which the star is marked in the celestial charts, or maps of the constellations; the 5th shews the magnitude of the stars; the 6th and 7th contain the right ascension in time, reckoned from Aries, and the yearly variation in right ascension; the 8th and 9th contain the declinations and the yearly variation in declination; where those marked + are augmented by the yearly variation; but those having the mark — annexed, are to be diminished by the variation: by the help of these yearly variations the right ascensions and declinations of these stars may be fitted for any distant year.

Precepts for finding the culminating of the stars, are at articles 133, 134.

290.

VIII. *Tables of the Equation of Time.*

In page 318 are three tables, articles 313, 314, 315: Article 313 is a table of the Sun's right ascension in degrees, to each degree of longitude in the first quadrant of the ecliptic; and also the differences between those longitudes and right ascensions. The table, art. 314, contains the said differences turned into time (132), of minutes, seconds, and the tenth part of seconds: the numbers in this table are the differences between the mean and true noons, arising from the obliquity of the ecliptic (267); and the table, art. 315, is nothing more than the equations of the center, table art. 305, converted into time; and are the differences between the times of the mean and true noons, arising from the eccentricity of the Earth's orbit: these two equations of time, properly put together, constitute another table, art. 316, of the absolute equation of time relative to the place of the Sun's apogee.

291. *To construct the table 316, of the absolute Equation of Time.*

1st. To the given time, find the Sun's true place, or assume a place.
 2d. The difference between that place, and the place of the apogee, gives the Sun's true anomaly.

3d. From the true anomaly find the mean.

4th. In table I. 314, seek the equation of time to the Sun's place.

5th. In table II. 315, seek the equation of time to the mean anomaly.

6th. The sum, or difference of these equations, according as their titles, or signs direct, will be the absolute equation of time to the Sun's place found at first, or to the corresponding time.

The tables, articles 314, 315, are made only to whole degrees of longitude and anomaly; so the proportional parts of the differences are to be taken for minutes or seconds, above whole degrees of the Sun's longitude and anomaly.

(294)

292. EXAM. I. *What is the Equation of Time, when the Sun's longitude is $7^{\circ} 12' 0''$?*

In table, art. 316, against 12° in the outside column, and under m, or 7 f., stands -16 m. 12 f., which shews that 16 m. 12 f. is to be subtracted from the apparent time; or the time the Sun comes to the meridian, to give the time of mean noon, or 12 hours, shewn by a good clock.

293. EXAM. II. *What is the Equation of Time when the Sun's longitude is $4^{\circ} 24' 30'' 42''$?*

The difference between the equation in table, art. 316, to $4^{\circ} 24'$ and $4^{\circ} 25'$, is om. $13'$ decreasing; and $30' 42'' = 30,7$. Then $60' : 30,7 :: 13' : 6,65$ or $7'$, the proportional part decreasing. And $+ 3$ m. 46 f. $- 7$ l. $= + 3$ m. 39 f., the equation sought. So the apparent time, or time of solar noon, increased by 3 m. 39 f., will give the time of mean noon.

If the time was given, viz. the month, day, hour, &c., to find the Equation.

To the given time find the Sun's longitude.

Then to this longitude find the equation of time, as above.

(278)

294. *To find the mean anomaly, from the true being given.*

SOLUTION, Say, As the square root of the perihelion distance,

To the square root of the aphelion distance;

So the tangent of half the true anomaly,

To the tangent of half the excentric anomaly.

And

As Radius, to the sine of the excentric anomaly;

So the degrees in an arc equal in length to the excentricity.

To the degrees &c. in the arc of correction.

The correction added to the excentric anomaly, gives the mean anomaly.

295. REMARKS. 1st. The greatest equation of the center being taken at $1^{\circ} 55' 39''$, the excentricity (249) will be 0,01682; the aphelion distance will be 1,01682, and the perihelion 0,98318.

Hence the ratio of the square root of the perihelion distance to the square root of the aphelion distance will be expressed by the logarithm 0,00731; which constant logarithm, added to the logarithmic tangent of $\frac{1}{2}$ the true anomaly, will give the logarithm tangent of $\frac{1}{2}$ the excentric anomaly.

296. 2d. In the 2d proportion, the arc equal to the length of the excentricity 1682 is a constant quantity.

Now the radius, or mean distance, is equal to the length of an arc of $57^{\circ} 29' 578$ (249); then $100000 : 1682 :: 57^{\circ} 29' 578 : 0^{\circ} 96' 375$, the length of the excentricity in degrees; whose constant logarithm is 9,98396, which added to the logarithmic sine of the excentric anomaly, abating 10 in the index of the sum, gives the logarithm of an arc, the degrees, minutes, and seconds of which being added to the excentric anomaly, gives the mean anomaly.

297. IX. *Ta.*

297. IX. *Table of corrections for the middle time, between equal altitudes of the Sun.* Art. 317.

This table, which is fitted to the latitudes of 30° , 40° , 50° , and 60° , will also serve, nearly, to all latitudes between 25° and 65° ; by entering the table with the nearest latitude to that given, and the given declination in degrees. It is constructed by art. 216.

EXAM. I. *In latitude 50° N., when the Sun's declination is 16° N., and the interval between the morning and afternoon observations is 5 hours: what correction must be applied to the middle time, to give the time of solar noon?*

In the table, art. 317, against 16° of declination taken in the outside column, and under 50° latitude, and 5 hours; with N. declination, stands 12 seconds; which 12 s. applied to the middle time between the observations, gives the time the Sun was on the meridian.

The correction is applied to the middle time by the precepts at the bottom of the table.

298. EXAM. II. *In latitude 50° N., on November 16th, 1761, observations at equal altitudes of the Sun were taken at the following times shown by a clock, the equal altitude instrument having three horizontal wires.*

Morning observations.

⊙ preceding limb ⊙ following limb

$9^h 28^m 55^s$ $9^h 35^m 21^s$
 $9 32 46\frac{1}{2}$ $9 39 23\frac{1}{2}$
 $9 36 44\frac{1}{2}$ $9 43 30$

Now $9^h 28^m 55^s + 2^h 1^m 20\frac{1}{2} + 12^h$

$9 32 46\frac{1}{2} + 1 57 28\frac{1}{2} + 12$

$9 36 44\frac{1}{2} + 1 53 30\frac{1}{2} + 12$

2

Again $9 35 21\frac{1}{2} + 1 54 56 + 12$

$9 39 23\frac{1}{2} + 1 50 53\frac{1}{2} + 12$

$9 43 30 + 1 46 43\frac{1}{2} + 12$

2

Afternoon observations.

⊙ preceding limb ⊙ following limb

$1^h 46^m 43^s$ $1^h 53^m 30^s$
 $1 50 53\frac{1}{2}$ $1 57 28\frac{1}{2}$
 $1 54 56$ $2 1 20\frac{1}{2}$

the mean = $11^h 45^m 7\frac{1}{2}$
 } 7,6 by the preceding limb.

the mean = $11^h 45^m 8^s$
 } 8 by the following limb.

The mean time of observation from both limbs, is $11^h 45^m 7,8$ s. The declination on the day of observation is 19° S. nearly; the interval between the observations is about 4 hours; and these give + 14 seconds for the correction of the middle time.

So the Sun was on the meridian when the clock shewed 1 rh. 45 m. 21,8 s. The Sun's place, at that time, was $7^{\circ} 24' 26''$ nearly.

In table 316, to $7^{\circ} 24'$ the tabular difference is 12", decreasing.

bet - 14.56 for 2A Then
 4 14.44 15
 12

Then $60' : 26,75' :: 12^s : 5,35^s$; and $+14\text{ m. } 56\text{ s.} - 51\text{ s.} = +14\text{ m. } 50\text{ s.}$, which is the equation of time: hence $11\text{ h. } 45\text{ m. } 21,8\text{ s.} + 14\text{ m. } 50,6\text{ s.} = 12\text{ h. } 0\text{ m. } 12,4\text{ s.}$

Which shews that the clock was 12 s. , nearly, too fast.

299. X. *Tables of Refraction and of the Sun's parallax.* (318,319)

These tables are the result of the experience of some of the most eminent Astronomers. By the refraction of the atmosphere, objects appear more elevated than they really are, and therefore the apparent altitude is to be diminished by the refraction, which is greatest near the horizon, and gradually diminishes towards the zenith, where there is no refraction. The parallax of altitude being the difference between the altitude of an object, as seen from the center and surface of the Earth, that from the center being the true altitude, and the greatest, except at the zenith, when parallax vanishes; therefore the apparent altitude is to be augmented by the parallax.

EXAM. The Sun's apparent altitude was observed to be $18^{\circ} 34' 48''$; what was his true altitude?

Apparent altitude	$18^{\circ} 34' 48''$	} Refraction	$2' 47'' -$
Correction is	$2\ 38$		} Parallax

Sun's true altitude $18\ 32\ 10$.



300.

ASTRONOMICAL TABLES

Fitted, in general, to the meridian of GREENWICH.

	Art.
I. Mean motions of the Sun and Apogee for years.	(301)
II. Mean motions of the Sun and Apogee for radical years.	(302)
III. Mean motions of the Sun for hours, minutes, and seconds.	(303)
IV. Mean motions of the Sun and Apogee for months and days.	(304)
V. Equations of the Sun's center.	(305)
VI. Sun's declination to signs and degrees.	(306)
VII. Sun's right ascension in time to signs and degrees.	(307)
VIII. Sun's longitude to each day for the years 1776, 1777, 1778, 1779.	(308)
IX. Sun's declination to each day for the same four years.	(309)
X. Sun's right ascension to each day for the same four years.	(310)
XI. To fit the tables VIII. IX. X. to any meridian.	(311)
XII. Right ascensions and declinations to 120 fixed stars.	(312)
XIII. Sun's right ascension in degrees &c. to signs and degrees of longitude.	(313)
XIV. Equation of time, on the obliquity of the ecliptic.	(314)
XV. Equation of time, on the eccentricity of the Earth's orbit.	(315)
XVI. Absolute equation of time, to the Sun's longitude.	(316)
XVII. Correction of the middle time between equal altitudes.	(317)
XVIII. Correction of altitude for refractions.	(318)
XIX. Correction of the Sun's altitude for his parallax.	(319)

The tables of the Sun's place, declination, and right ascension, are fitted to the years, 1776, 1777, 1778, 1779; and will serve in most nautical operations as well, for the four years preceding, viz. 1772, 1773, 1774, 1775, and also for the four years following, viz. 1780, 1781, 1782, 1783, as they are denoted at the heads of those tables. But as a Nautical Almanack is published yearly under the direction of the commissioners of longitude, the tables contained therein should be consulted in cases where the utmost precision is necessary.

sign - 0.	sign - 10.
♈ Aries - 0	♎ Libra - 6 -
♉ Taurus - 1	♏ Scorpio - 7
♊ Gemini - 2	♐ Sagittarius - 8
♋ Cancer - 3 X	♑ Capricorn - 9 TABLES
♌ Leo - 4	♒ Aquarius - 10
♍ Virgo - 5 -	♓ Pisces - 11 -

add. q. contains 30 Degrees - 0. 30. 30. the follow. Table
1. 14. 30. 33 - signatus 44. 30. 33 from Aries

For hours, minutes, and secs.		H.		M.		Lon. °	
1	2	3	4	5	6	7	8
16	27	51	31	16	23	31	16
18	51	32	4	55	42	32	18
19	51	33	6	38	39	39	19
20	42	47	8	33	36	36	20
21	37	42	10	28	34	40	21
22	32	38	12	24	30	41	22
23	27	34	14	19	26	42	23
24	22	30	16	14	21	43	24
25	17	25	18	9	16	44	25
26	12	20	20	4	11	45	26
27	7	15	22	0	8	46	27
28	0	10	24	0	4	47	28
29	0	5	26	0	0	48	29
30	0	0	27	0	0	49	30
31	0	0	28	0	0	50	31
32	0	0	29	0	0	51	32
33	0	0	30	0	0	52	33
34	0	0	31	0	0	53	34
35	0	0	32	0	0	54	35
36	0	0	33	0	0	55	36
37	0	0	34	0	0	56	37
38	0	0	35	0	0	57	38
39	0	0	36	0	0	58	39
40	0	0	37	0	0	59	40
41	0	0	38	0	0	60	41
42	0	0	39	0	0	1	42
43	0	0	40	0	0	2	43
44	0	0	41	0	0	3	44
45	0	0	42	0	0	4	45
46	0	0	43	0	0	5	46
47	0	0	44	0	0	6	47
48	0	0	45	0	0	7	48
49	0	0	46	0	0	8	49
50	0	0	47	0	0	9	50
51	0	0	48	0	0	10	51
52	0	0	49	0	0	11	52
53	0	0	50	0	0	12	53
54	0	0	51	0	0	13	54
55	0	0	52	0	0	14	55
56	0	0	53	0	0	15	56
57	0	0	54	0	0	16	57
58	0	0	55	0	0	17	58
59	0	0	56	0	0	18	59
60	0	0	57	0	0	19	60

TABLES of the MEAN MOTIONS of the SUN and his APOGEE to MEAN SOLAR TIME.

303.

For the radical years; new stile.		M. lon. °		M. lon. ap.		Years.	
1	2	3	4	5	6	7	8
15	34	9	10	15	34	9	10
16	34	9	10	15	34	9	10
17	34	9	10	15	34	9	10
18	34	9	10	15	34	9	10
19	34	9	10	15	34	9	10
20	34	9	10	15	34	9	10
21	34	9	10	15	34	9	10
22	34	9	10	15	34	9	10
23	34	9	10	15	34	9	10
24	34	9	10	15	34	9	10
25	34	9	10	15	34	9	10
26	34	9	10	15	34	9	10
27	34	9	10	15	34	9	10
28	34	9	10	15	34	9	10
29	34	9	10	15	34	9	10
30	34	9	10	15	34	9	10
31	34	9	10	15	34	9	10
32	34	9	10	15	34	9	10
33	34	9	10	15	34	9	10
34	34	9	10	15	34	9	10
35	34	9	10	15	34	9	10
36	34	9	10	15	34	9	10
37	34	9	10	15	34	9	10
38	34	9	10	15	34	9	10
39	34	9	10	15	34	9	10
40	34	9	10	15	34	9	10
41	34	9	10	15	34	9	10
42	34	9	10	15	34	9	10
43	34	9	10	15	34	9	10
44	34	9	10	15	34	9	10
45	34	9	10	15	34	9	10
46	34	9	10	15	34	9	10
47	34	9	10	15	34	9	10
48	34	9	10	15	34	9	10
49	34	9	10	15	34	9	10
50	34	9	10	15	34	9	10
51	34	9	10	15	34	9	10
52	34	9	10	15	34	9	10
53	34	9	10	15	34	9	10
54	34	9	10	15	34	9	10
55	34	9	10	15	34	9	10
56	34	9	10	15	34	9	10
57	34	9	10	15	34	9	10
58	34	9	10	15	34	9	10
59	34	9	10	15	34	9	10
60	34	9	10	15	34	9	10

302.

For years absolute.		M. lon. °		M. lon. ap.		Years.	
1	2	3	4	5	6	7	8
40	45	11	29	31	20	2	11
41	45	11	29	31	20	2	11
42	45	11	29	31	20	2	11
43	45	11	29	31	20	2	11
44	45	11	29	31	20	2	11
45	45	11	29	31	20	2	11
46	45	11	29	31	20	2	11
47	45	11	29	31	20	2	11
48	45	11	29	31	20	2	11
49	45	11	29	31	20	2	11
50	45	11	29	31	20	2	11
51	45	11	29	31	20	2	11
52	45	11	29	31	20	2	11
53	45	11	29	31	20	2	11
54	45	11	29	31	20	2	11
55	45	11	29	31	20	2	11
56	45	11	29	31	20	2	11
57	45	11	29	31	20	2	11
58	45	11	29	31	20	2	11
59	45	11	29	31	20	2	11
60	45	11	29	31	20	2	11

301.

Day.	Decem-ber.	Janu-ary.	Febru-ary.	March.	April.	May.	June.	July.	Augst.	Septem-ber.	October.	Novem-ber.	Decem-ber.
1	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
2	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
3	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
4	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
5	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
6	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
7	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
8	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
9	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
10	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
11	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
12	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
13	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
14	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
15	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
16	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
17	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
18	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
19	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
20	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
21	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
22	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
23	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
24	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
25	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
26	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
27	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
28	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
29	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
30	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11
31	10 39 2	1 32 27	1 29 8	1 29 8	3 29 15	4 29 49	5 29 23	6 29 16	8 29 56	9 29 56	10 29 56	11 30 11	12 30 11

X

304. TABLE of the MEAN MOTIONS of the SUN and APOGEE to MONTHS and DAYS.

305. TABLE of the EQUATION of the SUN'S CENTER to each sign and degree of mean Anomaly.

Deg.	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	Diff.		
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
I	46	48	49	50	51	52																												

308 No. 1
Sun's longitude
No. 296 - An. 277 - 278

A TABLE OF THE SUN'S LONGITUDE FOR THE YEARS 1772, 1776, AND 1780, BEING LEAP YEARS.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	9 10 45 46	10 12 18 26	11 11 32 41	12 19 35	11 36 23	12 11 25 48	13 10 3 42	4 9 39 32	5 9 31 4	6 8 50 49	7 9 40 50	8 9 58 15
2	9 11 46 57	10 13 19 16	11 12 32 45	12 13 32 37	11 12 32 23	12 12 23 16	13 11 58 54	4 10 37 1	5 10 29 17	6 9 49 59	7 10 41 16	8 10 59 11
3	9 12 48 7	10 14 20 5	11 13 32 48	12 14 17 37	11 13 32 31	12 13 20 36	13 11 58 54	4 10 37 1	5 10 29 17	6 9 49 59	7 10 41 16	8 10 59 11
4	9 13 49 17	10 15 20 53	11 14 32 48	12 15 16 35	11 14 30 33	12 14 17 58	13 12 55 16	4 11 32 30	5 12 25 47	6 10 48 25	7 11 41 32	8 11 59 13
5	9 14 50 26	10 16 21 39	11 15 32 47	12 16 15 31	11 15 28 33	12 15 15 20	13 13 52 28	4 13 29 31	5 13 24 4	6 12 47 42	7 13 41 49	8 14 59 13
6	9 15 51 36	10 17 22 23	11 16 32 44	12 17 14 25	11 16 26 31	12 16 12 41	13 14 49 40	4 14 27 3	5 14 22 23	6 13 47 0	7 14 42 8	8 15 59 13
7	9 16 52 45	10 18 23 6	11 17 32 39	12 18 13 17	11 17 24 28	12 17 10 2	13 15 46 52	4 15 24 37	5 15 20 44	6 14 46 21	7 15 42 28	8 16 59 13
8	9 17 53 54	10 19 23 48	11 18 32 31	12 19 12 7	11 18 22 24	12 18 7 22	13 16 43 5	4 16 22 12	5 16 19 7	6 15 45 44	7 16 42 50	8 17 59 13
9	9 18 55 3	10 20 24 29	11 19 32 22	12 20 10 55	11 19 20 18	12 19 4 41	13 17 41 17	4 17 19 48	5 17 17 31	6 16 45 9	7 17 43 14	8 18 59 13
10	9 19 56 11	10 21 25 8	11 20 32 11	12 21 9 41	11 20 18 11	12 20 1 59	13 18 38 31	4 18 17 25	5 18 15 58	6 17 44 36	7 18 43 40	8 19 59 13
11	9 20 57 19	10 22 25 46	11 21 31 58	12 22 8 25	11 21 16 2	12 20 59 17	13 19 35 43	4 19 15 4	5 19 14 26	6 18 44 5	7 19 44 7	8 20 59 13
12	9 21 58 26	10 23 26 23	11 22 31 43	12 23 7 7	11 22 13 51	12 21 56 34	13 20 32 51	4 20 12 44	5 20 11 57	6 19 43 36	7 20 44 35	8 21 59 13
13	9 22 59 33	10 24 26 58	11 23 31 26	12 24 5 47	11 23 13 39	12 22 53 50	13 21 30 11	4 21 10 25	5 21 11 29	6 20 43 9	7 21 45 5	8 22 59 13
14	9 24 0 39	10 25 27 31	11 24 31 7	12 25 4 25	11 24 9 25	12 23 51 6	13 22 27 25	4 22 8 3	5 22 10 4	6 21 42 44	7 22 45 57	8 23 59 13
15	9 25 1 45	10 26 28 3	11 25 30 46	12 26 3 1	11 25 7 10	12 24 48 21	13 23 24 40	4 23 5 52	5 23 8 4	6 22 42 21	7 23 46 11	8 24 59 13
16	9 26 2 50	10 27 28 32	11 26 30 23	12 27 2 35	11 26 4 53	12 25 45 36	13 24 21 55	4 24 3 37	5 24 7 18	6 23 42 0	7 24 46 46	8 25 59 13
17	9 27 3 55	10 28 29 1	11 27 29 58	12 28 1 37	11 27 2 35	12 26 42 50	13 25 19 11	4 25 5 23	5 25 5 58	6 24 41 42	7 25 47 14	8 26 59 13
18	9 28 4 59	10 29 29 28	11 28 29 31	12 29 57 6	11 28 58 37	12 27 40 4	13 26 16 27	4 26 5 11	5 26 4 40	6 25 41 25	7 26 48 1	8 27 59 13
19	9 29 6 3	10 30 29 54	11 29 29 2	12 30 57 29	11 29 57 6	12 28 37 17	13 27 13 44	4 27 3 23	5 27 3 23	6 26 41 10	7 27 48 40	8 28 59 13
20	9 30 7 7	10 31 30 18	11 30 28 31	12 31 57 32	11 30 55 35	12 29 34 30	13 28 11 2	4 28 2 9	5 28 2 9	6 27 40 57	7 28 49 21	8 29 59 13
21	9 31 8 11	10 32 30 40	11 31 27 57	12 32 57 1	11 31 53 57	12 30 31 4	13 29 8 20	4 29 1 56	5 29 1 56	6 28 40 36	7 29 50 47	8 30 59 13
22	9 32 9 15	10 33 31 18	11 32 27 32	12 33 57 12	11 32 52 57	12 31 31 12	13 30 8 24	4 30 1 28	5 30 1 28	6 29 40 36	7 30 50 47	8 31 59 13
23	9 33 10 18	10 34 31 41	11 33 27 5	12 34 57 16	11 33 52 59	12 32 31 16	13 31 8 28	4 31 1 31	5 31 1 31	6 30 40 36	7 31 50 47	8 32 59 13
24	9 34 11 21	10 35 32 6	11 34 27 28	12 35 57 20	11 34 53 1	12 33 31 20	13 32 8 32	4 32 1 34	5 32 1 34	6 31 40 36	7 32 50 47	8 33 59 13
25	9 35 12 24	10 36 32 29	11 35 27 51	12 36 57 24	11 35 53 4	12 34 31 24	13 33 8 36	4 33 1 37	5 33 1 37	6 32 40 36	7 33 50 47	8 34 59 13
26	9 36 13 27	10 37 32 52	11 36 27 54	12 37 57 28	11 36 53 7	12 35 31 28	13 34 8 40	4 34 1 40	5 34 1 40	6 33 40 36	7 34 50 47	8 35 59 13
27	9 37 14 30	10 38 33 15	11 37 27 57	12 38 57 32	11 37 53 10	12 36 31 32	13 35 8 44	4 35 1 44	5 35 1 44	6 34 40 36	7 35 50 47	8 36 59 13
28	9 38 15 33	10 39 33 38	11 38 27 60	12 39 57 36	11 38 53 13	12 37 31 36	13 36 8 48	4 36 1 48	5 36 1 48	6 35 40 36	7 36 50 47	8 37 59 13
29	9 39 16 36	10 40 34 1	11 39 27 63	12 40 57 40	11 39 53 16	12 38 31 40	13 37 8 52	4 37 1 52	5 37 1 52	6 36 40 36	7 37 50 47	8 38 59 13
30	9 40 17 39	10 41 34 24	11 40 27 66	12 41 57 44	11 40 53 19	12 39 31 44	13 38 8 56	4 38 1 56	5 38 1 56	6 37 40 36	7 38 50 47	8 39 59 13
31	9 41 18 42	10 42 34 47	11 41 27 69	12 42 57 48	11 41 53 22	12 40 31 48	13 39 8 60	4 39 1 60	5 39 1 60	6 38 40 36	7 39 50 47	8 40 59 13

Days.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
January.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
February.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
March.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
April.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
May.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
June.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
July.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
August.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
September.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
October.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
November.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
December.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	10 12 49 43	11 11 32	11 11 32	11 11 32	11 11 32	11 11 32	11 11 32	11 11 32	11 11 32	11 11 32	11 11 32	11 11 32
2	10 16 23 2	10 17 53 36	11 16 3 38	11 16 45 52	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28
3	9 19 33 3	10 14 51 20	11 13 3 40	11 13 49 2	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25
4	9 14 20 43	10 15 52 7	11 14 3 41	11 14 48 0	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2
5	9 15 21 53	10 16 52 52	11 15 3 40	11 15 46 57	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2
6	9 16 23 2	10 17 53 36	11 16 3 38	11 16 45 52	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28
7	9 19 33 3	10 14 51 20	11 13 3 40	11 13 49 2	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25
8	9 14 20 43	10 15 52 7	11 14 3 41	11 14 48 0	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2
9	9 15 21 53	10 16 52 52	11 15 3 40	11 15 46 57	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2
10	9 16 23 2	10 17 53 36	11 16 3 38	11 16 45 52	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28
11	9 19 33 3	10 14 51 20	11 13 3 40	11 13 49 2	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25
12	9 14 20 43	10 15 52 7	11 14 3 41	11 14 48 0	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2
13	9 15 21 53	10 16 52 52	11 15 3 40	11 15 46 57	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2
14	9 16 23 2	10 17 53 36	11 16 3 38	11 16 45 52	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28
15	9 19 33 3	10 14 51 20	11 13 3 40	11 13 49 2	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25
16	9 14 20 43	10 15 52 7	11 14 3 41	11 14 48 0	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2
17	9 15 21 53	10 16 52 52	11 15 3 40	11 15 46 57	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2
18	9 16 23 2	10 17 53 36	11 16 3 38	11 16 45 52	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28
19	9 19 33 3	10 14 51 20	11 13 3 40	11 13 49 2	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25
20	9 14 20 43	10 15 52 7	11 14 3 41	11 14 48 0	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2
21	9 15 21 53	10 16 52 52	11 15 3 40	11 15 46 57	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2
22	9 16 23 2	10 17 53 36	11 16 3 38	11 16 45 52	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28
23	9 19 33 3	10 14 51 20	11 13 3 40	11 13 49 2	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25
24	9 14 20 43	10 15 52 7	11 14 3 41	11 14 48 0	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2
25	9 15 21 53	10 16 52 52	11 15 3 40	11 15 46 57	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2
26	9 16 23 2	10 17 53 36	11 16 3 38	11 16 45 52	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28
27	9 19 33 3	10 14 51 20	11 13 3 40	11 13 49 2	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25
28	9 14 20 43	10 15 52 7	11 14 3 41	11 14 48 0	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2	11 14 28 2
29	9 15 21 53	10 16 52 52	11 15 3 40	11 15 46 57	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2	11 15 20 2
30	9 16 23 2	10 17 53 36	11 16 3 38	11 16 45 52	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28	11 15 58 28
31	9 19 33 3	10 14 51 20	11 13 3 40	11 13 49 2	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25	11 13 42 25

A TABLE of the SUN'S LONGITUDE for the Years 1774, 1778, and 1782, being the second after Leap Year.

June from 1774 to 1782

308. 1773

30	10 10 47 50	10 11 48 51
31	10 11 48 51	

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	9 11 2 21	10 12 34 56	11 10 48 57	0 11 36 38	1 10 54 12	2 10 44 53	3 9 22 13	4 8 57 51	5 8 48 56	6 8 7 48	7 8 57 2	8 9 13 51
2	9 13 4 41	10 14 35 46	11 11 49 2	0 13 35 42	1 12 52 18	2 11 41 34	3 10 19 24	4 9 55 18	5 9 47 1	6 9 6 57	7 9 57 12	8 10 14 46
3	9 14 5 51	10 15 37 21	11 13 49 8	0 14 33 44	1 13 48 25	2 13 36 21	3 12 13 47	4 11 50 16	5 11 43 28	6 11 5 21	7 11 57 38	8 11 16 39
4	9 15 7 1	10 16 38 7	11 14 49 8	0 15 32 40	1 14 46 27	2 14 13 44	3 13 10 59	4 12 47 47	5 12 41 44	6 12 4 36	7 12 57 54	8 13 17 37
5	9 16 8 10	10 17 38 51	11 15 49 6	0 16 31 36	1 15 44 27	2 15 31 6	3 14 8 11	4 13 45 18	5 13 40 2	6 13 3 53	7 13 58 12	8 14 18 36
6	9 17 9 19	10 18 39 34	11 16 49 3	0 17 30 30	1 16 42 25	2 16 28 25	3 15 5 23	4 14 38 21	5 14 33 51	6 14 3 12	7 14 58 31	8 15 19 35
7	9 18 10 28	10 19 40 16	11 17 48 57	0 18 29 22	1 17 40 21	2 17 25 47	3 16 4 15	4 15 40 25	5 15 36 42	6 15 3 15	7 15 58 52	8 16 20 35
8	9 19 11 37	10 20 40 57	11 18 48 49	0 19 28 11	1 18 38 16	2 18 23 7	3 16 59 47	4 16 38 0	5 16 35 5	6 16 1 56	7 16 59 15	8 17 21 36
9	9 20 12 4	10 21 41 36	11 19 48 39	0 20 26 58	1 19 36 10	2 19 20 25	3 17 57 0	4 17 35 36	5 17 33 30	6 17 1 21	7 17 59 40	8 18 22 38
10	9 21 13 54	10 22 42 14	11 20 48 28	0 21 25 43	1 20 34 2	2 20 17 43	3 18 54 13	4 18 33 14	5 18 31 57	6 18 0 49	7 18 59 0	8 19 23 41
11	9 22 15 1	10 23 42 50	11 21 48 15	0 22 24 26	1 21 31 53	2 21 15 0	3 19 51 27	4 19 30 5	5 19 30 26	6 19 0 18	7 19 59 49	8 20 24 45
12	9 23 16 8	10 24 43 22	11 22 48 0	0 23 23 8	1 22 29 42	2 22 12 17	3 20 48 41	4 20 28 33	5 20 28 57	6 20 0 19	7 20 59 22	8 21 25 49
13	9 24 17 14	10 25 43 57	11 23 47 43	0 24 21 48	1 23 27 30	2 23 9 33	3 21 45 55	4 21 26 15	5 21 27 30	6 21 0 59	7 21 58 22	8 22 26 54
14	9 25 18 19	10 26 44 20	11 24 47 23	0 25 20 25	1 24 25 16	2 24 6 49	3 22 43 10	4 22 23 58	5 22 26 5	6 22 0 58	7 22 58 7	8 23 27 59
15	9 26 19 24	10 27 44 59	11 25 47 1	0 26 19 1	1 25 23 1	2 25 4 4	3 23 40 25	4 23 21 42	5 23 24 24	6 23 0 58	7 23 58 36	8 24 29 5
16	9 27 20 29	10 28 45 27	11 26 46 37	0 27 17 35	1 26 20 44	2 26 58 26	3 24 37 40	4 24 17 27	5 24 23 20	6 24 0 56	7 24 57 56	8 25 30 12
17	9 28 21 33	10 29 45 54	11 27 46 11	0 28 16 17	1 27 18 26	2 27 58 34	3 25 34 56	4 25 17 14	5 25 22 0	6 25 0 52	7 25 57 39	8 26 31 19
18	9 29 22 36	10 30 46 19	11 28 45 44	0 29 14 37	1 28 16 7	2 27 55 47	3 26 32 12	4 26 15 2	5 26 20 42	6 26 0 42	7 26 57 27	8 27 32 27
19	9 30 23 38	10 31 46 43	11 29 45 14	0 30 13 5	1 29 13 46	2 28 53 1	3 27 29 29	4 27 12 52	5 27 19 19	6 27 0 56	7 27 57 24	8 28 33 35
20	10 1 24 40	11 1 46 43	11 30 45 14	0 31 12 31	1 30 11 24	2 29 50 14	3 28 26 47	4 28 10 43	5 28 18 13	6 28 0 52	7 28 57 12	8 29 34 43
21	10 2 25 41	11 2 47 5	11 31 44 8	0 32 11 9	1 31 9 1	2 30 47 27	3 29 24 5	4 29 8 36	5 29 17 1	6 29 0 53	7 29 56 18	8 30 35 51
22	10 3 26 41	11 3 47 25	11 32 43 33	0 33 10 55	1 32 8 17	2 31 44 39	3 29 24 5	4 29 18 45	5 29 26 6	6 30 0 15	7 30 55 28	8 31 37 0
23	10 4 27 40	11 4 47 43	11 33 42 56	0 34 10 33	1 33 7 42	2 32 41 51	3 30 24 4	4 30 18 44	5 30 26 14	6 31 0 44	7 31 54 47	8 32 38 9
24	10 5 28 38	11 5 48 14	11 34 42 16	0 35 9 57	1 34 6 16	2 33 39 4	3 31 24 4	4 31 12 45	5 31 20 23	6 32 0 26	7 32 54 41	8 33 39 25
25	10 6 29 35	11 6 48 35	11 35 41 33	0 36 9 33	1 35 5 42	2 34 36 16	3 32 27 4	4 32 20 4	5 32 28 6	6 33 0 22	7 33 53 7	8 34 39 26
26	10 7 30 31	11 7 48 39	11 36 40 49	0 37 9 10	1 36 5 17	2 35 33 27	3 33 27 4	4 33 20 5	5 33 28 6	6 34 0 18	7 34 53 7	8 35 39 27
27	10 8 31 26	11 8 48 39	11 37 40 1	0 38 8 45	1 37 4 42	2 36 30 39	3 34 27 4	4 34 20 5	5 34 28 6	6 35 0 13	7 35 53 7	8 36 39 28
28	10 9 32 20	11 9 48 26	11 38 39 15	0 39 8 19	1 38 3 16	2 37 27 50	3 35 27 4	4 35 20 5	5 35 28 6	6 36 0 8	7 36 53 7	8 37 39 29
29	10 10 33 13	11 10 48 11	11 39 38 26	0 40 7 54	1 39 2 41	2 38 24 16	3 36 26 4	4 36 20 5	5 36 28 6	6 37 0 3	7 37 53 7	8 38 39 30
30	10 11 34 5	11 11 47 5	11 40 37 33	0 41 7 29	1 40 1 16	2 39 21 43	3 37 23 4	4 37 16 4	5 37 19 13	6 38 0 27	7 38 53 7	8 39 39 31
31	10 12 35 11	11 12 46 11	11 41 36 40	0 42 6 54	1 41 0 41	2 40 18 50	3 38 22 4	4 38 16 4	5 38 19 13	6 39 0 32	7 39 53 7	8 40 39 32

308. A TABLE OF THE SUN'S LONGITUDE FOR THE YEARS 1775, 1779, AND 1783, BEING THE THIRD AFTER LEAP YEAR.

309. A TABLE of the SUN'S DECLINATION for the Years 1772, 1776, and 1780, being Leap Years.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	13 1 45	17 7 36	14 29	4 52 35	15 19 55	22 10 37	23 5 4	7 51 7	0 35	3 30 40	14 43 48	1 58 14
2	22 56 34	16 50 24	51 34	5 15 36	15 37 43	22 18 17	23 0 33	17 35 37	7 38 35	3 53 57	15 2 44	22 6 59
3	32 50 57	16 32 54	28 33	5 38 31	15 55 13	22 25 34	22 55 37	17 19 51	4 17 12	15 21 26	15 18	32 23 11
4	42 44 52	16 15 7	6 1 21	6 1 21	16 12 35	22 32 27	22 50 17	17 3 48	54 14	4 40 25	15 39 53	22 23 11
5	52 38 20	15 57 3	5 42 15	6 24 4	16 29 34	22 38 47	22 44 33	16 47 28	6 31 54	5 3 34	15 58 4	22 30 38
6	62 31 21	15 38 42	5 18 58	6 46 41	16 46 19	22 45 3	22 38 25	16 30 52	9 28	5 26 39	16 15 59	22 37 38
7	72 23 56	15 20 54	5 55 37	7 9 10	17 2 48	22 50 46	22 31 54	16 13 59	5 46 55	5 49 40	16 33 38	22 44 12
8	82 16 5	15 1 13	7 31 13	7 31 32	17 19 6	22 56 5	22 24 59	15 56 31	5 24 16	6 12 36	16 51 0	22 50 19
9	92 7 47	14 42 5	8 45	7 53 46	17 34 54	23 1 9	22 17 42	15 39 28	1 32	6 35 27	17 8	22 55 59
10	10 1 58	4 22 43	8 15 14	8 15 52	17 40 30	23 5 4	22 10 1	15 21 50	4 38 42	6 58 13	17 24 52	23 1 12
11	11 21 49	53 14 3	21 40	8 37 51	18 5 49	23 9 36	22 1 58	15 3 57	4 15 47	7 20 53	17 41 21	23 5 57
12	12 21 40	18 13 17	5 58	8 59 42	18 20 50	23 13 17	21 53 32	15 45 49	5 52 48	7 43 28	17 57 32	23 10 14
13	13 21 30	13 13 1	9 21 24	9 21 24	18 35 57	23 19 24	21 35 34	14 27 27	8 29 45	8 5 56	18 13 24	23 14 4
14	14 21 19	52 13 2	10 48	9 42 56	18 49 57	23 25 57	21 44 44	14 14 45	16 38	8 28 17	18 28 57	23 17 26
15	15 21 9	2 12 42	1 47	10 4 18	19 4 1	23 21 51	21 26 1	13 40 2	2 43 28	8 50 31	18 44 11	23 20 21
16	16 20 57	48 12 21	45 1 23	10 25 30	19 17 46	23 23 53	21 16 6	13 30 59	2 20 14	9 12 37	18 59 4	23 22 47
17	17 20 46	9 12 0	51 0	10 46 32	19 31 11	23 26 30	21 6 50	13 11 41	5 56	9 34 36	19 13 36	23 24 45
18	18 20 34	7 11 39	46 0	11 7 23	19 44 16	23 27 27	20 44 13	12 52 16	3 33 39	9 56 26	19 27 47	23 26 15
19	19 20 21	41 11 18	31 0	12 20	11 23 4	23 27 31	20 44 13	12 32 35	1 10 18	10 18 7	19 41 38	23 27 17
20	20 20 8	53 10 57	5	N. 11 21	11 48 11	23 27 54	20 32 53	12 12 42	0 46 55	10 40 39	19 55 8	23 27 50
21	21 19 55	42 10 35	29 0	12 8 50	20 21 52	23 27 53	20 21 13	11 52 57	0 23 36	11 1 1	20 8 16	23 27 56
22	22 19 42	9 10 13	43 0	12 28 56	20 33 16	23 27 27	19 12 0	11 32 21	0 5	11 22 13	20 21 1	23 27 33
23	22 19 28	14 9 51	47 1	12 48 49	20 44 38	23 26 36	19 56 51	11 11 55	5 23 21	11 43 15	20 33 24	23 26 42
24	22 19 13	58 19 13	58	13 8 29	20 55 39	23 25 26	19 44 9	10 51 18	0 46 48	12 4 7	20 45 24	23 25 21
25	25 18 59	21 9 7	3 9	13 27 57	21 6 19	23 23 46	18 31 8	10 20 31	1 10 15	12 24 48	20 57 1	23 23 33
26	26 18 44	23 8 45	3 2	13 47 12	21 16 37	23 21 35	19 17 47	10 9 33	1 33 42	12 45 17	21 8 14	23 21 16
27	27 18 29	4 8 22	32	14 6 13	21 26 33	23 19 12	18 50 8	9 48 26	1 57 7	13 5 34	21 19 3	23 18 32
28	28 18 13	24 8 0	33	14 25 0	21 45 19	23 12 57	18 35 50	9 5 44	2 43 58	13 45 31	21 39 2	23 11 40
29	29 17 57	25 7 37	21	15 1 51	21 54 8	23 9 11	18 21 12	8 44 9	3 7 20	14 5 10	21 49 4	23 7 32
30	30 17 41	7 17 41	7	16 18 1	21 54 8	23 9 11	18 21 12	8 44 9	3 7 20	14 5 10	21 49 4	23 7 32
31	31 17 24	31 17 24	31	17 2 34	22 2 34	23 2 34	18 6 16	8 22 26	14 24 35	14 24 35	23 2 50	31

309. 1772

309. A TABLE of the SUN'S DECLINATION for the Years 1773, 1777, and 1781, being the first after Leap Years.

Day.	Decemb.	Novemb.	October.	Septemb.	August.	July.	June.	May.	April.	March.	February.	January.
1	11	10	9	8	7	6	5	4	3	2	1	0
2	11	10	9	8	7	6	5	4	3	2	1	0
3	11	10	9	8	7	6	5	4	3	2	1	0
4	11	10	9	8	7	6	5	4	3	2	1	0
5	11	10	9	8	7	6	5	4	3	2	1	0
6	11	10	9	8	7	6	5	4	3	2	1	0
7	11	10	9	8	7	6	5	4	3	2	1	0
8	11	10	9	8	7	6	5	4	3	2	1	0
9	11	10	9	8	7	6	5	4	3	2	1	0
10	11	10	9	8	7	6	5	4	3	2	1	0
11	11	10	9	8	7	6	5	4	3	2	1	0
12	11	10	9	8	7	6	5	4	3	2	1	0
13	11	10	9	8	7	6	5	4	3	2	1	0
14	11	10	9	8	7	6	5	4	3	2	1	0
15	11	10	9	8	7	6	5	4	3	2	1	0
16	11	10	9	8	7	6	5	4	3	2	1	0
17	11	10	9	8	7	6	5	4	3	2	1	0
18	11	10	9	8	7	6	5	4	3	2	1	0
19	11	10	9	8	7	6	5	4	3	2	1	0
20	11	10	9	8	7	6	5	4	3	2	1	0
21	11	10	9	8	7	6	5	4	3	2	1	0
22	11	10	9	8	7	6	5	4	3	2	1	0
23	11	10	9	8	7	6	5	4	3	2	1	0
24	11	10	9	8	7	6	5	4	3	2	1	0
25	11	10	9	8	7	6	5	4	3	2	1	0
26	11	10	9	8	7	6	5	4	3	2	1	0
27	11	10	9	8	7	6	5	4	3	2	1	0
28	11	10	9	8	7	6	5	4	3	2	1	0
29	11	10	9	8	7	6	5	4	3	2	1	0
30	11	10	9	8	7	6	5	4	3	2	1	0
31	11	10	9	8	7	6	5	4	3	2	1	0

309. A TABLE of the SUN'S DECLINATION for the Years 1774, 1778, and 1782, being the second after Leap Years.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	12 59 16	6 58 50	25 37	4 41 24	15 11 13	22 6 49	23 7 8	17 58 33	8 11 11	3 19 22	14 34 31	21 53 50
2	12 53 52	16 41 28	7 2 43	5 4 28	15 29 9	22 14 40	23 2 48	17 49 13	7 49 16	3 42 41	14 53 35	22 1 47
3	12 48 0	16 23 48	6 39 44	5 27 27	15 46 49	22 22 8	23 8 22	17 43 12	7 27 13	4 5 58	15 12 24	22 1 19
4	12 41 41	16 5 51	6 16 40	5 50 20	16 4 15	22 29 12	23 15 56	17 11 38	7 5 3	4 29 12	15 30 59	22 19 24
5	12 34 54	15 47 38	5 53 31	6 13 6	16 21 25	22 35 53	23 22 23	16 47 23	6 42 45	4 52 22	15 49 18	22 27 4
6	12 27 42	15 29 10	5 30 17	6 35 43	16 38 13	22 42 10	23 41 27	16 38 58	5 20 21	4 15 28	16 7 21	22 34 18
7	12 20 3	15 10 26	5 6 58	6 58 16	16 54 13	22 48 3	23 5 13	16 22 13	5 57 51	4 38 31	16 25 38	22 41 5
8	12 11 58	14 51 26	4 43 35	7 20 42	17 11 54	22 53 33	23 12 26	16 5 13	5 35 15	4 1 29	16 42 38	22 47 25
9	12 3 26	14 32 11	4 20 9	7 43 2	17 27 15	22 58 39	23 19 20	15 47 57	5 12 33	4 24 23	16 59 52	22 53 18
10	12 1 54 28	14 12 41	3 56 38	8 5 13	17 43 0	23 3 20	23 26 13	15 30 27	4 49 46	4 47 12	17 16 48	23 44
11	12 1 45 5	13 52 57	3 33 6	8 27 15	17 58 27	23 7 3	23 5 58	15 12 39	4 26 54	7 9 55	17 33 26	23 3 43
12	12 1 35 17	13 33 0	3 9 32	8 49 9	18 13 36	23 11 30	23 14 43	14 54 38	4 3 57	7 32 32	17 49 45	23 8 14
13	12 1 25 3	13 12 50	2 45 55	9 10 54	18 28 28	23 14 58	23 21 40	14 36 23	3 40 56	7 55 3	18 5 46	23 12 18
14	12 1 14 25	12 52 26	2 22 16	9 32 3	18 43 0	23 18 2	23 28 18	14 17 54	3 17 51	8 17 27	18 21 28	23 15 56
15	12 1 3 22	12 31 50	1 58 36	9 53 5	18 57 13	23 20 42	23 30 43	13 59 11	2 54 42	8 39 45	18 36 54	23 19 2
16	12 0 51 56	12 11 3	1 34 55	10 15 15	19 1 7	23 22 56	23 32 59	13 40 14	2 31 30	9 1 55	18 51 54	23 21 42
17	12 0 40 5	11 50 4	1 11 14	10 36 22	19 24 46	23 24 46	23 34 21	13 21 5	2 8 15	9 23 58	19 6 37	23 23 54
18	12 0 27 51	11 28 54	0 47 33	10 57 18	19 37 57	23 26 11	23 36 26	12 42 8	1 44 57	9 45 52	19 20 59	23 25 38
19	12 0 15 14	11 7 32	0 23 51	11 18 3	19 50 52	23 27 12	23 38 26	12 22 20	1 21 37	10 7 37	19 35 0	23 26 53
20	12 0 2 14	10 46 0	0 0 9	11 38 37	20 3 28	23 27 48	23 40 38	12 2 58	0 58 15	10 29 13	19 48 40	23 27 41
21	12 0 48 52	10 24 18	N. 23 32	11 59 0	20 27 37	23 27 45	23 42 5	12 2 23	0 34 51	10 50 40	20 1 58	23 28 0
22	12 0 35 8	10 2 27	0 47 12	12 12 19	20 15 4	23 27 45	23 44 15	11 42 13	0 11 26	11 11 58	20 14 54	23 27 51
23	12 0 21 2	9 40 27	1 10 50	12 39 11	20 39 9	23 27 6	23 46 20	11 21 52	5 12 0	11 33 6	20 27 28	23 27 13
24	12 0 6 34	9 18 18	1 34 26	12 58 58	20 50 21	23 26 2	23 48 19	11 20 0	0 35 27	11 54 3	20 39 39	23 26 7
25	12 0 51 46	8 56 1	1 58 0	13 18 13	21 1 12	23 24 33	23 50 38	10 40 37	0 58 54	12 14 49	20 51 27	23 24 32
26	12 0 36 38	8 33 36	2 21 31	13 37 54	21 11 41	23 22 40	23 52 17	10 19 44	1 22 21	12 35 24	21 2 51	23 22 30
27	12 0 21 8	8 11 4	2 55 0	13 57 2	21 21 49	23 20 22	23 54 19	9 58 41	1 45 47	12 55 47	21 13 51	23 19 59
28	12 0 5 18	7 48 24	3 8 25	14 15 56	21 31 34	23 17 39	23 56 57	9 37 29	2 9 13	13 15 58	21 24 27	23 17 1
29	12 0 17 49 10	6 31 46	3 31 46	14 34 37	21 40 57	23 14 31	23 58 48	9 16 7	2 32 37	13 35 56	21 34 40	23 13 34
30	12 0 3 32	5 18 5	3 14 53	14 53 1	21 49 57	23 11 0	23 60 21	8 32 58	2 56 0	13 55 41	21 44 28	23 9 30
31	12 0 17 32 43	4 18 10	3 55 3	15 10 4	21 58 35	23 8 35	23 62 21	8 13 36	2 56 0	14 15 13	21 55 41	23 5 16

309. A TABLE OF THE SUN'S DECLINATION FOR THE YEARS 1775, 1779, AND 1783, BEING THE THIRD AFTER LEAP YEARS.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	23 0 26	17 2 58	7 31 8	4 35 48	15 6 51	22 4 50	28 8 4	18 2 12	8 16 28	3 13 42	14 29 50	21 51 34
2	22 55 7	16 45 40	8 16	4 58 55	15 42 50	22 20 26	17 46 55	7 54 34	3 48 55	4 0 18	15 48 58	22 9 14
3	22 49 22	16 28 5	6 45 19	5 21 53	15 42 30	22 20 20	17 31 20	7 32 32	3 37 1	4 0 18	15 26 29	22 9 14
4	22 43 9	16 10 13	6 22 17	5 45 46	16 0	22 27 30	17 54 10	7 15 29	4 23 32	4 23 32	15 26 29	22 17 26
5	22 36 30	15 52 4	5 59 9	6 7 33	16 17 15	22 34 17	17 48 44	6 59 21	4 46 44	4 46 44	15 44 52	22 25 12
6	22 29 24	15 33 39	5 35 5	6 30 14	16 34 12	22 40 40	17 42 54	6 42 57	5 25 46	5 9 52	16 2 59	22 32 32
7	22 21 51	15 14 59	5 12 37	6 52 49	17 50 53	22 52 15	17 36 41	6 26 16	5 35 56	5 35 56	16 20 49	22 39 25
8	22 13 52	14 56 3	4 49 15	7 15 17	17 37 38	22 57 27	17 22 3	6 10 15	5 55 56	5 18 51	16 55 41	22 51 51
9	22 5 26	14 36 51	4 25 49	7 37 38	17 23 23	22 57 27	17 7 5	5 52 7	5 18 51	5 18 51	16 55 41	22 51 51
10	21 56 35	14 17 25	4 2 10	8 21 55	17 54 43	23 2 15	16 57 4	5 16 57	4 32 26	7 4 25	17 29 25	23 2 30
11	21 47 18	13 57 45	3 38 48	8 43 51	18 9 57	23 10 37	16 40 48	4 59 0	7 27 4	17 45 49	18 1 55	23 7 8
12	21 37 36	13 37 51	3 15 14	9 5 38	18 24 53	23 14 11	16 22 22	4 46 30	7 49 37	18 1 55	18 1 55	23 11 18
13	21 27 28	13 17 44	2 51 37	9 27 17	18 39 31	23 17 21	16 4 14	4 32 59	8 12 3	18 17 42	18 3 9	23 15 14
14	21 16 56	12 57 24	2 27 59	9 48 46	18 53 50	23 20 7	15 42 16	4 22 22	8 34 22	18 17 42	18 3 9	23 18 16
15	21 5 59	12 36 51	2 4 20	9 48 46	18 53 50	23 20 7	15 42 16	4 22 22	8 34 22	18 17 42	18 3 9	23 18 16
16	20 54 38	12 16 6	1 40 39	10 10 6	19 7 49	23 22 28	15 23 19	4 44 49	8 56 34	18 48 17	18 3 9	23 21 3
17	20 42 53	11 55 9	1 16 58	10 31 15	19 21 29	23 24 24	15 13 53	4 32 43	9 18 37	19 3 5	19 17 38	23 23 21
18	20 30 45	11 34 1	1 53 16	10 52 13	19 35 48	23 25 55	15 2 55	4 24 1	9 40 34	19 17 38	19 17 38	23 26 34
19	20 18 14	11 12 43	1 29 34	11 13 2	19 47 48	23 27 42	15 52 12	4 6 52	10 2 21	19 31 38	20 4 5	23 29 30
20	20 5 19	10 51 14	1 5 53	11 33 39	20 0 28	23 27 42	15 40 41	3 56	10 24 0	19 45 22	20 4 5	23 32 10
21	19 52 2	10 29 35	N. 17 48	11 54 5	20 12 47	23 27 58	15 29 42	3 40 32	10 45 30	19 58 45	20 4 5	23 35 21
22	19 38 23	10 7 46	41 28	12 14 19	20 24 47	23 27 49	15 17 55	3 24 17	11 6 50	20 11 46	20 4 5	23 38 27
23	19 24 22	9 45 48	5 6	12 34 22	20 36 26	23 27 15	15 5 48	3 10 47	11 26 47	20 24 25	20 4 5	23 41 24
24	19 10 1	9 23 41	1 28 43	12 54 13	20 47 43	23 26 17	14 53 21	2 59 18	11 6 18	20 36 41	20 4 5	23 44 29
25	18 55 18	9 1 26	1 52 17	13 13 51	20 58 39	23 24 44	14 40 34	2 45 39	12 9 48	20 48 35	20 4 5	23 47 28
26	18 40 14	8 39 3	2 15 49	13 33 16	21 9 13	23 23 7	14 27 28	2 30 2	12 30 2	21 0	20 4 5	23 50 26
27	18 24 50	8 16 32	3 39 18	13 52 27	21 19 25	23 20 56	14 14 3	2 14 49	12 50 51	21 11 12	20 4 5	23 53 20
28	18 9 6	7 53 54	3 2 44	14 11 25	21 29 15	23 18 20	14 0 18	2 0 39	13 11 5	21 21 54	20 4 5	23 56 14
29	17 53 2	7 30 6	3 26 6	14 30 8	21 38 42	23 15 19	13 46 15	1 49 2	13 31 6	21 32 12	20 4 5	23 59 14
30	17 36 40	7 7 2	3 49 23	14 48 44	21 47 4	23 11 53	13 31 53	1 32 50	13 50 2	21 42 5	20 4 5	24 0 36
31	17 19 59	6 40	3 49 23	14 48 44	21 47 4	23 11 53	13 31 53	1 32 50	13 50 2	21 42 5	20 4 5	24 0 36

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31	17 15 57		4 18 10		21 58 35	18 13 36	8 32 58	14 15 13	14 15 13	14 15 13
30	17 32 43	3 55	4 53	21 49 27	21 58 35	18 13 36	8 32 58	14 15 13	14 15 13	14 15 13
29	17 12 43	3 55	4 53	21 49 27	21 58 35	18 13 36	8 32 58	14 15 13	14 15 13	14 15 13

310. A TABLE of the SUN'S RIGHT ASCENSION in Time for the Years 1772, 1776, and 1780, being Leap Years.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	18 46 49	20 59 6	22 51 55	0 45 20	2 36 40	4 39 32	6 43 47	8 48 25	10 44 21	12 32 30	14 29 5	16 33 18
2	18 51 14	21 3 39	22 55 22	0 48 59	2 40 19	4 43 38	6 47 55	8 52 18	10 47 58	12 36 8	14 33 2	16 37 38
3	18 55 39	21 7 13	22 59 22	0 52 37	2 44 19	4 47 45	6 52 2	8 56 10	10 51 10	12 40 13	14 37 25	16 41 59
4	19 0 4	21 11 16	23 3 5	0 56 15	2 48 9	4 51 52	6 56 10	8 59 10	10 54 13	12 43 25	14 40 57	16 45 20
5	19 4 28	21 15 18	23 6 47	0 59 54	2 52 0	4 55 59	7 0 17	9 3 51	10 58 50	12 47 4	14 44 56	16 50 42
6	19 8 51	21 19 19	23 10 29	1 3 33	2 55 52	5 0 6	7 4 24	9 7 41	11 2 27	12 50 44	14 48 56	16 55 5
7	19 13 13	21 23 19	23 14 11	1 7 13	2 59 45	5 4 13	7 8 30	9 11 31	11 6 3	12 54 23	14 52 57	16 59 28
8	19 17 35	21 27 18	23 17 52	1 10 52	3 3 38	5 8 21	7 12 36	9 15 20	11 9 39	12 58 3	14 56 58	17 3 51
9	19 21 57	21 31 16	23 21 33	1 14 32	3 7 31	5 12 29	7 16 42	9 19 8	11 13 15	13 1 58	14 59 58	17 8 15
10	19 26 18	21 35 13	23 25 13	1 18 12	3 11 25	5 16 37	7 20 47	9 22 56	11 16 51	13 5 26	14 55 5	17 12 39
11	19 30 38	21 39 10	23 28 54	1 21 54	3 15 20	5 20 46	7 24 51	9 26 44	11 20 27	13 9 7	15 9 7	17 17 4
12	19 34 58	21 43 6	23 32 33	1 25 32	3 19 15	5 24 55	7 28 55	9 30 31	11 24 3	13 12 50	15 13 12	17 21 29
13	19 39 18	21 47 1	23 36 11	1 29 13	3 23 10	5 29 4	7 32 59	9 34 16	11 27 39	13 16 32	15 16 17	17 25 54
14	19 43 37	21 50 55	23 39 53	1 32 54	3 27 6	5 33 12	7 37 2	9 38 1	11 31 14	13 20 15	15 21 24	17 30 20
15	19 47 55	21 54 49	23 43 32	1 36 36	3 31 3	5 37 22	7 41 5	9 41 46	11 34 50	13 23 59	15 25 31	17 34 46
16	19 52 12	21 58 42	23 47 10	1 40 18	3 35 1	5 41 31	7 45 7	9 45 31	11 38 25	13 27 44	15 29 40	17 39 12
17	19 56 28	22 2 34	23 50 49	1 44 0	3 38 59	5 45 40	7 49 9	9 49 15	11 42 1	13 31 29	15 33 49	17 43 38
18	20 0 45	22 6 26	23 54 28	1 47 43	3 42 57	5 49 49	7 53 9	9 52 58	11 45 36	13 35 14	15 37 59	17 48 4
19	20 5 0	22 10 17	23 58 6	1 51 26	3 46 56	5 53 59	7 57 10	9 56 41	11 49 12	13 39 0	15 42 10	17 52 31
20	20 9 14	22 14 7	0 1 44	1 55 9	3 50 56	5 58 8	8 1	10 0 24	11 52 48	13 42 47	15 46 21	17 56 57
21	20 13 28	22 17 56	0 5 22	1 58 53	3 54 56	6 2 18	8 5	10 4 7	11 56 23	13 46 35	15 50 33	18 1 24
22	20 17 41	22 21 45	0 9 0	1 62 38	3 58 57	6 6 28	8 9	10 8 12	12 0 7	13 50 23	15 54 46	18 5 51
23	20 21 53	22 25 34	0 12 39	1 66 23	4 2 59	6 10 38	8 13	10 11 29	12 4 36	13 54 12	15 58 0	18 10 18
24	20 26 5	22 29 22	0 16 17	1 70 8	4 6 59	6 14 47	8 17	10 15 9	12 8 11	13 58 1	16 3 16	18 14 44
25	20 30 16	22 33 9	0 19 54	1 73 53	4 11 3	6 18 56	8 21	10 18 49	12 11 48	14 1 52	16 7 31	18 19 10
26	20 34 25	22 36 55	0 23 32	1 77 40	4 15 5	6 23 4	8 25	10 22 29	12 15 24	14 5 43	16 11 47	18 23 36
27	20 38 33	22 40 41	0 27 10	1 81 27	4 19 8	6 28 31	8 29	10 26 14	12 19 9	14 9 38	16 15 34	18 27 28
28	20 42 41	22 44 26	0 30 48	1 85 15	4 23 13	6 33 17	8 33	10 30 3	12 23 15	14 13 31	16 19 21	18 31 29
29	20 46 49	22 48 11	0 34 26	1 89 4	4 27 17	6 38 3	8 37	10 34 39	12 27 52	14 17 20	16 23 8	18 35 55
30	20 50 55	22 51 55	0 38 4	1 93 51	4 31 22	6 43 9	8 41	10 39 10	12 32 38	14 21 14	16 27 59	18 40 1
31	20 55 1	22 55 5	0 41 42	1 97 42	4 35 27	6 48 6	8 45	10 43 32	12 37 5	14 25 0	16 32 59	18 44 20

310. A TABLE of the SUN'S RIGHT ASCENSION in Time for the Years 1773, 1777, and 1781, being the first after Leap Years.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	Septemb.	October.	November.	Decemb.	Days.
1	18 50 10	22 51 0	44 27 2	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	1
2	18 54 35	21 6 14	44 22 54	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	2
3	18 58 59	21 10 17	44 18 22	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	3
4	19 3 23	21 14 19	44 14 23	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	4
5	19 7 46	21 18 20	44 10 23	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	5
6	19 12 5	21 22 20	44 6 23	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	6
7	19 16 32	21 26 20	44 2 23	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	7
8	19 20 54	21 30 18	44 23 16	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	8
9	19 25 15	21 34 16	44 18 16	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	9
10	19 29 35	21 38 13	44 13 16	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	10
11	19 33 55	21 42 9	44 8 13	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	11
12	19 38 15	21 46 5	44 3 10	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	12
13	19 42 34	21 50 1	44 23 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	13
14	19 46 52	21 53 53	44 18 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	14
15	19 51 10	21 57 46	44 13 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	15
16	19 55 26	22 1 38	44 8 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	16
17	19 59 43	22 5 30	44 3 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	17
18	20 3 58	22 9 21	44 23 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	18
19	20 8 12	22 13 11	44 18 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	19
20	20 12 26	22 17 1	44 13 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	20
21	20 16 39	22 20 50	44 8 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	21
22	20 20 52	22 24 38	44 3 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	22
23	20 25 4	22 28 26	44 23 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	23
24	20 29 15	22 32 13	44 18 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	24
25	20 33 25	22 36 0	44 13 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	25
26	20 37 34	22 39 46	44 8 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	26
27	20 41 41	22 43 32	44 3 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	27
28	20 45 49	22 47 16	44 23 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	28
29	20 49 56	22 51 0	44 18 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	29
30	20 54 2	22 54 44	44 13 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	30
31	20 58 7	22 57 36	44 8 5	35 44 4	46 23 6	42 46 6	47 29 10	43 28 12	31 37 14	28 31 37	14 28 8	16 32 15	31

310. A TABLE of the SUN'S RIGHT ASCENSION for the Years 1774, 1778, and 1782, being the second after Leap Years.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	18 49 5	21 1 12	22 50 6	23 43 35	24 34 38	25 34 47	26 34 55	27 35 0	28 35 5	29 36 10	30 36 25	31 36 40
2	18 53 30	21 5 15	22 53 50	23 47 13	24 38 18	25 38 28	26 38 38	27 39 45	28 40 50	29 41 55	30 42 10	31 42 25
3	18 57 35	21 9 18	22 57 34	23 50 50	24 42 28	25 42 38	26 42 48	27 43 55	28 45 0	29 46 5	30 47 10	31 47 25
4	19 1 40	21 13 21	23 1 40	23 55 5	24 47 13	25 47 23	26 47 33	27 48 40	28 49 45	29 50 50	30 51 5	31 51 20
5	19 5 45	21 17 24	23 5 45	23 59 10	24 50 30	25 50 40	26 50 50	27 51 55	28 53 0	29 54 5	30 55 10	31 55 25
6	19 9 50	21 21 27	23 9 50	24 3 10	24 54 35	25 54 45	26 54 55	27 56 0	28 57 5	29 58 10	30 59 15	31 59 30
7	19 13 55	21 25 30	23 13 55	24 7 15	24 58 40	25 58 50	26 59 0	27 60 5	28 61 10	29 62 15	30 63 20	31 63 35
8	19 17 50	21 29 33	23 17 50	24 11 20	25 1 5	26 1 15	27 1 25	28 2 30	29 3 35	30 4 40	31 5 45	
9	19 21 55	21 33 36	23 21 55	24 15 25	25 5 10	26 5 20	27 5 30	28 6 35	29 7 40	30 8 45	31 9 50	
10	19 25 50	21 37 39	23 25 50	24 19 30	25 9 15	26 9 25	27 9 35	28 10 40	29 11 45	30 12 50	31 13 55	
11	19 29 55	21 41 42	23 29 55	24 23 35	25 13 20	26 13 30	27 13 40	28 14 45	29 15 50	30 16 55	31 17 50	
12	19 33 50	21 45 45	23 33 50	24 27 40	25 17 25	26 17 35	27 17 45	28 18 50	29 19 55	30 21 0	31 22 5	
13	19 37 55	21 49 48	23 37 55	24 31 45	25 21 30	26 21 40	27 21 50	28 22 55	29 24 0	30 25 5	31 26 10	
14	19 41 50	21 53 51	23 41 50	24 35 50	25 25 35	26 25 45	27 25 55	28 27 0	29 28 5	30 29 10	31 30 15	
15	19 45 55	21 57 54	23 45 55	24 39 55	25 29 40	26 29 50	27 29 60	28 31 5	29 32 10	30 33 15	31 34 20	
16	19 49 50	22 1 57	23 49 50	24 43 50	25 33 45	26 33 55	27 34 5	28 35 10	29 36 15	30 37 20	31 38 25	
17	19 53 55	22 5 60	23 53 55	24 47 55	25 37 50	26 38 0	27 38 10	28 39 15	29 40 20	30 41 25	31 42 30	
18	19 57 50	22 9 63	23 57 50	24 51 50	25 41 55	26 42 5	27 42 15	28 43 20	29 44 25	30 45 30	31 46 35	
19	19 51 55	22 13 66	24 1 55	24 55 55	25 45 50	26 46 10	27 46 20	28 47 25	29 48 30	30 49 35	31 50 40	
20	19 55 50	22 17 69	24 5 50	24 59 50	25 49 55	26 50 15	27 50 25	28 51 30	29 52 35	30 53 40	31 54 45	
21	19 59 55	22 21 72	24 9 55	25 3 55	25 53 50	26 54 10	27 54 20	28 55 25	29 56 30	30 57 35	31 58 40	
22	19 53 50	22 25 75	24 13 50	25 7 50	25 57 55	26 58 15	27 58 25	28 59 30	29 60 35	30 61 40	31 62 45	
23	19 57 55	22 29 78	24 17 55	25 11 55	26 1 50	27 2 10	28 2 20	29 3 25	30 4 30	31 5 35		
24	19 51 50	22 33 81	24 21 50	25 15 50	26 5 55	27 6 15	28 6 25	29 7 30	30 8 35	31 9 40		
25	19 55 55	22 37 84	24 25 55	25 19 55	26 9 50	27 10 10	28 10 20	29 11 25	30 12 30	31 13 35		
26	19 59 50	22 41 87	24 29 50	25 23 50	26 13 55	27 14 15	28 14 25	29 15 30	30 16 35	31 17 40		
27	19 53 55	22 45 90	24 33 55	25 27 55	26 17 50	27 18 10	28 18 20	29 19 25	30 20 30	31 21 35		
28	19 57 50	22 49 93	24 37 50	25 31 50	26 21 55	27 22 15	28 22 25	29 23 30	30 24 35	31 25 40		
29	19 51 55	22 53 96	24 41 55	25 35 55	26 25 50	27 26 10	28 26 20	29 27 25	30 28 30	31 29 35		
30	19 55 50	22 57 99	24 45 50	25 39 50	26 29 55	27 30 15	28 30 25	29 31 30	30 32 35	31 33 40		
31	19 59 55	23 1 102	24 49 55	25 43 55	26 33 50	27 34 10	28 34 20	29 35 25	30 36 30	31 37 35		

310. A TABLE OF THE SUN'S RIGHT ASCENSION IN TIME FOR THE YEARS 1775, 1779, AND 1783, BEING THE THIRD AFTER LEAP YEARS.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	18 48	12 01	12 49	12 00	11 42	11 33	11 24	11 15	11 06	10 57	10 48	10 39
2	18 52	12 04	12 52	12 03	11 45	11 36	11 27	11 18	11 09	10 59	10 50	10 41
3	19 56	12 07	12 55	12 06	11 48	11 39	11 30	11 21	11 12	11 03	10 54	10 45
4	19 59	12 10	12 58	12 09	11 51	11 42	11 33	11 24	11 15	11 06	10 57	10 48
5	19 59	12 12	12 59	12 11	11 52	11 43	11 34	11 25	11 16	11 07	10 58	10 49
6	19 10	12 15	13 02	12 14	11 54	11 45	11 36	11 27	11 18	11 09	10 59	10 50
7	19 14	12 18	13 05	12 17	11 57	11 48	11 39	11 30	11 21	11 12	11 03	10 54
8	19 18	12 21	13 08	12 20	11 59	11 50	11 41	11 32	11 23	11 14	11 05	10 56
9	19 23	12 24	13 11	12 23	12 02	11 53	11 44	11 35	11 26	11 17	11 08	10 59
10	19 27	12 27	13 14	12 26	12 05	12 00	11 51	11 42	11 33	11 24	11 15	11 06
11	19 31	12 30	13 17	12 29	12 08	12 03	11 54	11 45	11 36	11 27	11 18	11 09
12	19 36	12 33	13 20	12 32	12 11	12 06	11 57	11 48	11 39	11 30	11 21	11 12
13	19 40	12 36	13 23	12 35	12 14	12 09	11 59	11 50	11 41	11 32	11 23	11 14
14	19 44	12 39	13 26	12 38	12 17	12 12	12 02	11 53	11 44	11 35	11 26	11 17
15	19 49	12 42	13 29	12 41	12 20	12 15	12 05	11 56	11 47	11 38	11 29	11 20
16	19 53	12 45	13 32	12 44	12 23	12 18	12 08	11 59	11 50	11 41	11 32	11 23
17	19 57	12 48	13 35	12 47	12 26	12 21	12 11	12 02	11 53	11 44	11 35	11 26
18	20 01	12 51	13 38	12 50	12 29	12 24	12 14	12 05	11 56	11 47	11 38	11 29
19	20 06	12 54	13 41	12 53	12 32	12 27	12 17	12 08	11 59	11 50	11 41	11 32
20	20 10	12 57	13 44	12 56	12 35	12 30	12 20	12 11	12 02	11 53	11 44	11 35
21	20 14	13 00	13 47	12 59	12 38	12 33	12 23	12 14	12 05	11 56	11 47	11 38
22	20 18	13 03	13 50	13 02	12 41	12 36	12 26	12 17	12 08	11 59	11 50	11 41
23	20 23	13 06	13 53	13 05	12 44	12 39	12 29	12 20	12 11	12 02	11 53	11 44
24	20 27	13 09	13 56	13 08	12 47	12 42	12 32	12 23	12 14	12 05	11 56	11 47
25	20 31	13 12	13 59	13 11	12 50	12 45	12 35	12 26	12 17	12 08	11 59	11 50
26	20 35	13 15	14 02	13 14	12 53	12 48	12 38	12 29	12 20	12 11	12 02	11 53
27	20 39	13 18	14 05	13 17	12 56	12 51	12 41	12 32	12 23	12 14	12 05	11 56
28	20 43	13 21	14 08	13 20	12 59	12 54	12 44	12 35	12 26	12 17	12 08	11 59
29	20 47	13 24	14 11	13 23	13 02	12 57	12 47	12 38	12 29	12 20	12 11	12 02
30	20 52	13 27	14 14	13 26	13 05	13 00	12 50	12 41	12 32	12 23	12 14	12 05
31	20 56	13 30	14 17	13 29	13 08	13 03	12 53	12 44	12 35	12 26	12 17	12 08

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311. TABLE for fitting the TABLES of the SUN'S LONG. DECL. or Rt. ASCEN. to any meridian : Or, for finding either quantity at any hour. Degrees of longitude from the Meridian of London : Or, time before and after noon.

[illegible]

[illegible]

312. TABLE of the RIGHT ASCENSION and DECLINATION of sixty STARS in the Northern Hemisphere; to the year 1764.

Constellations.	Place of the Stars in the Constellations.	Names.	Mark.	Magn.	Rt. Ascen. in time. h. m. f.	Year. Var. f.	Declinat. North. ° ' "	Yearly Variat. " "
Pegasus	Ending of the Wing	Schedar	γ	2	0 1 07	3.08	13 52 17	20.94+
Cassiopea	Breast		α	3	0 27 14	3.30	55 14 18	20.00+
Pole Star			α	2	0 45 3	10.05	88 2 38	19.69+
Andromeda	Girdle	Mirach	β	2	0 56 34	3.29	34 21 48	19.43+
Aries	Preceding Horn		α	3	1 41 38	3.28	19 38 48	18.12+
Andromeda	Foot	Almaach	γ	2	1 49 30	3.61	41 11 10	17.82+
Aries	Following Horn		α	3	1 53 55	3.34	22 20 16	17.63+
Whale	Following in Cheek		γ	3	2 31 6	3.12	2 13 53	15.87+
	Jaw		α	2	2 49 58	3.13	3 9 0	14.81+
Medusa	Head	Algol	β	2	2 52 54	3.84	40 1 43	14.64+
Perseus	Brightest	Algenib	α	2	3 7 36	4.19	48 59 59	13.74+
Taurus	Brightest in	Pleiades	η	3	3 33 29	3.54	23 21 24	12.01+
Perseus	Knee		ε	3	3 42 5	3.94	38 18 21	11.41+
Taurus	First of the	Hyades	γ	3	4 6 23	3.39	15 2 22	9.59+
	Northern Eye		α	3	4 14 52	3.48	18 38 19	8.93+
	Southern Eye	Aldebaran	α	1	4 22 24	3.43	16 1 0	8.33+
Auriga	In the Goat	Capella	α	1	4 59 18	4.40	45 43 51	5.33+
Taurus	Northern Horn		β	2	5 11 23	3.78	28 23 6	4.29+
Orion	West Shoulder		γ	2	5 12 29	3.22	6 6 54	4.19+
	East Shoulder		α	1	5 42 25	3.25	7 20 33	1.60+
Auriga	In the Hand	Betelgeuse	α	3	5 43 38	4.08	37 10 13	1.51+
Gemini	Foot of Pollux		α	3	6 24 5	3.48	16 34 49	2.02-
	Knee of Pollux		γ	3	6 50 5	3.58	20 53 45	4.27-
	Brightest in Head	Castor	α	2	7 19 30	3.87	32 23 0	6.73-
Little Dog	Brightest	Procyon	α	1	7 26 57	3.21	5 48 59	7.37-
Gemini	Head of Pollux	Pollux	β	2	7 30 51	3.75	28 34 35	7.66-
Great Bear	North Paw		α	1	8 42 56	4.25	48 56 57	13.00-
Cancer	In the Claw	Acubens	α	3	8 42 55	3.31	12 45 31	13.01-
Great Bear	Preceding Knee		β	3	9 16 58	4.22	52 44 11	15.11-
Leo	North in the Head		α	3	9 39 17	3.47	27 6 20	16.33-
	In the Heart	Regulus	α	1	9 55 47	3.24	13 6 44	17.13-
Great Bear	Lower Pointer		β	2	10 47 26	3.75	38 25 19	19.01-
Leo	Upper Pointer	Dubhe	α	2	10 48 57	3.89	63 1 7	19.05-
Great Bear	In the Tail		β	2	11 37 1	3.12	15 53 28	19.93-
	S. following in the Square		γ	2	11 41 17	3.24	55 0 21	19.98-
	Left in the Square		δ	2	12 3 38	3.05	58 20 39	20.04-
	First in the Tail	Aliath	ε	2	12 43 34	2.69	57 14 39	19.69-
	Middle of the Tail		ζ	2	13 14 27	2.45	56 9 40	19.01-
	Left in the Tail	Benetnaish	η	2	13 38 14	2.41	50 29 50	18.24-
Bootes	The brightest	Arcturus	α	1	14 4 55	2.82	20 25 39	17.16-
	In following Thigh	Mirach	ε	3	14 34 46	2.63	28 4 48	15.67-
	In following Shoulder		δ	3	15 6 7	2.44	14 12 27	13.82-
Crown	The brightest		α	2	15 24 36	2.05	27 31 22	12.60-
Serpens	Following South in Neck		γ	3	15 45 33	2.75	14 27 5	11.14-
Hercules	Preceding Shoulder		β	3	16 20 6	2.59	22 1 8	8.51-
	Following at the Side		α	3	16 51 15	2.29	31 17 18	5.96-
Ophiurus	The Head	Raf. Algethi	α	2	17 3 53	2.74	14 40 36	4.91-
	The Head		α	2	17 23 59	2.78	12 45 3	3.17-
Hercules	The Knee		β	3	17 48 11	2.06	37 17 41	1.08-
Harp	The brightest	Vega	α	1	18 28 57	2.06	38 34 38	2.48+
	Following in Lozange		δ	3	18 46 15	2.11	36 36 44	3.97+
Eagle	Preceding Wing		δ	3	19 13 36	3.02	2 39 46	6.26+
	The brightest	Altair	α	2	19 39 16	2.91	8 15 41	8.36+
Swan	The Breast		γ	3	20 13 45	2.16	39 10 47	11.00+
	The Tail	Deneb	α	2	20 33 24	2.05	44 26 48	12.39+
Cepheus	Preceding Shoulder		α	3	21 12 54	1.43	61 35 25	14.92+
Pegasus	The Neck		γ	3	22 29 41	2.09	9 36 23	18.46+
	The Thigh	Scheat	β	2	22 52 21	2.80	26 48 21	19.15+
	The Wing	Markab.	α	2	22 53 2	2.90	13 56 24	19.17+
Andromeda	The Head		α	2	23 56 14	3.06	7 47 13	20.04+

312. TABLE of the RIGHT ASCENSION and DECLINATION of Sixty STARS in the Southern Hemisphere; to the Year 1764.

Constellations.	Place of the Stars in the Constellations.	Names.	Magn. p.	Rt. Ascen. in time. h. m. s.	Year. Var. f.	Declinat. South. ° ' "	Yearly Variat. " "
Phenix	The Head		2	0 14 34	3.01	43 35	3 20.00
Whale	Brightest in Tail		2	0 31 44	3.01	19 17	13 19.85
Phenix	Thigh		3	0 55 31	2.73	48 0	10 19.46
	Following Wing		3	1 18 6	2.67	44 31	54 18.90
Eridanus	Source of the River	Achernar	1	1 28 55	2.26	58 26	32 18.56
Whale	Preceding in Jaw		3	2 27 25	3.08	0 42	5 16.07
Eridanus	Near the Whale		3	3 4 24	2.91	9 42 36	13.93
	The following		3	31 58	2.88	10 34 43	12.10
	The fourth Bend		3	3 47 2	2.79	14 11 47	11.02
Goldfish	In the Tail		3	4 28 55	1.28	55 32 24	7.78
Orion	Bright Foot	Rigel	1	5 3 13	2.89	8 29 28	4.97
	Preceding in Belt		2	5 19 59	3.07	0 29 33	3.54
	Middle of Belt		2	5 24 16	3.05	1 22 17	3.17
	Left in the Belt		2	5 28 53	3.04	2 5 10	2.77
Dove	Preceding of the brightest		2	5 31 8	2.20	34 12 47	2.56
Orion	In the Knee		3	5 36 35	2.85	9 46 9	2.10
Dove	Following of brightest		3	5 42 39	2.11	35 52 17	1.65
Argo	The brightest	Canopus	1	6 18 44	1.34	51 34 31	1.60
	The brightest		1	6 34 47	2.69	16 24 18	2.97
Great Dog	In the Back	Syrus	2	6 58 48	2.45	26 2 6	5.03
	In the Tail		2	7 14 45	2.39	28 51 29	6.37
	In the Poop		2	7 55 18	2.12	39 20 54	9.62
Argo	Preceding in the Hull		2	8 2 17	1.85	46 38 57	10.16
	Brightest in Middle		2	8 38 10	1.61	53 51 0	12.73
	Bright among the Oars		1	9 10 32	0.75	68 44 53	14.79
Female Hydra	The Heart		2	9 16 0	2.90	7 38 44	15.08
Argo	Northern in Section		2	10 35 58	2.37	58 27 0	18.68
Centaur	Preceding in the Crupper		3	11 56 10	3.05	49 24 20	20.03
Crofs	Preceding Arm		3	12 2 45	3.10	57 26 11	20.04
	The Foot		1	12 13 41	3.22	61 47 27	20.01
	The Head		2	12 18 41	3.36	70 49 34	19.97
	Following Arm		2	12 34 7	3.42	58 23 46	19.83
Centaur	Top of the Crupper		2	12 28 38	3.27	47 39 31	19.89
Female Hydra	The Tail		3	13 6 8	3.22	11 55 11	19.22
Virgo	The Sheaf		1	13 12 47	3.15	9 55 18	19.06
Centaur	Preceding Leg		2	13 47 24	4.09	59 13 12	17.91
	South in the Shield		3	14 20 37	3.75	41 6 19	16.43
	Bright in the Foot		1	14 23 52	4.41	59 50 57	16.26
	Following Hand		3	14 43 55	3.84	41 8 18	15.17
Libra	Southern Scale		2	14 37 51	3.30	15 2 47	15.50
	Northern Scale		2	15 4 21	3.30	8 29 46	13.54
Southern A	The Vertex		3	15 34 34	5.06	62 40 22	11.96
Scorpio	Middle of the Forehead		3	15 46 24	3.52	21 55 53	11.09
	N. in the Forehead		2	15 51 45	3.50	19 8 25	10.69
Ophiucus	Preceding Hand		3	16 2 0	3.14	3 4 5	9.92
Scorpio	The Heart	Antares	1	16 14 59	3.66	25 53 13	8.93
	First Joint in Tail		3	16 34 56	3.90	33 50 24	7.34
Ophiucus	Following Knee		2	16 56 52	3.44	15 24 46	5.52
Altar	In the Middle		3	17 13 38	4.61	49 39 29	4.12
Scorpio	Bright at Tail's End		3	17 17 37	4.08	36 54 21	3.77
	Sixth Knot in Tail		3	17 31 06	4.18	40 0 25	2.60
Sagittarius	In the Hand		3	18 5 52	3.85	29 54 12	0.43
	Preceding Shoulder		3	18 40 37	3.74	26 34 3	3.45
	Following in Head		3	18 55 13	3.59	21 22 43	4.70
	The Eye		2	20 6 49	4.89	57 28 1	10.44
Peacock	The Wing		3	20 23 24	5.65	67 1 25	11.64
Capricorn	Preceding in Tail		3	21 26 36	3.35	17 43 3	15.66
Crane	Preceding Wing		2	21 53 17	3.98	48 5 23	17.00
Aquarius	Following Arm		3	22 9 27	3.09	2 34 6	17.71
Sea Fish	The Brightest	Fomalhaut	1	22 44 34	3.34	30 52 0	18.94

313. TABLE of the Sun's Right Ascension, in Degrees and Paris, to each Deg. of Lon. in the four Quadrants: And of the Diff. between Lon. and Rt. Ascen.

[illegible]

313. TABLE of the Sun's Right Ascension, in Degrees and Paris, to each Deg. of Lon. in the four Quadrants: And of the Diff. between Lon. and Rt. Ascen.

316. TABLE of the Absolute Equation of Time, fitted to each sign and degree of the Ecliptic.

Place of the Apogee ϖ 9°. Obliquity of ecliptic $23^{\circ} 28'$.

	0	1	2	3	4	5	6	7	8	9	10	11
♈	7 36	1 9	3 51	1 13	5 57	2 20	7 38	15 31	13 33	1 11	11 28	14 19
♉	7 17	1 23	3 47	1 26	5 59	2 4	7 58	15 39	13 17	0 42	11 45	14 13
♊	6 58	1 36	3 41	1 40	6 0	1 48	8 19	15 46	13 0	— 12	12 1	14 6
♋	6 39	1 43	3 37	1 53	6 1	1 31	8 40	15 52	12 42	+	17 12	17 13
♌	6 20	2 0	3 32	2 7	6 1	1 14	9 1	15 57	12 23	0 46	12 32	13 59
♍	5 6	2 11	3 26	2 20	6 0	0 56	9 21	16 2	12 4	1 16	12 46	13 43
♎	5 42	2 22	3 19	2 33	5 59	0 38	9 41	16 6	11 44	1 45	12 59	13 34
♏	5 24	2 32	3 12	2 45	5 57	0 20	10 1	16 9	11 23	2 14	13 12	13 24
♐	5 5	2 42	3 4	2 58	5 54	+	10 20	16 11	11 1	2 43	13 24	13 14
♑	4 47	2 51	2 56	3 11	5 51	— 18	10 39	16 13	10 39	3 11	13 35	13 3
♒	4 28	3 0	2 47	3 23	5 47	0 37	10 57	16 13	10 16	3 39	13 45	12 51
♓	4 9	3 8	2 38	3 35	5 42	0 57	11 15	16 13	9 53	4 7	13 54	12 39
♈	3 50	3 16	2 29	3 46	5 37	1 17	11 33	16 12	9 29	4 35	14 2	12 27
♉	3 32	3 23	2 19	3 58	5 31	1 38	11 51	16 10	9 5	5 2	14 9	12 14
♊	3 13	3 30	2 8	4 9	5 24	1 58	12 8	16 7	8 40	5 29	14 16	12 0
♋	2 55	3 36	1 57	4 19	5 17	2 19	12 25	16 4	8 14	5 56	14 22	11 46
♌	2 37	3 41	1 46	4 29	5 9	2 40	12 41	16 0	7 48	6 22	14 27	11 31
♍	2 19	3 46	1 35	4 39	5 1	3 1	12 57	15 55	7 22	6 48	14 31	11 16
♎	2 1	3 50	1 23	4 48	4 52	3 22	13 12	15 49	6 55	7 13	14 35	11 1
♏	1 43	3 53	1 11	4 57	4 43	3 44	13 27	15 42	6 28	7 37	14 38	10 46
♐	1 26	3 56	0 59	5 5	4 33	4 5	13 42	15 35	6 0	8 1	14 40	10 30
♑	1 9	3 58	0 46	5 13	4 22	4 26	13 56	15 26	5 32	8 24	14 41	10 14
♒	0 52	4 0	0 34	5 20	4 11	4 47	14 9	15 17	5 4	8 47	14 42	9 58
♓	0 36	4 1	0 21	5 27	3 59	5 9	14 21	15 7	4 36	9 9	14 41	9 41
♈	0 20	4 1	— 8	5 33	3 46	5 30	14 33	14 56	4 8	9 31	14 40	9 24
♉	+	4 1	+	5 39	3 33	5 52	14 44	14 44	3 39	9 53	14 39	9 6
♊	— 11	4 0	0 19	5 44	3 19	6 13	14 53	14 31	3 10	10 10	14 37	8 48
♋	0 26	3 59	0 31	5 48	3 4	6 35	15 5	14 17	2 41	10 34	14 34	8 30
♌	0 40	3 57	0 46	5 52	2 50	6 56	15 14	14 3	2 11	10 53	14 30	8 12
♍	0 53	3 54	0 56	5 55	2 35	7 17	15 23	13 48	1 41	11 11	14 25	7 54
♎	1 9	3 51	1 13	5 57	2 20	7 28	15 31	13 33	1 11	11 28	14 19	7 36
♏	1 27	3 48	1 12	5 50	2 7	7 40	15 38	13 24	0 14	11 35	14 10	7 28
♐	2 7	3 40	1 4	5 42	2 0	7 52	15 45	13 15	0 21	11 42	14 1	7 20
♑	2 17	3 32	1 16	5 34	1 11	8 4	15 52	13 6	0 28	11 49	14 0	7 12
♒	2 27	3 24	1 26	5 26	0 56	9 16	16 0	12 57	0 35	11 56	13 51	7 4
♓	2 37	3 16	1 36	5 18	0 40	10 28	16 7	12 48	0 42	12 4	13 42	6 56

The equations with +, are to be added to the apparent time, to have the mean time; those with —, are to be subtracted from apparent for mean time.

The preceding mark, whether + or —, at the head of any column, belongs to all the equations in that column until the sign changes; and those columns having two signs at the head, shew that the preceding sign changes to the following somewhere in that column.

Equal Altitudes of the Sun.

Latitude 30 Degr.			Latitude 40 Degr.			Latitude 50 Degr.			Latitude 60 Degr.		
Hours bet. Observ.			Hours bet. Observ.			Hours bet. Observ.			Hours bet. Observ.		
6	5	4	6	5	4	6	5	4	6	5	4
N. decl.			N. decl.			N. decl.			N. decl.		
sec.	sec.	sec.	sec.	sec.	sec.	sec.	sec.	sec.	sec.	sec.	sec.
0	9	8	13	14	13	18	20	19	28	27	28
1	9	8	14	13	13	19	19	18	28	27	29
2	9	8	13	12	14	19	18	20	28	27	26
3	8	7	12	12	14	18	18	17	28	27	26
4	8	7	12	12	14	18	17	20	27	26	25
5	8	7	12	12	13	18	17	20	27	26	25
6	8	7	12	11	13	17	16	20	27	25	24
7	7	6	12	11	13	17	16	20	26	25	24
8	7	6	11	11	13	16	16	20	26	24	23
9	7	6	11	10	15	16	15	20	25	23	23
10	7	6	11	10	15	16	15	20	24	23	22
11	6	5	10	10	14	15	15	19	23	22	22
12	6	5	10	9	14	15	14	19	22	21	21
13	6	5	10	9	14	14	14	19	21	21	21
14	5	5	10	9	14	14	13	18	20	20	19
15	5	4	9	8	14	13	12	18	19	19	18
16	5	4	9	8	14	13	12	18	18	18	17
17	4	4	9	7	13	12	11	17	17	17	16
18	4	3	9	7	12	12	10	16	16	15	15
19	3	3	8	6	12	12	10	15	15	15	14
20	3	2	8	6	11	11	9	14	15	14	13
21	2	1	7	5	10	10	8	13	14	13	12
22	1	0	6	4	9	9	7	12	11	11	10
23	0	0	3	3	2	2	5	5	9	8	8
24	0	0	3	3	2	2	5	5	5	5	4

The correction is subtractive from December 22 to June 21, or in ascending signs; and additive from June 21 to December 22, or in descending signs.

218. A TABLE OF REFRACTIONS.

Ref.	sec.	30
min.	0	29
Alt. Deg.	62	63
Ref.	sec.	48
min.	0	46
Alt. Deg.	64	65
Ref.	sec.	26
min.	0	25
Alt. Deg.	66	67
Ref.	sec.	24
min.	0	23
Alt. Deg.	68	69
Ref.	sec.	21
min.	0	20
Alt. Deg.	70	71
Ref.	sec.	19
min.	0	18
Alt. Deg.	72	73
Ref.	sec.	17
min.	0	16
Alt. Deg.	74	75
Ref.	sec.	15
min.	0	14
Alt. Deg.	76	77
Ref.	sec.	13
min.	1	12
Alt. Deg.	78	79
Ref.	sec.	11
min.	1	10
Alt. Deg.	80	81
Ref.	sec.	9
min.	1	8
Alt. Deg.	82	83
Ref.	sec.	7
min.	1	6
Alt. Deg.	84	85
Ref.	sec.	5
min.	1	4
Alt. Deg.	86	87
Ref.	sec.	3
min.	1	2
Alt. Deg.	88	89
Ref.	sec.	1
min.	1	0
Alt. Deg.	90	91

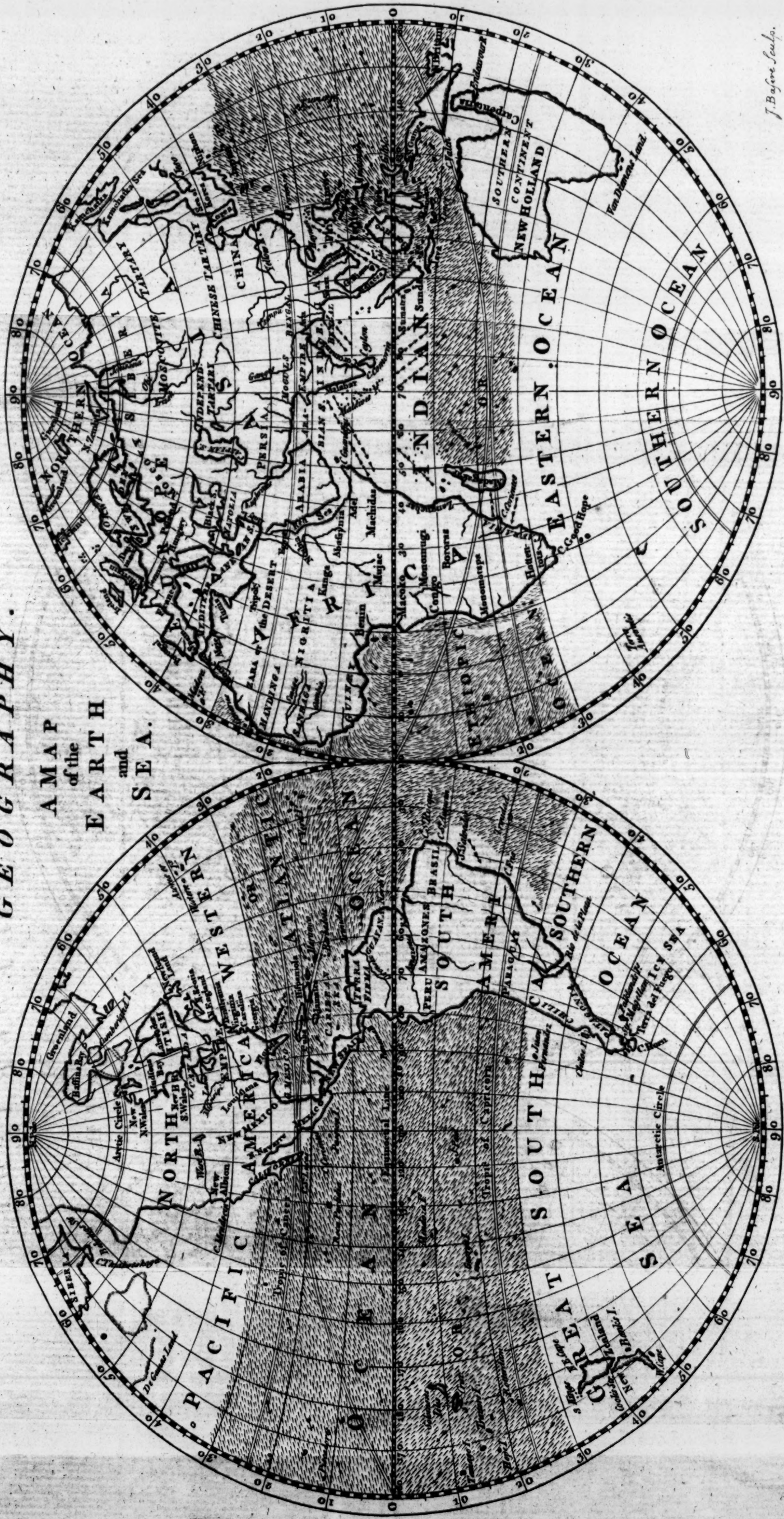
Of the Sun's Parallax

Altitudes 0°. 10° 20° 30° 40° 50° 60° 70° 80° 90°.
Parallax 9" 8" 8" 8" 6" 5" 3" 2" 1" 0".

END OF BOOK V.

GEOGRAPHY.

A MAP
of the
EARTH
and
SEA.





THE ELEMENTS OF NAVIGATION.

BOOK VI. OF GEOGRAPHY.

SECTION I. *Definitions and Principles.*

1. **G**EOGRAPHY is the art of describing the figure, magnitude, and positions of the several parts of the surface of the Earth.

2. The EARTH is a spherical or globular figure*; and is usually called the *terraqeous globe*. *See Description p. xxv*

3. There are two points on the surface of the *terraqeous globe*, called the **POLES OF THE EARTH**, which are diametrically opposite to one another: one is called the *north pole*, and the other, the *south pole*.

* For in ships at sea, the first parts of them that become visible are the upper sails; and as they approach nearer, the lower sails appear; and so on until they shew their hulls.

Also ships, in sailing from high capes, or head lands, lose sight of those eminences gradually from the lower parts, until the top vanishes.

Now, as these appearances are the objects of our senses in all parts of the Earth,

Therefore the surface of the Earth must be convex.

And this convexity is, at sea, observed to be every where uniform.

But a body, whose surface is every where uniformly convex, is a globe.

Therefore the figure of the Earth is globular.

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In order to describe the positions of places, Geographers have found it necessary to imagine certain circles drawn on the surface of the Earth, to which they have given the names of Equator, Meridian, Horizon, Parallels of latitude, &c.

4. The EQUATOR is a great circle on the Earth, equally distant from each pole, dividing the terraqueous globe into two equal parts; one called the *northern hemisphere*, in which is the north pole; and the other containing the south pole, is called the *southern hemisphere*.

5. MERIDIANS are imaginary circles on the Earth, passing through both the poles, and cutting the equator at right angles.

Every point on the surface of the Earth has its proper meridian.

6. LATITUDE is the distance of a place from the equator, reckoned in degrees and parts of degrees on a meridian.

On the north side of the equator it is north latitude; and on the south side it is south latitude.

As latitude begins at the equator, where it is nothing; so it ends at the poles, where the latitude is greatest, or 90 degrees.

7. PARALLELS OF LATITUDE are circles parallel to the equator.

Every place on the Earth has its parallel of latitude.

DIFFERENCE OF LATITUDE is an arc of a meridian, or the least distance of the parallels of latitude of two places; shewing how far one of them is to the northward, or southward, of the other.

The difference of latitude can never exceed 180 degrees.

8. In north latitudes, if about the middle of the months of March and September, a person looks towards the Sun at noon, the south is before him, the north behind; the west on the right hand, and the east on the left: and in south latitudes, if the face is turned towards the Sun, at the same times, the north is before, the south behind, the east to the right, and the west on the left.

In latitudes greater than $23\frac{1}{2}$ degrees, these positions found at noon, will hold good on any day of the year.

9. LONGITUDE of any place on the Earth is expressed by an arc of the equator, shewing the east or west distance of the meridian of that place from some fixed meridian, where longitude is reckoned to begin.

10. DIFFERENCE OF LONGITUDE is an arc of the equator, intercepted between the meridians of two places, shewing how far one of them is to the eastward, or westward of the other.

As longitude begins at the meridian of some place, and is counted from thence both eastward and westward, till they meet at the same meridian on the opposite point; therefore the difference of longitude can never exceed 180 degrees.

11. When two places have latitudes both north, or both south; or have longitudes both east, or both west, they are said to be of the same, or of like name: but when one has north latitude, and the other south, or if one has east longitude, and the other west, then they are said to have contrary, or different, or unlike names.

12. The HORIZON is that apparent circle which limits, or bounds, the view of a spectator on the sea, or on an extended plain; the eye of the spectator being always supposed in the center of his horizon.

When

When the Planets or Stars come above the eastern part of the horizon, they are said to rise ; and when they descend below the western part, they are said to set.

When a ship is under the equator, both the poles appear in the horizon ; and in proportion as she falls towards either, or increases her latitude, that pole is seen proportionally higher above the horizon, and the other disappears as much : but when a ship is sailing towards the equator, or decreases her latitude, she depresses the elevated pole ; that is, its distance from the horizon decreases.

Of the division of the Earth into Zones.

13. A ZONE is a broad space on the Earth, included between two parallels of latitude.

There are five zones ; namely, one *Torrid*, two *Frigid*, and two *Temperate* : these names arise from the quality of the heat and cold, to which their situations are liable.

14. The *TORRID ZONE* is that portion of the Earth, over every part of which the Sun is perpendicular at one time of the year or other.

This zone is about 47 degrees in breadth, extending to about $23\frac{1}{2}$ degrees on each side of the equator ; the parallel of latitude terminating the limits in the northern hemisphere, is called the *Tropic of Cancer* ; and in the southern hemisphere, the limiting parallel is called the *Tropic of Capricorn*.

15. The *FRIGID ZONES* are those regions about the poles, where the Sun does not rise for some days, nor set for some days, of the year.

These zones extend round the poles to the distance of about $23\frac{1}{2}$ degrees : that in the northern hemisphere is called the north frigid zone, and is bounded by a parallel of latitude, called the *Arctic polar circle* ; and the other, in the southern hemisphere, the south frigid zone ; the parallel of latitude bounding it, being called the *Antarctic polar circle*.

16. The *TEMPERATE ZONES* are those spaces between the *Torrid* and the *Frigid* zones.

Of the division of the Earth by Climates.

17. A CLIMATE, in a geographical sense, is that space of the Earth contained between two parallels of latitude, where the difference between the longest day in each parallel is half an hour.

These climates are narrower the farther they are from the equator ; therefore, supposing the equator the beginning of the first climate, the polar circle will be the end of the 24th climate ; for afterwards the longest day increases not by half hours, but by days and months.

SECTION II.

Of the natural division of the Earth.

18. By the natural division of the Earth, is meant the parts on its surface formed by nature; such as *Continents, Oceans, Islands, Seas, Rivers, Mountains, &c.*

The surface of the Earth is naturally divided into Land and Water.

Land is divided into {
 1. Continents.
 2. Islands.
 3. Peninsulas.

4. Isthmuses.
 5. Promontories.
 6. Mountains.

Water is divided into {
 1. Oceans.
 2. Seas.
 3. Gulfs.

4. Straits.
 5. Lakes.
 6. Rivers.

19. A CONTINENT, or, as it frequently called, the *Terra Firma*, or, *main land*, is a very large tract, comprehending several contiguous Countries, Kingdoms, and States.

20. An OCEAN is a vast collection of salt-water, separating the continents from one another, and washing their borders or shores.

21. An ISLAND is a part of dry land, surrounded with water.

22. A SEA is a branch of the Ocean, flowing between some parts of the continent, or separating an Island from the Continent.

23. A PENINSULA is a part of dry land encompassed by water, except a narrow neck which joins it to some other land.

24. An ISTHMUS is the neck joining the peninsula to the adjacent land, and forms the passage between them.

25. A MOUNTAIN is a part of the land more elevated than the adjacent country, whereby it is seen at a greater distance than the neighbouring lower lands.

26. A PROMONTORY is a mountain stretching itself into the sea; the extremity whereof is called a *Cape*, or *Head-land*.

27. A HILL is a small kind of mountain: A *Cliff* is a steep shore, hill, or mountain: And *Rocks* are great stones, rising like hills above the dry land, or above the bottom of the sea.

28. A GULF, or BAY, is a part of the Ocean, or Sea, contained between two shores; and is every where environed with land, except its entrance, where it communicates with other Bays, Seas, or Oceans.

29. A STRAIT is a narrow passage, by which there is a communication between a Gulf and its neighbouring sea, or joining one part of the sea, or ocean, with another.

30. A LAKE is a collection of Waters contained in some hollow, or cavity, in an inland place, of a large extent, and every where surrounded with land; having no visible communication with the Ocean.

31. RIVERS are streams of Water, flowing chiefly from the Mountains, and running in long narrow channels, or cavities, through the land, 'till they either fall into the sea, or into other rivers, which at last disembogue, or run into the sea.

32. There

32. There are generally reckoned four Continents, namely, EUROPE, ASIA, AFRICA, and AMERICA.

To these may be added the *Terra artica*, or northern continent, and the *Terra antarctica*, or southern continent, or lands detached from *Asia*.

The continent of *America* is usually divided into two parts, called *north* and *south America*; they are joined together by the *Isthmus of Darien*. Also the continents of *Asia* and *Africa* are joined together by the *Isthmus of Sues*.

The *Terra artica*, with *Europe* and *Asia*, lie all within the northern hemisphere; and also part of *Africa* and *America*: The other parts of these two continents, together with the *Terra antarctica*, lie in the southern hemisphere.

33. There are five Oceans, namely, the NORTHERN, the ATLANTIC, the PACIFIC, the INDIAN, and the SOUTHERN.

The *Atlantic ocean* is usually divided into two parts, one called the *north Atlantic ocean*, and the other the *south Atlantic*, or *Ethiopic ocean*.

The *Northern ocean* stretches to the northward of Europe, Asia, and America, towards the north pole.

The *Atlantic ocean* lies between the continents of Europe and Africa on the east, and America on the west.

That part of the north Atlantic ocean, lying between Europe and America, is frequently called the *Western ocean*.

The *Pacific ocean*, or, as it is sometimes called, the *South Sea*, is bounded by the western and north-west shores of America, and by the eastern and north-east shores of Asia.

The *Indian ocean* washes the shores of the eastern coasts of Africa, and the south of Asia; and is bounded on the east by the Indian islands and the southern continent.

The *Southern ocean* extends to the southward of Africa and America towards the south pole.

The northern and southern continents not being sufficiently known to Geographers; all that need be said of them, is, that the *Terra Artica*, or land to the northward of Hudson's Bay and Greenland, is in general too cold for the residence of mankind; and that the lands supposed to be parts of the southern continent, are found to be very large islands: *viz.* New Zealand is much larger than Great Britain, and has a strait dividing it into two islands. New Holland is an island as large as Europe. New Guinea is a very large island; and New Britain is thought to be a cluster of large and small islands, and most probably are the islands hitherto called the Solomon's islands.

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SECTION

SECTION III.

Of the Political division of the Earth.

34. By the political division of the Earth, is meant the different Countries, Empires, Kingdoms, States, and other denominations established by men, either from the ambition of tyrants, or for the sake of good government.

OF EUROPE.

Europe is bounded on the north by the northern, or frozen, ocean; on the east by Asia; on the south by the Mediterranean Sea, separating Europe from Africa; and by the north Atlantic, or western ocean, on the west: It lies between the latitudes of 36 and 72 degrees of north latitude; and between the longitudes of 10 degrees west, and 65 degrees east from London; is about 3000 miles long, reckoning from the N. E. to the S. W. and about 2500 miles broad.

35. The countries, their position with regard to the middle parts of Europe, the chief cities, principal rivers, with their courses, and the most noted mountains, and what quarter of the country they are in, are exhibited in the following table; where E stands for empire, K for kingdom, R for republic, Nd. for northward, &c.

Countries.	Position.	Chief Cities.	Rivers.	Course.	Mountains.
E. Turkey	S. E.	Constantinople	Danube	E.	Argentum Nd.
K. Poland	Mid.	Warsaw	Vistula	N. N. W.	Carpathian Sd.
E. Muscovy.	N. E.	{ Moscow { Petersburg	Volga	E. to S	Boglowy Sd.
K. Sweden	N.	Stockholm	Neiper	S.	Riphean Wd.
K. Norway	N. N. W.	Bergen	Dalecarlia	E.	Dofrine Wd.
K. Denmark	N. W.	Copenhagen	Glama	S.	Dofrine Ed.
K. Hungary	Mid.	Presburg	Eyder	W.	Carpathian Nd.
E. Germany	Mid.	Vienna	Danube	S. E.	Alps Sd.
Italy	S.	Rome	{ Po { Tyber	E.	Alps Nd.
R. Switzerland	Mid.	Bern	Rhine	S.	Apennine Mid.
Netherlands	W.	Brussels	Maele	W.	Alps Sd.
R. Dutchland	W.	Amsterdam	Rhine	N.	
K. France	W.	Paris	{ Loire { Rhone	N. N. W.	Pyrenees S. W.
K. Spain	S. W.	Madrid	Tagus	N. to W.	Alps Ed.
K. Portugal	S. W.	Lisbon	Tagus	S.	Pyrenees N. E.
K. England	W.	London	Thames	W.	C. Rocca W.
K. Scotland	W.	Edinburgh	Foith	E.	Malvern N. W.
K. Ireland	W.	Dublin	Shannon	E.	Grampian Nd.
				S. W.	Knockpatrick

There are in Europe four Kingdoms beside those enumerated above; but they are contained in the forenamed Countries.

The Kingdom of Prussia, which is part of Poland; the King's residence is at Berlin, a city in Germany.

The

The Kingdom of Bohemia, a part of Germany; the ch. city is Prague.
The Kingdom of Sardinia, an Italian island; the King resides at Turin, a city in Italy.

The Kingdom of the Sicilies, appending to Italy; the King resides at Naples, a city in Italy.

In some of the forenamed countries, are several dominions independent one of the other; particularly in Germany and Italy.

The principal states in Germany are the following 12: Where D stands for duchy, El. for electorate, P for principality.

States	D. Austria	K. Bohemia	El. Bavaria	El. Brandenburg
Ch. cities	Vienna	Prague	Munich	Berlin
States	El. Saxony	El. Hanover	El. Palatine	El. Mentz
Ch. cities	Dresden	Hanover	Manheim	Mentz
States	El. Triers	El. Cologne	PHesse-Cassel	D. Wurtemberg
Ch. cities	Triers	Cologne	Cassel	Stutgard

The principal states in Italy are the following 12.

States	D. Savoy	P. Piedmont	D. Milanese	D. Parmesan
Ch. cities	Chambery	Turin	Milan	Parma
States	D. Modenese	D. Mantuan	R. Venice	R. Genoa
Ch. cities	Modena	Mantua	Venice	Genoa
States	D. Luscany	Patriarchate	R. Lucca	K. Naples
Ch. cities	Florence	Rome	Lucca	Naples

36. *The principal Seas, Gulfs and Bays in Europe, are*

The Mediterranean Sea, having Europe on the N. and Africa on the S.

The Adriatic Sea, between Italy and Turkey.

The Euxine, or Black Sea, in Turkey, between Europe and Asia.

The White Sea, in the N. N. W. parts of Muscovy.

The Baltic Sea, between Sweden, Denmark, and Poland.

The German Ocean, or Sea, between Germany and Britain.

The English Channel, between England and France.

St. George's Channel, between Britain and Ireland.

The Bay of Biscay, formed between France and Spain.

The Gulf of Bothnia, in the N. E. parts of Sweden.

The Gulf of Finland, between Sweden and Russia.

The Gulf of Venice, the N. W. end of the Adriatic Sea.

37. *The principal Islands in Europe, are*

The British Isles, viz. Great Britain, Ireland, Orkneys, and Western Isles.

The Spanish Isles; Majorca, Minorca, Ivisa, in the Medit. Sea.

Italian Isles; Sicily, Sardinia, Corfica, Lipari, in the Medit. Sea,

Turkish Isles; Candia, Archipelago Isles, in the Medit. Sea

Swedish Isles; Gothland, Oeland, Alan, Rugen, in the Baltic Sea.

Danish Isles; Zeland in the Baltic Sea; and Hieland, Faro Isles, E. and

W. Greenland in the Northern ocean.

Azores Isles in the Atlantic ocean, belonging to Portugal.

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OF ASIA.

38. The continent of Asia is bounded on the north by the Northern, or frozen ocean, on the east by the Pacific Ocean, on the south by the Indian ocean, and by Africa and Europe on the west: It lies, including its Islands, between the latitudes of 10 degrees south, and 72 degrees north; and is between the longitudes of 25 and 148 degrees east of London: Its length, exclusive of the isles, being about 4800 miles, and breadth about 4300 miles.

39. The positions and names of the chief countries, cities, rivers, and mountains, are contained in the following table.

Countries.	Position.	Chief Cities.	Rivers	Courses.	Mountains.
E. China	S. E.	{ Pekin Nankin Canton	Yellow R. Kiam Ta	S. to E. E. S. E.	Ottorocoran Damafian Wd.
K. Korea	E.	Kingkitau	Yalu	S.	Shanalin
Chinese Tartary	E.	Chynian	Yamour	N. E. to E.	Fongwhanshan
Mongalia	Mid.	Kudak	Yellow R.	N. to W.	
K. Thibet	Mid.	Esferdu	Yaru	E.	Kantes
Bukharia, or Ulbs	{ Mid. Mid.	Samarkand	Amu	N. W.	Belurlag
Karazm	Mid.	Ujenz	Amu	S. W.	Irder
Kalmucks	Mid.		Tekis		Tubratubusluk
E. Siberia	N.	{ Tobolski Astracan	Oby Jenika	N. N. N. W.	Stolp
E. Turkey	W.	Smyrna	Euphrates	S. E.	Taurus
K. Syria	W.	Aleppo	Euphrates	S.	Lebanon
Arabia	S. W.	Medina	Euphrates	S. E.	Gabel el ared
E. Persia	S.	Ispahan	{ Oxus Araxes	W. S. W.	Caucausus
India	{ S. S.	Agra Delli	Indus	S. W.	Taurus
Wst. the Ganges	{ S. S.	Ava Pegu	Ganges	S. W.	Caucausus
India	{ S. S.	Siam	Domea	S.	Balagate
East the Ganges	{ S. S.	Cambodia	Mecon Menan Ava	S. S. S.	{ Damascene

Some of these countries contain several others.

Asiatic Turkey contains

Countries	Georgia N.	Turcomania E.	Curdistan E.	Diabec E.
Ch. cities	Teflis	or Armenia Erzerum	or Assyria Betlis	Moufol
Countries	Eyraca S. E.	Arabia desert S.	Natolia W.	Syria W.
Ch. cities	Bagdat		Smyrna	Aleppo

India

India west the Ganges contains

Northern Parts.	Countries Indofan		N. Cambaya	S. W. Bengala SE.
	Ch. cities	Delli	or Guzarat	Patna.
Malabar Coast.	Countries Decan		Bijnagar	W.
	Ch. cities	or Viſapour	or Carnate	
Coromandel Coast	Countries Bijnagar		K. Golconda	K. Oriza
	Ch. cities	Goa	Caleut and Cochin	Oriza

India east the Ganges contains

Countries Ch. cities	K. Ava	N. W. K. Pegu	W. K. Siam	Sd. K. Malacca S.
	Ava	Pegu	Siam	Malacca
Countries Ch. cities	K. Cambodia S.	K. Cochinchina E.	K. Laos N.	K. Tonquin N. E.
	Cambodia	Thoonoa	Lanchan	Keccio

40. *The principal Seas, Gulfs, and Bays in Asia, are*
Caspian Sea, quite surrounded by Siberia on the north, Korazm east, by Persia on the south, and by Georgia on the west.
Korean Sea, between Korea and the islands of Japan.
Yellow Sea, between China and the Japan isles.
Gulf of Cochinchina, on the borders of Tonquin and Cochin China.
Bay of Siam, formed by the countries of Siam and Malacca.
Bay of Bengal, between India east, and India west the Ganges.
Gulf of Persia, having Persia on the N. E. and Arabia on the S. W.

41. *The principal Islands belonging to Asia, are*
Ladrone, or Marian Isles, whose chief island is Guam.

<i>Japan Isles</i>	Ch. isles	Japan	Bongo	Tonfa
	Ch. cities	Jeddo	Bongo	Tonfa
<i>Philippines</i>	Ch. isles	Luconia	Mindanao	Samar
	Ch. cities	Manila	Mindanao	
<i>Chinese Isles</i>	Ch. isles	Formosa	Ainan	Makao
	Ch. cities	Taywanfu	Tan	Makao
<i>Moluccas</i>	Ch. isles	Celebes	Gilolo	Ceram
	Ch. cities	Macasser	Gilolo	Ambay
<i>Sunda isles</i>	Ch. isles	Borneo	Sumatra	Java
	Ch. cities	Banjar	Achin	Batavia

The *Andaman Isles* to the west of Siam.

Nicobar Isles west of Malacca.

Maldiva Isles to the S. W. of Bijnagar.

The *Island of Ceylon* S. E. of Bijnagar; the chief city is Candy, or Candy Uta.

OF AFRICA.

42. This large continent is a peninsula joined to Asia by the isthmus of Suez. On the N. E. it is separated from Asia by the Red Sea; it has the Indian Ocean on the east, the Southern on the south, the Atlantic on the west, and the Mediterranean Sea on the north, which separates it from Europe. It is situated between the latitudes of 37 degrees N., and 35 degrees S.; and between the longitude of 18 degrees W. and 50 degrees E. from London; is about 4300 miles long, and 4200 miles broad.

43. The positions and names of the chief countries, cities, rivers, and mountains, are contained in the following table.

Countries.	Position.	Chief Cities.	Rivers.	Course.	Mountains.
E. Morocco	N. W.	Fez	Mulvia	N.	Atlas.
K. Algiers	N.	Algiers	Saffran	N.	Atlas.
K. Tunis	N.	Tunis	Megarada	N.	Atlas.
K. Tripoli	N.	Tripoli	Salines	N. E.	Atlas.
K. Barca	N.	Docra	Nile	N.	Meis
K. Egypt	N. E.	Cairo	Nile	N.	Gianadel
E. Abyssinia } or Ethiopia }	E.	Ambamarjam	Nile	N.	
Ajan	E.	Adea	Madadoxa	S. S. E.	
Zanguebar.	E.	Melinda	Cuama	E. S. E.	Amara
Sofala	S. E.	Sofala	Amara	E.	Amara
Terra de Natal	S. E.	Natal	St. Esprit	E.	Table
Cafraria	S. S.	Cape Town	St. Christopher	E.	Sunda
Mataman	S. S. W.		Angri	W.	Sunda
K. Benguela	S. S. W.	Benguela	Negros	W.	Sunda
K. Angola	S. W.	Loando	Coanza	W.	Sunda
K. Congo	S. W.	St. Salvador	Zaara	S. W.	Sunda
K. Loango	S. W.	Loango	Zette	S. W.	St. Esprit
Biafara	S. W.	Biafara	Camerones	S. W.	St. Esprit
K. Benin	S. W.	Benin	Formola	S. W.	
Guinea	S. W.	Cape Coast	Volta	S.	Sierra Leona
Mandinga	W.	James Fort	Gambia	W.	C. Verd.
Sanhaga	W.	Sanhaga	Senegal	N. W.	Atlas
Biledulgerid	Mid. N.	Dara	Dara	S.	
Zara	Mid.	Zuenzega	Nubia	E.	
Nubia	Mid.	Nubia	Nubia	E.	
Negroland	Mid.	Tombute	Niger	W.	
Ethiopia inter.	Mid.	Chaxumo	Niger	W.	Luna
Monomugi	Mid. S.	Merango	Cuama	E. S. E.	Luna
E. Monomotapa	Mid. S.	Mogar	Amara	S. E.	Amara

Many parts of the coasts of Africa are subject to the European nations: Thus the Kingdoms of Algiers, Tunis, Tripoli, Barca, and Egypt, are either subject to the Ottoman, or Turkish Empire, or acknowledge themselves under its protection.

Abyssini

Abyssinia is governed by its own Emperor.

Ajan or Anian is peopled by a few wild Arabs.

In Zanguebar and Sofala, the Portuguese have many black Princes tributary to them.

Caffraria, or the country of the Hottentots, belongs to the Dutch.

The sea coasts of Guinea are usually distinguished by the names of the *Slave Coast*, *Gold Coast*, *Ivory Coast*, *Grain Coast*, and *Sierra Leone*.

The English, Dutch, French, Portuguese and others, have several settlements along these coasts, and even many miles up the country, particularly the English on the rivers Gambia and Senegal.

In the general table, the countries are taken in a very large sense; for many of them contain a great number of states independent one of the other, the particulars whereof are not known to Geographers.

44. *The principal Seas, Gulfs and Bays in Africa, are*

The *Red Sea*, between Africa and Asia: It washes the coast of Arabia on the Asiatic side, and the coasts of Egypt and Abyssinia on the African side.

Mosambique Sea, between Africa and the island of Madagascar eastward.

Saldanna Bay in Caffraria, on the Ethiopic Ocean.

Bight of Benin on the coast of Guinea, in the Ethiopic Ocean.

45. *The principal African Islands, are*

	Chief Isle.	Chief Town.	Situation.
<i>Madeira isles</i>	Madeira	Funchal	N. Atlantic Ocean
<i>Canary isles</i>	Canaria	Palma	N. Atlantic Ocean
<i>C. Verd isles</i>	St. Jago	St. Jago	N. Atlantic Ocean
<i>Ethiopian isles</i>	St. Helena		Ethiopic Ocean
<i>Kimora isles</i>	Johanna	Demani	Indian Ocean
<i>Sokotora isles</i>	Zocotora	Calanfia	Indian Ocean
<i>Amirante isles</i>	But little known		Indian Ocean

The island of *Madagascar*, one of the largest in the world, lies in the Indian Ocean: It is divided into a multitude of little states; some of them formed by the European privateers, and their successors descended from mixing with the natives.

The islands of *Bourbon* and *Mauritius* lie in the Indian Ocean, to the east of Madagascar: These belong to the French.

The Madeiras, and Cape de Verd Isles, belong to the Portuguese.

The Canary Isles to Spain; St. Helena to England.

OF AMERICA.

46. This vast continent, called by some the new world, having been discovered by the Europeans since the year 1492, is usually divided into two parts, one called North, and the other South America; being joined to one another by the Isthmus of Darien.

North America lies between the latitudes of 10 degrees and 80 degrees north; and chiefly between the longitudes of 50 degrees and 130 degrees west of London; is about 4200 miles from north to south, and about 4800 from east to west. It is bounded on the east by the north Atlantic Ocean, by the Gulf of Mexico on the south, on the west by the Pacific ocean, and by the Northern continent and ocean to the northward.

South America is bounded on the east by the south Atlantic Ocean, by the Southern Ocean to the south, by the Pacific Ocean on the west, and on the north by the Caribbean Sea: It lies between the latitudes of 12 degrees north, and 58 degrees south; and between the longitudes of 45 degrees and 83 degrees west from London; is about 4200 miles long, and about 2200 miles in breadth.

47. The positions and names of the chief countries, cities, rivers, and mountains, in North America, are in the following table.

Countries.	Position.	Chief Cities.	Rivers.	Course.	Mountains.
California	W.	St. Juan	N. River	S. S. E.	
New Mexico	S.	Sante Fe	Panuco	E.	
Old Mexico	S. W.	Mexico	Mississipi	S.	
Louisiana	S.	New Orleans	St. John	N. to E.	Apalachian
Florida	S.	St. Augustin	Altamaha	E. S. E.	Apalachian
Georgia	S. S. E.	Savannah	Ashly	S. E.	Apalachian
Carolina	S. E.	Charles Town	Powtomack	S. E.	Apalachian
Virginia	E.	James Town	Powtomack	S. E.	Apalachian
Maryland	E.	Annapolis	Delawar	S.	Apalachian
Pennsilvania	E.	Philadelphia	Albany	S.	
Jerseys	E.	New York	Connecticut	S.	
New England	E.	Boston	S. John	S. S. E.	Ladies
Nova Scotia	N. E.	Hallifax	St. Lawrence	N. E.	
Canada	Mid.	Quebec	Rupert	W.	
New Britain	N. N. E.	Fort Rupert	Nelson	W.	
New Wales	N.	Fort Nelson			

California, Old Mexico, and New Mexico, belong to Spain.

Louisiana, to the west of the river Mississipi, was possessed by the French, at the end of the late war; but is now transferred to Spain.

All the other countries are in the hands of the English.

In South America, the position and names of the chief countries, cities, rivers, and mountains, are as follow.

Countries.	Position.	Chief Cities.	Rivers.	Courſe.	Mountains.
Terra Firma	N.	Panama	Oronoque	N. E.	
Peru	W.	Lima	Chuquimayo	W. N. W.	Andes
Chili	S. W.	St. Jago	Valpariſo	W.	Andes
Patagonia	S.		Defaguadero	S.	Andes
La Plata	S. E.	Buenos Ayres	La Plata	S.	Andes
Paraguay	Mid.	Aſſumption	Paragua	S.	
Braſil	E.	St. Salvador	Rio Real	N. E.	
Amazonia	Mid.		Amazons	E.	
Guiana	N. E.	Surinam	Elquebe	N. N. E.	

Terra Firma, Peru, Chili, La Plata, and Paraguay, are in the poſſeſſion of the Spaniards.

Braſil belongs to the Portuguese.

Patagonia, Amazonia, and Guiana, are poſſeſſed by the native Indians, except ſome parts of the coaſts of Guiana, in the hands of the Dutch and French.

48. *The principal Seas, Gulfs, and Bays in America.*

The *Caribbean Sea*, bounded by Terra Firma on the ſouth, and a range of iſlands on the north and eaſt.

Gulf of Mexico, formed by Old Mexico, Louiſiana, and Florida.

Bay of Campeachy, part of the Gulf of Mexico, on the Mexican coaſt.

Bay of Honduras, part of the Caribbean Sea, next to Mexico.

Bay of Panama, in the Pacific Ocean, next the Iſthmus of Darien.

Bay of California, in the Pacific Ocean, having California on the weſt.

Bay of Fundy, in Nova Scotia, north Atlantic Ocean.

Gulf of St. Lawrence, in the North Atlantic Ocean, bounded by Nova Scotia, New Britain, and ſome iſlands eaſtward and ſouth eaſtward.

Hudſon's Bay, between New Britain E. and New Wales W.

49. *The chief American Iſlands in the Atlantic Ocean.*

Newfoundland, and *Cape Breton*, eaſt of the Gulf of St. Lawrence.

Bermudas, or *Summer Iſlands*, eaſt of Carolina.

Bahama Iſles, ſouth eaſt of Florida.

Great Antilles { Cuba ch. town Havana } lying E. of the
Hispaniola ch. town St. Domingo } Mexican Gulf, and
Jamaica ch. town Kingſton } N. of the Car. Sea

Caribbee Iſles, bounding the Caribbean Sea on the E. and N. E.

Leſſer Antilles, on the N. N. E. of Terra Firma, in the Caribbee Sea.

Terra del Fuego, on the ſouth of Patagonia, in the Southern Ocean.

Galapago Iſles, lying N. W. of Peru in the Pacific Ocean.

SECTION IV.

Geographical Problems.

50. PROBLEM I. *Given the latitudes of two places :
Required their difference of latitude ?*

CASE I. When the latitudes of the given places have the same name.

RULE. Subtract the lesser latitude from the greater, the remainder is the difference of latitude.

EXAM. I. *What is the difference of latitude between London and Rome ?*

London's lat. $51^{\circ} 32' \text{ N.}$
Rome's lat. $41^{\circ} 54' \text{ N.}$

Diff. lat. $\begin{array}{r} 9 \ 38 \\ 60 \\ \hline \end{array}$

$\underline{578 \text{ miles.}}$

EXAM. II. *What is the difference of latitude between the Lizard and the Island of Madeira ?*

Lizard's lat. $49^{\circ} 57' \text{ N.}$
Madeira's lat. $32^{\circ} 38'$

Diff. lat. $\begin{array}{r} 17 \ 19 \\ \hline \end{array} = 1039 \text{ m.}$

EXAM. III. *What is the difference of latitude between the Island of St. Helena and the Cape of Good Hope ?*

C. Good Hope's lat. $33^{\circ} 55' \text{ S.}$
St. Helena's lat. $16^{\circ} 00' \text{ S.}$

Diff. lat. $\begin{array}{r} 17 \ 55 \\ \hline \end{array} = 1075 \text{ m.}$

EXAM. IV. *A ship from the latitude of $43^{\circ} 18' \text{ N.}$ is come to the lat. of $34^{\circ} 49' \text{ N.}$ Required the diff. of latitude ?*

Lat. from $43^{\circ} 18' \text{ N.}$
Lat. in $34^{\circ} 49' \text{ N.}$

Diff. lat. $\begin{array}{r} 8 \ 29 \\ \hline \end{array} = 509 \text{ m.}$

The situation of about 1400 particular places are contained in a Geographical Table, art. 137, at the end of this book ; wherein the latitudes and longitudes of places are to be sought, as they follow in alphabetical order.

CASE II. When the latitudes of the given places have contrary names.

RULE. Add the latitudes together, and the sum will be the difference of latitude.

EXAM. I. *Required the diff. of lat. between C. Finisterre and C. St. Roque ?*
C. Finisterre lat. $43^{\circ} 15' \text{ N.}$
C. St. Roque lat. $5^{\circ} 00' \text{ S.}$

Diff. lat. $\begin{array}{r} 48 \ 15 \\ 60 \\ \hline \end{array}$

$\underline{2895 \text{ miles.}}$

EXAM. II. *What is the difference of latitude between the Island of Barbados and C. Negro ?*

I. of Barbados's lat. $13^{\circ} 00' \text{ N.}$
C. Negro's lat. $16^{\circ} 30' \text{ S.}$

Diff. lat. $\begin{array}{r} 29 \ 30 \\ \hline \end{array} = 1770 \text{ m.}$

EXAM. III. *Required the difference of latitude between Cape Horn and Cape Corientes in Mexico ?*

Cape Horn's lat. $55^{\circ} 42' \text{ S.}$
C. Corientes's lat. $20^{\circ} 18' \text{ N.}$

Diff. lat. $\begin{array}{r} 76 \ 00 \\ \hline \end{array} = 4560 \text{ m.}$

EXAM. IV. *A ship from the lat. of $8^{\circ} 28' \text{ S.}$ has sailed north to the lat. $6^{\circ} 45' \text{ N.}$ Required the diff. of latitude ?*

Lat. from $8^{\circ} 28' \text{ S.}$
Lat. in $6^{\circ} 45' \text{ N.}$

Diff. lat. $\begin{array}{r} 15 \ 13 \\ \hline \end{array} = 913 \text{ m.}$

51. PROBLEM II. *Given the latitude of one place and the difference of latitude between it and another place: Required the latitude of the other?*

CASE I. When the given latitude and difference of latitude have the same name.

RULE. To the given latitude, add the degrees and minutes in the diff. of latitude, that sum is the other latitude, of the same name.

EXAM. I. *A ship from the latitude of 38° 14' N. sails north till her difference of latitude is 12° 32': What latitude is she come to?*
 Lat. from 38° 14' N.
 Diff. lat. 12 32 N.

 Lat. in 50 46 N.

EXAM. II. *A ship from the island of Ascension runs south till her diff. of latitude is 5° 37': What is the present latitude of the ship?*
 I. Ascension's lat. 7° 57' S.
 Diff. lat. 5 37 S.

 Ship's lat. 13 34 S.

EXAM. III. *A ship from the Island of Madeira sails N. 675 miles: What lat. is she in?*
 I. Madeira's lat. 32° 38' N.
 Diff. lat. $\frac{675}{60}$ (I. 22) = 11 15 N.

 Ship's lat. 43 53 N.

EXAM. IV. *Four days ago we were in the latitude of the Cape of Good Hope, and have run each day 92 miles directly S. What is our present latitude?*

C. Good Hope's lat. 33° 55' S.
 Diff. lat. $\frac{92 \times 4}{60}$ (I. 22) = 4 36 S.

 Present latitude 38 31 S.

CASE II. When the given latitude and difference of latitude have contrary names.

RULE. Take the difference between the given latitude and the degrees and minutes in the diff. of latitude, the remainder is the other latitude, of the same name with the greater.

EXAM. I. *A ship from the latitude of 38° 14' N. sails south till her difference of latitude is 12° 32': What latitude is she come to?*
 Lat. from 38° 14' N.
 Diff. lat. 12 32 S.

 Lat. in 25 42 N.

EXAM. II. *A ship from the Island of Ascension runs north till her diff. lat. is 5° 37': What is the present latitude of the ship?*
 I. Ascension's lat. 7° 57' S.
 Diff. lat. 5 37 N.

 Ship's lat. 2 20 S.

EXAM. III. *A ship from Sierra Leona sails S. 839 miles: What latitude is she in?*
 Sierra Leona's lat. 8° 30' N.
 Diff. lat. $\frac{839}{60}$ (I. 22) = 13 59 S.

 Ship's lat. 5 29 S.

EXAM. IV. *Four days ago we were in the latitude of the Island of St. Matthew, and sailed due north 6 miles an hour: What latitude is the ship in?*

St. Matthew's lat. 1° 23' S.
 Diff. lat. $\frac{6 \times 24 \times 4}{60}$ (I. 22) = 9 36 N.

 Ship's latitude 8 13 N.

52. PROBLEM III. *Given the longitude of two places:
Required the difference of longitude?*

RULE. If the longitudes are of the same name, their difference is the difference of longitude required.

But if the longitudes are of different names, their sum gives the difference of longitude.

And if this sum exceeds 180 degrees, take it from 360 degrees, and there remains the difference of longitude.

EXAM. I. *Required the difference of longitude between London and Naples.*

London's long. $00^{\circ} 00' E.$
Naples' long. $14^{\circ} 45' E.$

Diff. longitude

$14^{\circ} 45'$
 60

885 miles.

EXAM. II. *A ship in longitude $14^{\circ} 45' W.$ is bound to a port in longitude $48^{\circ} 18' west$: What diff. of longitude must she make?*

Ship's long. $14^{\circ} 45' W.$
Long. bound to $48^{\circ} 18' W.$

Diff. longitude

$33^{\circ} 33'$
 60

2013 minutes,

EXAM. III. *What is the difference of longitude between Cape Gardafuir and Cape Comorin?*

C. Gardafuir's long. $50^{\circ} 25' E.$
C. Comorin's long. $78^{\circ} 17' E.$

Diff. longitude

$27^{\circ} 52'$
 60

1672 minutes.

Sometimes the diff. lon. between two places is estimated by the diff. of time, allowing an hour to every 15 degrees of longitude, and one min. of time for every 15 min. of a deg. or a deg. for every 4 min. of time.

EXAM. *Having at 6 h. 48 m. P. M. observed at sea a certain appearance in the heavens, which I know was seen the same instant at 3 h. 25 m. P. M. in London: Required the diff. longitude between the places of observation?*

From 6 h. 48 m.

Take $3^h 35^m$

Leaves $3^h 13^m =$ diff. time.

And because the hour of appearance at London was least, therefore I know myself to be to the eastward of London.

53. PRO-

EXAM. IV. *Required the difference of longitude between St. Christopher's and Cape Negro?*

St. Christopher's long. $62^{\circ} 50' W.$
C. Negro's long. $11^{\circ} 30' E.$

Diff. longitude

$74^{\circ} 20'$
 60

4460 minutes.

EXAM. V. *A ship in longitude $140^{\circ} 20' W.$ is bound to a place in longitude $139^{\circ} 35' E.$ what diff. of longitude must she make?*

Ship's long. $140^{\circ} 20' W.$
Long. bound to $139^{\circ} 35' E.$

Diff. longitude

$80^{\circ} 05'$

EXAM. VI. *What is the difference of longitude between Cape Horn and Manila?*

C. Horn's long. $66^{\circ} 00' W.$
Manila's long. $124^{\circ} 00' E.$

Diff. longitude

$190^{\circ} 00'$
 $360^{\circ} 00'$

$170^{\circ} 00'$

53. PROBLEM IV. *Given the longitude of one place, and the difference of longitude between that and another : Required the longitude of the second place ?*

RULE. If the given longitude and difference of longitude are of a contrary name, their difference is the longitude required ; and is of the same name with the greater.

But if the given longitude and difference of longitude are of the same name, the sum is the longitude sought, of the same name with the given place.

And if the sum is greater than 180 degrees, take it from 360 degrees, remains the longitude required, of a contrary name with that of the given place.

EXAM. I. *A ship from the longitude of $41^{\circ} 12' E.$ sails westward until her difference of longitude is $15^{\circ} 47'$: What is her present longitude ?*

Ship's longitude	$41^{\circ} 12' E.$
Diff. long.	$15 \quad 47 \quad W.$
Pres. long.	$25 \quad 25 \quad E.$

EXAM. II. *A ship from Cape Charles in Virginia sails eastward until she has altered her longitude 22° 53': What longitude is she in ?*

C. Charles's long.	$76^{\circ} 07' W.$
Diff. long.	$22 \quad 53 \quad E.$
Ship's long.	$53 \quad 14 \quad W.$

EXAM. IV. *A ship from Cape Finisterre sails westward, and finds she has altered her longitude 587 miles: What longitude is she arrived in ?*

C. Finisterre's long.	$9^{\circ} 20' W.$
Diff. long.	$587 \quad 60$
Long. in	$19 \quad 07 \quad W.$

EXAM. V. *A ship from Cape St. Lucar in California has made $87^{\circ} 18'$ of west longitude: What longitude is she in ?*

C. Lucar's long.	$111^{\circ} 35' W.$
Diff. long.	$87 \quad 18 \quad W.$
	$198 \quad 53 \quad W.$
	$360 \quad 00$
Ship's longitude	$161 \quad 07 \quad E.$

EXAM. III. *Four days ago I departed from C. St. Sebastian in Madagascar, and I have made each day 75 miles of east longitude: Required the longitude the ship is in ?*

75	
4	C. Sebas. long. $49^{\circ} 13' E.$
	Diff. long.
	$5 \quad 00 \quad E.$
6,0)30,0	Ship's long.
	$54 \quad 13 \quad E.$
50	

EXAM. VI. *Seven days ago my longitude was $172^{\circ} 17' W.$ and I have made each day 132 miles of west longitude: Required my present longitude ?*

132	Departed long. $172^{\circ} 17' W.$
	7 D.f. long.
	$15 \quad 24 \quad W.$
6,0)92,4	
	$187 \quad 41$
	$360 \quad 00$
15 24	Present long. $172 \quad 19 \quad E.$

54.

SECTION V.

Of the Use of the Globes.

By the Globes here are meant two spherical bodies, whose convex surfaces are supposed to give a true representation of the earth and heavens, as visible by observation, and are called the Terrestrial and Celestial Globes.

The TERRESTRIAL GLOBE has delineated on its convexity, the whole surface of the *earth* and *sea*, in their relative size, form, and situation.

The CELESTIAL GLOBE has drawn on its surface, the images of the several constellations and stars: The relative magnitude and position which the stars are observed to have in the heavens, being preserved on this globe.

To the globes are fitted certain appurtenances, whereby a great variety of useful problems are neatly solved.

The BRAZEN MERIDIAN is that ring, or hoop, wherein the globe hangs on its axis; which is represented by two wires passing through its poles. This circle is divided into four quarters of 90° each; in one semicircle the divisions begin at each pole, and end at 90° , where they meet: In the other semicircle, the divisions begin at the middle, and proceed thence towards each pole, where they end at 90° degrees. The graduated side of this brazen circle serves as a meridian for any point on the surface of the earth, the globe being turned about till that point comes under the circle.

The HOUR CIRCLE is a small circle of brass, which is divided into 24 hours, the quarters and half quarters: It is fixed on the brazen meridian, equally distant from the north end of the axis, to which an index is fitted, that points out the divisions of the hour circle as the globe is turned about.

The HORIZON is represented by the upper surface of the wooden circular frame encompassing the globe about its middle. On this wooden frame is a kind of perpetual calendar, contained in several concentric circles: The inner one is divided into four quarters of 90° degrees each; the next circle is divided into the twelve months, with the days in each according to the new stile; the next contains the 12 equal signs of the zodiac, each being divided into 30° degrees; the next is the 12 months and days according to the old stile; and there is another circle, containing the 32 winds, with their halves and quarters. Although these circles are on all horizons, yet their disposition by the workmen are not always the same.

The QUADRANT OF ALTITUDE is a thin freight slip of brass, one edge of which is graduated into 90° degrees and their quarters, equal to those of the meridian: To one end of this is fixed a brass nut and screw, whereby it is put on, and fastened to, the meridian: which, if in the zenith, or pole of the horizon, then the graduated edge represents a vertical circle passing through any point.

Besides these, there are several circles described on the surfaces of both globes, such as the equinoctial, ecliptic, circles of longitude and right ascension, the tropics, polar circles, parallels of lat. and decl., on the celestial globe; and on the terrestrial, the equator, ecliptic, tropics, polar circles, parallels of latitude, hour circles, or meridians to every 15 degrees, and the spiral rhumbs flowing from several centers, called Flies.

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29 01-

55. PROBLEM I.

To find the latitude and longitude of any place on the terrestrial globe.

- 1st. Bring the given place under that side of the graduated brazen meridian where the degrees begin at the equator, by turning the globe about.
- 2d. Then the degree of the meridian over it, shews the latitude.
- 3d. And the degree of the equator under the merid., shews the lon.

On some globes the longitude is reckoned, on the equator, from the meridian where it begins, only eastward until it ends at 360° : On such globes, when the longitude of a place exceeds 180° , take it from 360° , and call the remainder the longitude westward.

56. PROBLEM II.

To find any place on the globe whose latitude and longitude are given.

- 1st. Bring the given longitude, found on the equator, to the meridian.
- 2d. Then under the given latitude, found on the meridian, is the place sought.

57. PROBLEM III.

To find the distance and bearing of any two given places on the globe.

- 1st. Lay the graduated edge of the quadrant of altitude over both places, the beginning, or 0 degrees, being on one of them, and the degrees between them shew their distance; these degrees multiplied by 60 give sea miles, and by 70 give the distance in land miles nearly.
- 2d. Observe, while the quadrant lies in this position, what rhumb of the nearest fly, or compass, runs mostly parallel to the edge of the quadrant, and that rhumb shews the bearing sought nearly.

58. PROBLEM IV.

To find the Sun's place and declination on any day.

- 1st. Seek the given day in the circle of months on the horizon, and sight against it, in the circle of signs, is the Sun's place.

Thus it will be found that the Sun enters.

The spring signs, Aries, March 20. Taurus, April 20. Gemini, May 21.
The summer signs, Cancer, June 21. Leo, July 23. Virgo, Aug. 23.
Autumnal signs, Libra, Sept. 22. Scorpio, Oct. 23. Sagittar. Nov 22.
The winter signs, Capric. Dec. 21. Aquarius, Jan. 20. Pisces, Feb. 18.
 2d. Seek the Sun's place in the ecliptic on the globe, bring that place to the meridian, and the division it stands under is the Sun's declination on the given day.

On the globes, the ecliptic is readily distinguished from the equator, not only by the different colours wherewith they are stained, but also by the ecliptic's approaching towards the poles, after its interfection with the equator. The marks of the signs are also put along the ecliptic, one at the beginning of every successive 30 degrees.

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59. PRO-

59.

PROBLEM V.

To rectify the globe for the latitude, zenith, and noon.

1st. Set the globe upon an horizontal plane, with its parts answering to those of the world; move the meridian in its notches, by raising or depressing the pole, until the degrees of latitude cut the horizon; then is the globe rectified for the latitude.

2d. Reckon the latitude from the equator towards the elevated pole, there screw the bevil edge of the nut belonging to the quadrant of altitude, and the rectification is made for the zenith.

3d. Bring the Sun's place (found by the last problem) to the meridian, set the index to the XII at noon, or upper XII, and the globe is rectified for the Sun's southing, or noon.

60.

PROBLEM VI.

To find where the Sun is vertical at any given time.

1st. Bring the Sun's place, found for the given day (58), to the meridian, note the degree over it, and set the index to the given hour.

2d. Turn the globe till the index comes to 12 at noon, then the place under the said noted degree has the Sun in the zenith at that time.

3d. All the places that pass under that degree, by turning the globe round, will have the Sun vertical to them on that day.

61.

PROBLEM VII.

To find on what days the Sun will be vertical at any given place in the torrid zone.

1st. Note the latitude of the given place on the meridian.

2d. Turn the globe, and note what points of the ecliptic pass under the latitude noted on the meridian

3d. Seek those points of the ecliptic in the circle of signs on the horizon, and right against them in the circle of months stand the days required.

In this manner it will be found, that the Sun will be vertical to the Island of St. Helena on the 6th of November, and on the 4th of February. And at Barbadoes on the 24th of April, and the 18th of August.

62.

PROBLEM VIII.

At any given hour in a given place, to find what hour it is in any other place.

1st. Bring the place where the time is given to the meridian, and set the index to the given hour.

2d. Bring the other given place to the meridian, and the index shews the hour corresponding to the given time.

63. PROBLEM IX.

At any given time, to find all those places of the Earth where the Sun is then rising or setting, where mid-day or midnight.

Find the place where the Sun is vertical at the given time (60), rectify the globe for the latitude of that place, and bring it to the meridian.

Then all those places, that are in the west half of the horizon, have the Sun rising; and those in the eastern half have it setting.

Those under the meridian above the horizon have the Sun culminating, or noon; and those under the meridian below the horizon have midnight.

Those above the horizon have day; those below it have night.

64. PROBLEM X.

To find the angle of position of two places; or, the angle made by the meridian of one place, and a great circle passing through both places.

Rectify for the latitude of one of the given places, and bring it to the meridian; there fix the quadrant of altitude, and set its graduated edge to the other place: Then will that edge of the quadrant cut the horizon in the degree of position sought.

Thus, the angle of position at the Land's End to Barbadoes is south 71° westerly: But the angle of position at Barbadoes to the Land's End is north $37^{\frac{1}{2}}$ degrees easterly.

Hence neither of those positions can be the true bearing; for the rhumb passing through both places, will be opposite one way to what it is the other.

65. PROBLEM XI.

The latitude of any place not within the polar circle being given, to find the time of sun-rising and setting, and the length of the day and night.

Rectify for the latitude and the noon; bring the Sun's place to the eastern side of the horizon, and the index shews the time of rising: The Sun's place brought to the western side of the horizon, the index gives the setting.

Or, The time of rising taken from 12 hours gives the time of setting. The time of setting being doubled, gives the length of the day.

And the time of rising being doubled, gives the length of the night.

Thus, at London, on April 15th, the day is $13^{\frac{1}{2}}$ hours; the night $10^{\frac{1}{2}}$ hours.

66. PROBLEM XII.

To find the length of the longest and shortest days in any given place.

Rectify for the latitude; bring the solstitial point of that hemisphere to the eastern part of the horizon, set the index to 12 at noon, turn the globe till the solstitial point comes to the western side of the horizon, the hours past over by the index give the length of the longest day, or night; and its complement to 24 hours gives the length of the shortest night, or day.

B b 3

67. PRO.

67.

P R O B L E M XIII.

A place being given in either frigid or frozen zone, to find the time when the Sun begins to appear at, or depart from, that place: Also how many successive days he is present to, or absent from, that place.

Rectify for the latitude, turn the globe, and observe what degrees in the first and second quadrants of the ecliptic are cut by the north point of the horizon, the latitude being supposed north.

Find those degrees in the circle of signs on the horizon, and their corresponding days of the month; and all the time between those days the Sun will not set in that place.

Again. Observe what degree in the third and fourth quadrants of the ecliptic will be cut by the south point of the horizon, and the days answering: Then the Sun will be quite absent from the given place during the intermediate days; that day in the 3d quadrant, shews when he begins to disappear; and that in the 4th quadrant, shews when he begins to shine in the place proposed.

Thus, at the North cape, in lat. 71° , the Sun never sets from May 15 to July 28, which is 74 days; and never rises from November 16 to January 24, which is 69 days.

68.

P R O B L E M XIV.

To find the antæci, perieci, and antipodes of any place.

Bring the given place to the meridian, tell as many degrees of latitude on the contrary side of the equator, and it gives the place of the antæci; that is, of those who have opposite seasons of the year, but the same times of the day.

The given place being under the meridian, set the index to 12 at noon, turn the globe till the index points to 12 at night, and the point under the meridian in the given latitude is the place of the perieci; that is, of those who have the same seasons of the year, but opposite times of the day.

The globe remaining in this position, seek on the contrary side of the equator for the degrees of latitude given, and the point under the meridian thus found will be the antipodes to the given place; that is, of such whose seasons of the year and times of the day are directly opposite to those of the given place.

69.

P R O B L E M XV.

To find the beginning and end of the twilight in any place.

Rectify the globe for the latitude, zenith, and noon.

Seek the point of the ecliptic opposite to the Sun's place, turn the globe and quadrant of altitude, till the said opposite point of the ecliptic stands against 18 degrees on the quadrant of altitude; then will the index shew the beginning or end of the twilight: That is, the beginning in the morning, when those points meet in the western hemisphere or the end in the evening, when the said points meet in the eastern hemisphere.

70. P R O.

PROBLEM XVI.

70.

The latitude of a place and day of the month being given; to find the Sun's declination, meridian altitude, right ascension, amplitude, oblique ascension, ascensional difference; and thence the time of rising, setting, length of the day and night.

Rectify for the latitude and noon. Then,

The degree of the meridian over the Sun's place is the declination.

The meridian altitude is shewn by the degrees the Sun is above the horizon; and is equal to the sum or diff. of the co-lat. and decl.

The Sun's right ascen. is that degree of the equator under the meridian.

Bring the Sun's place to the eastern part of the horizon. Then,

The amplitude is that degree of the horizon opposite the Sun.

The oblique ascension is that degree of the equator cut by the horizon.

The ascen. diff. is the diff. between the right and oblique ascensions.

The ascen. diff. converted into time, will give the time the Sun rises before or after the hour of six, according as his amplitude is to the northward or southward of the east point of the horizon.

PROBLEM XVII.

71.

Given the latitude of the place and day of the month; to find the Sun's altitude and azimuth, either when he is due east or west, at 6 o'clock, or at any other hour while he is above the horizon.

Rectify the globe for the latitude, zenith, and noon.

Set the quadrant of altitude to the east point of the horizon, turn the globe till the Sun's place comes to the quadrant's edge, and it shews the altitude, his azimuth being now 90° , and the index shews the hour.

Turn the globe till the index points at 6, there stay it, and move the quadrant till its edge cuts the Sun's place; then the degrees at the Sun shews its altitude, and the degrees cut by the quadrant in the horizon shews the azimuth reckoning from the north.

In like manner, the globe being turned till the index is against any other hour, suppose 10 in the forenoon. Then,

The graduated edge of the quadrant of altitude being turned to cut the Sun's place, will give both the altitude and azimuth at that time.

PROBLEM XVIII.

72.

Given the latitude, day of the month, and Sun's altitude; to find the azimuth and hour of the day.

Rectify the globe for the latitude, zenith, and noon.

Turn the globe and quadrant until the Sun's place coincide with the altitude on the graduated edge of the quadrant.

Then will that edge of the quadrant cut in the horizon the degrees of azimuth, reckoned from the north; and the index will shew the hour of the day.

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Find those degrees in the circle of signs on the horizon, and their corresponding days of the month; and all the time between those days the Sun will not set in that place.

Again. Observe what degree in the third and fourth quadrants of the ecliptic will be cut by the south point of the horizon, and the days answering: Then the Sun will be quite absent from the given place during the intermediate days; that day in the 3d quadrant, shews when he begins to disappear; and that in the 4th quadrant, shews when he begins to shine in the place proposed.

Thus, at the North cape, in lat. 71° , the Sun never sets from May 15 to July 28, which is 74 days; and never rises from November 16 to January 24, which is 69 days.

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Bring the given place to the meridian, tell as many degrees of latitude on the contrary side of the equator, and it gives the place of the antæci; that is, of those who have opposite seasons of the year, but the same times of the day.

The given place being under the meridian, set the index to 12 at noon, turn the globe till the index points to 12 at night, and the point under the meridian in the given latitude is the place of the peræci; that is, of those who have the same seasons of the year, but opposite times of the day.

The globe remaining in this position, seek on the contrary side of the equator for the degrees of latitude given, and the point under the meridian thus found will be the antipodes to the given place; that is, of such whose seasons of the year and times of the day are directly opposite to those of the given place.

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To find the beginning and end of the twilight in any place.

Rectify the globe for the latitude, zenith, and noon.

Seek the point of the ecliptic opposite to the Sun's place, turn the globe and quadrant of altitude, till the said opposite point of the ecliptic stands against 18 degrees on the quadrant of altitude; then will the index shew the beginning or end of the twilight: That is, the beginning in the morning, when those points meet in the western hemisphere, or the end in the evening, when the said points meet in the eastern hemisphere.

70. PRO.

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The Sun's right ascen. is that degree of the equator under the meridian.

Bring the Sun's place to the eastern part of the horizon. Then,

The amplitude is that degree of the horizon opposite the Sun.

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The ascen. diff. converted into time, will give the time the Sun rises before or after the hour of six, according as his amplitude is to the northward or southward of the east point of the horizon.

PROBLEM XVII.

71.

Given the latitude of the place and day of the month; to find the Sun's altitude and azimuth, either when he is due east or west, at 6 o'clock, or at any other hour while he is above the horizon.

Rectify the globe for the latitude, zenith, and noon.

Set the quadrant of altitude to the east point of the horizon, turn the globe till the Sun's place comes to the quadrant's edge, and it shews the altitude, his azimuth being now 90° , and the index shews the hour.

Turn the globe till the index points at 6, there stay it, and move the quadrant till its edge cuts the Sun's place; then the degrees at the Sun shews its altitude, and the degrees cut by the quadrant in the horizon shews the azimuth reckoning from the north.

In like manner, the globe being turned till the index is against any other hour, suppose 10 in the forenoon. Then,

The graduated edge of the quadrant of altitude being turned to cut the Sun's place, will give both the altitude and azimuth at that time.

PROBLEM XVIII.

72.

Given the latitude, day of the month, and Sun's altitude; to find the azimuth and hour of the day.

Rectify the globe for the latitude, zenith, and noon.

Turn the globe and quadrant until the Sun's place coincide with the altitude on the graduated edge of the quadrant.

Then will that edge of the quadrant cut in the horizon the degrees of azimuth, reckoned from the north; and the index will shew the hour of the day.

73.

PROBLEM XIX.

To represent the appearance of the heavens at any time in a given place.

Rectify the celestial globe for the latitude, zenith, and noon, and turn the globe till the index points at the given hour. Then,

Those stars in the eastern half of the horizon are rising; those in the western are setting; and those in the meridian are culminating.

The quadrant being set to any given star, will shew its altitude, and at the same time its azimuth, reckoned in the horizon.

Now by turning the globe round, it will readily appear what stars never set in that place, and those which never rise; those of perpetual apparition never go below the horizon, and those of perpetual absence never come above it.

74.

PROBLEM XX.

To find the latitude and longitude of any star.

Put the center of the quadrant of altitude on the pole of the ecliptic, and its graduated edge on the given star. Then,

The latitude is shewn by the degrees between the ecliptic and star.

The longitude is the degrees cut on the ecliptic by the quadrant.

75.

PROBLEM XXI.

To find the declination and right ascension of a star.

Bring the star to the meridian, the degree over it is the declination; and the degree of the equator under the meridian is the right ascension.

76.

PROBLEM XXII.

On any day, and in any given place, to find when a proposed star rises, sets, or culminates.

Rectify the globe for the latitude and noon.

Bring the star to the eastern side of the horizon, and the index shews the time of its rising.

Turn the globe till the star comes to the meridian, and the index shews the time of its culminating; and in like manner when it sets, the time will be shewn by the index.

Its meridian altitude, oblique ascension, and ascensional difference, are found in the same manner as for the Sun, at art. 70.

SECTION

SECTION VI.

Of Winds.

77. A FLUID is a body whose particles readily give way to any impressed force; and by this readiness of yielding, the particles are easily put into motion.

Thus, not only liquids, but steams or vapours, smoke or fumes, and others of a like kind, are reckoned as fluids.

From all parts of the Earth vapours and fumes are constantly arising to some distance from its surface.

This is known by observation; and is chiefly caused by the heat of the Sun, and sometimes from subterraneous fires arising from the accidental mixing of some bodies.

78. AIR is a fine invisible fluid surrounding the globe of the Earth, and extended to some miles above its surface.

The ATMOSPHERE is that collection of air, and the bodies contained in it, which circumscribes the Earth.

79. From a multitude of experiments, air is found to be both heavy and springy.

By its weight, it is capable of supporting other bodies, such as vapours and fumes, in the same manner as wood is supported by water.

By its springiness or elasticity, a quantity of air is capable of being expanded, or spreading itself so as to fill a larger space*; and of being compressed or confined in a smaller compass†.

80. Air is compressed or condensed by cold, and expanded or rarefied by heat.

This is evident from a multitude of experiments.

An alteration being made by heat or cold in any part of the atmosphere, its neighbouring parts will be put in motion by the endeavour which the air always makes to restore itself to its former state.

For experiments shew, that condensed or rarefied air will return to its natural state, when the cause; whatever it be, of that condensation or rarefaction is removed.

81. WIND is a stream or current of air which may be felt; and usually blows from one part of the horizon to its opposite part.

82. The *horizon*, besides being divided into 360 degrees, like all other circles, is by mariners supposed to be divided into four *quadrants*, called the north-east, north-west, south-east, and south-west quarters; each of these quarters they divide into eight equal parts, called points, and each point into four equal parts, called quarter points.

So that the horizon is divided into 32 points, which are called *rhumbs*, or *winds*; to each wind is assigned a name, which shews from what point of the horizon the wind blows.

The points of North, South, East, and West, are called *cardinal points*; and are at the distance of 90 degrees, or 8 points, from one another.

* Near 14000 times. *Wallis's Hydrof.* p. 13.

† Into $\frac{1}{16}$ part. *Phil. Trans.* N^o 181.

83. Winds are either constant or variable, general or particular.

Constant winds are such as blow the same way, at least, for one or more days; and *variable winds* are such as frequently shift within a day.

A *general wind* is that which blows the same way over a large tract of the Earth, almost the whole year.

A *particular wind* is that which blows, in any place, sometimes one way, and sometimes another, indifferently.

If the wind blows gently, it is called a breeze: if it blows harder, it is called a gale, or a stiff gale; and if it blows very hard, it is called a storm*.

84. The following observations on the wind have been made by skilful seamen; and particularly by the great Dr. Halley.

1st. Between the limits of 60 degrees, namely, from 30° of north latitude to 30° of south latitude, there is a constant east wind throughout the year, blowing on the Atlantic and Pacific Oceans; and this is called the *trade wind*.

FOR as the Sun, in moving from east to west, heats the air more immediately under him, and thereby expands it; the air to the eastward is constantly rushing towards the west to restore the equilibrium, or natural state of the atmosphere; and this occasions a perpetual east wind in those limits.

2d. The trade winds near their northern limits blow between the north and east; and near the southern limits they blow between the south and east.

FOR as the air is expanded by the heat of the Sun near the equator; therefore the air from the northward and southward will both tend towards the equator to restore the equilibrium: Now these motions from the north and south, joined with the foregoing easterly motion, will produce the motions observed near the said limits between the north and east, and between the south and west.

3d. These general motions of the wind are disturbed on the continents, and near their coasts.

FOR the nature of the soil may either cause the air to be heated or cooled; and hence will arise motions that may be contrary to the foregoing general one.

4th. In some parts of the Indian ocean there are periodical winds, which are called *MONSOONS*; that is, such as blow half the year one way, and the other half year the contrary way.

FOR air that is cool and dense, will force the warm and rarefied air in a continual stream upwards, where it must spread itself to preserve the equilibrium: So that the upper course or current of the air shall be contrary to the under current; for the upper air must move from those parts where the greatest heat is; and so, by a kind of circulation, the N. E. trade wind below will be attended with a S. W. above; and a S. E. below with a N. W. above: And this is confirmed by the experience of seamen, who, as soon as they get out of the trade winds, immediately find a wind blowing from the opposite quarter.

* The swiftness of the wind in a storm is not more than 50 or 60 miles in an hour; and a common brisk gale is about 15 miles an hour.

5th. In the Atlantic Ocean, near the coasts of Africa, at about 100 leagues from shore, between the latitudes of 28° and 10° north, seamen constantly meet with a fresh gale of wind blowing from the N. E.

6th. Those bound to the Caribbee Islands, across the Atlantic Ocean, find, as they approach the American side, that the said N. E. wind becomes easterly; or seldom blows more than a point from the east, either to the northward or southward.

These trade winds, on the American side, are extended to 30° , 31° , or even to 32° of N. latitude; which is about 4° farther than what they extend to on the African side: Also to the southward of the equator, the trade winds extend 3 or 4 degrees farther towards the coast of Brasil on the American side, than they do near the Cape of Good Hope on the African side.

7th. Between the latitudes of 4° North and 4° South, the wind always blows between the south and east: On the African side the winds are nearest the south; and on the American side nearest the east. In these seas Dr. Halley observed, that when the wind was eastward, the weather was gloomy, dark, and rainy, with hard gales of wind: But when the wind veered to the southward, the weather generally became serene, with gentle breezes next to a calm.

These winds are somewhat changed by the seasons of the year; for when the Sun is far northward, the Brasil S. E. wind gets to the south, and the N. E. wind to the east; and when the Sun is far south, the S. E. wind gets to the east, and the N. E. winds on this side of the equator veer more to the north.

8th. Along the coast of Guinea, from Sierra Leone to the Island of St. Thomas (under the equator), which is above 500 leagues, the southerly and south-west winds blow perpetually: For the S. E. trade wind having passed the equator, and approaching the Guinea coast within 80 or 100 leagues, inclines towards the shore, and becomes south, then S. E. and by degrees, as it comes near the land, it veers about to south, S. S. W. and in with the land it is S. W. and sometimes W. S. W. This tract is troubled with frequent calms, violent sudden gusts of wind, called Tornadoes, blowing from all points of the horizon.

The reason of the wind setting in west on the coast of Guinea, is in all probability owing to the nature of the coast, which being greatly heated by the Sun, rarefies the air exceedingly, and consequently the cool air from off the sea will keep rushing in to restore the equilibrium.

9th. Between the 4th and 10th degrees of north latitude, and between the longitudes of Cape Verd, and the easternmost of the Cape Verd Isles, there is a tract of sea which seems to be condemned to perpetual calms, attended with terrible thunder and lightnings, and such frequent rains, that this part of the sea is called the *rains*. Ships in sailing these 6 degrees have been sometimes detained whole months, as it is reported.

The cause of this seems to be, that the westerly winds setting in on this coast, and meeting the general easterly wind in this tract, balance each other, and so cause the calms; and the vapours carried thither by each wind meeting and condensing, occasion the almost constant rains.

The

The last three observations shew the reason of two things which mariners experience in sailing from Europe to India, and in the Guinea trade.

And first. The difficulty which ships in going to the southward, especially in the months of July and August, find in passing between the coast of Guinea and Brasil, notwithstanding the width of this sea is more than 500 leagues. This happens, because the S. E. winds at that time of the year commonly extend some degrees beyond the ordinary limits of 4° N. latitude; and besides coming so much southerly, as to be sometimes south, sometimes a point or two to the west; it then only remains to ply to windward: And if, on the one side, they steer W. S. W. they get a wind more and more easterly; but then there is a danger of falling in with the Brasilian coast, or shoals; and if they steer E. S. E. they fall into the neighbourhood of the coast of Guinea, from whence they cannot depart without running easterly as far as the island of St. Thomas; and this is the constant practice of all the Guinea ships.

Secondly. All ships departing from Guinea for Europe, their direct course is northward; but on this course they cannot go, because the coast bending nearly east and west, the land is to the northward: Therefore as the winds on this coast are generally between the S. and W. S. W. they are obliged to steer S. S. E. or south, and with these courses they run off the shore; but in so doing they always find the winds more and more contrary; so that when near the shore, they can lie south, at a greater distance they can make no better than S. E., and afterwards E. S. E.; with which courses they commonly fetch the Island of St. Thomas and Cape Lopez, where finding the winds to the eastward of the south, they sail westerly with it, till coming to the latitude of 4 degrees south, where they find the S. E. wind blowing perpetually.

On account of these general winds, all those that use the West-India trade, even those bound to Virginia, reckon it their best course to get as soon as they can to the southward, that so they may be certain of a fair and fresh gale to run before it to the westward: And for the same reason those homewards-bound from America endeavour to gain the latitude of 30 degrees, where they first find the winds begin to be variable; though the most ordinary winds in the north Atlantic Ocean come from between the south and west.

10th. Between the southern latitudes of 10 and 30 degrees in the Indian Ocean, the general trade wind about the S. E. by S. is found to blow all the year long in the same manner as in the like latitudes in the Ethiopic Ocean: and during the six months from May to December, these winds reach to within 2 degrees of the equator; but during the other six months, from November to June, a N. W. wind blows in the tract lying between the 3d and 10th degrees of southern latitude, in the meridian of the north end of Madagascar: and between the 2d and 12th degree of south latitude, near the longitude of Sumatra and Java.

11th. In

11th. In the tract between Sumatra and the African coast, and from 3 degrees of south latitude quite northward to the Asiatic coasts, including the Arabian Sea and the Gulf of Bengal, the Monsoons blow from September to April on the N. E.; and from March to October on the S. W. In the former half year the wind is more steady and gentle, and the weather clearer than in the latter six months: And the wind is more strong and steady in the Arabian Sea than in the Gulf of Bengal.

12. Between the Island of Madagascar and the coast of Africa, and thence northward as far as the equator, there is a tract, wherein from April to October there is a constant fresh S. S. W. wind; which to the northward changes into the W. S. W. wind, blowing at that time in the Arabian Sea.

13th. To the eastward of Sumatra and Malacca on the north of the equator, and along the coasts of Cambodia and China, quite through the Philippines as far as Japan, the Monsoons blow northerly and southerly; the northern setting in about October or November, and the southern about May: The winds are not quite so certain as those in the Arabian Seas.

14th. Between Sumatra and Java to the west, and New Guinea to the east, the same northerly and southerly winds are observed; but the first half year Monsoon inclines to the N. W. and the latter to the S. E. These winds begin a month or six weeks after those in the Chinese Seas set in, and are quite as variable.

15th. These contrary winds do not shift from one point to its opposite all at once; in some places the time of the change is attended with calms, in others by variable winds: And it often happens on the shores of Coromandel and China, towards the end of the Monsoons, that there are most violent storms, greatly resembling the hurricanes in the West Indies; wherein the wind is so vastly strong, that hardly any thing can resist its force.

All navigation in the Indian Ocean must necessarily be regulated by these winds; for if mariners should delay their voyages till the contrary Monsoon begins, they must either sail back, or go into harbour, and wait for the return of the trade wind.

SECTION

SECTION VII.

Of the Tides.

85. A TIDE is that motion of the water in the seas and rivers, by which they are found regularly to rise and fall.

The general cause of the tides was discovered by Sir Isaac Newton, and are deduced from the following considerations.

86. Daily experience shews, that all bodies thrown upwards from the Earth, fall down to its surface in perpendicular lines; and as lines perpendicular to the surface of a sphere tend towards the center, therefore the lines, along which all heavy bodies fall, are directed towards the Earth's center.

As these bodies apparently fall by their weight, or gravity; therefore the law by which they fall, is called the LAW OF GRAVITATION.

87. A piece of glass, amber, or sealing wax, and some other things, being rubbed against the palm of a hand, or against a woollen cloth, until it be warmed, will draw bits of paper, or other light substances towards it, when held sufficiently near those substances.

Also a magnet, or loadstone, being held near the filings of iron or steel, or other small pieces of these substances, will draw them to itself; and a piece of hammered iron or steel, that has been rubbed by a magnet, will have a like property of drawing iron or steel to itself.

And this property in some bodies of drawing others to themselves in the foresaid manner, is called ATTRACTION.

88. Now as bodies by their gravity fall towards the Earth, it is not improper to say the Earth attracts those bodies; and therefore in respect to the Earth, the words gravitation and attraction may be used one for the other, as by them is meant no more than the power, or law, by which bodies tend towards its center.

And it is likely that this is the cause why the parts of the Earth adhere and keep close to one another.

89. The incomparable Sir Isaac Newton, by a sagacity peculiar to himself, discovered from many observations that this law of gravitation or attraction was universally diffused throughout the solar system; and that the regular motions observed among the heavenly bodies were governed by this same principle; so that the Earth and Moon attracted each other, and both of them were attracted by the Sun: Also that the force of attraction exerted by these bodies one on the other was less and less as the distance increased, in proportion to the squares of those distances; that is, the power of attraction at double the distance was four times less, at triple the distance, nine times less, at quadruple the distance, sixteen times less, and so on.

90. Now as the Earth is attracted by the Sun and Moon, therefore all the parts of the Earth will not gravitate towards its center in the same manner as if those parts were not affected by such attractions: And it is very evident that, was the Earth intirely free from such actions of the Sun and Moon, the oceans being equally attracted towards its center, on all sides, by the force of gravity, would continue in a perfect stagnation without ever ebbing or flowing: But since the case is other-

otherwise, the oceans must needs rise higher in those places where the Sun and Moon diminish their gravity, or where the Sun and Moon have the greatest attraction.

As the force of gravity must be diminished most in those parts of the Earth to which the Moon is nearest, or in the ZENITH; that is, where she is vertical or perpendicularly over, and consequently where her attraction is most powerful; therefore the waters in such places will rise higher, and it will be *full sea or flood* in such places.

91. *The parts of the Earth directly under the Moon, and also those in their NADIR, viz. such places diametrically opposite to those where the Moon is in the Zenith, will have the flood, or high water, at the same time.*

For either half the Earth would gravitate equally towards the other half, were they free from all external attraction.

But by the action of the Moon, the gravitation of one half-earth towards its center is diminished, and of the other is increased.

Now in the half-earth next the Moon, the parts in the zenith being most attracted, and thereby their gravitation towards the Earth's center diminished; therefore the waters in these parts must be higher than in any other part of this half-earth.

And in the half-earth farthest from the Moon, the parts in the nadir being less attracted by the Moon than the parts nearer to her, do therefore gravitate less towards the Earth's center, and consequently the waters in these parts must be higher than they are in any other part of this half-earth.

92. *Those parts of the Earth, where the Moon appears in the horizon, or is 50 degrees distant from the zenith and nadir, will have the ebbs or lowest waters.*

For as the waters in the zenith and nadir rise at the same time, the waters in their neighbourhood will press towards those places to maintain the equilibrium; and to supply the places of these, others will move the same way, and so on to places 90° distant from the said zenith and nadir: Consequently in those places where the Moon appears in the horizon, the waters will have more liberty to descend towards the center; and therefore in those places they will be the lowest.

93. Hence, it plainly follows, that the ocean, did it cover the surface of the Earth, must put on a spheroidal, or egg-like figure; wherein the longest diameter passes through the place where the Moon is vertical; and the shortest diameter will be where she is in the horizon. And as the Moon apparently shifts her position from east to west in going round the Earth every day, the longer diameter of the spheroid following her motion, will occasion the two floods and ebbs observable in about every 25 hours, which is about the length of a lunar day; that is, the time spent between the Moon's leaving the meridian of any place, and coming to it again.

94. Hence, in any place, the greater the Moon's mid-day altitude, the greater the evening-tides will be; and the greater the mid-night depression, the greater the morning-tides will be.

Also the summer evening, and the winter morning-tides are highest: For the Moon's summer altitude, and winter depression, are greatest; especially

especially when her declination is north in summer and south in winter.

95. *The time of high water is not precisely at the time of the Moon's coming to the meridian, but about an hour or two after.*

For the Moon acts with some force after she has past the meridian, and thereby adds to the libratory, or waving motion, which she has put the water into, whilst she was in the meridian; in the same manner as a small force applied upwards to a ball already raised to some height, will raise it still higher.

96. *The tides are greater than ordinary twice every month; that is, about the times of new and full Moon: These are called SPRING-TIDES.*

FOR at these times the actions of both Sun and Moon concur to draw in the same right line; and therefore the sea must be more elevated: In conjunction, or when the Sun and Moon are on the same side of the Earth; they both conspire to raise the water in the zenith, and consequently in the nadir: And when the Sun and Moon are in opposition, that is, when the Earth is between them, whilst one makes high-water in the zenith and nadir, the other does the same in the nadir and zenith.

97. *The tides are less than ordinary twice every month; that is, about the times of first and last quarters of the Moon: And these are called NEAP TIDES.*

BECAUSE in the quarters of the Moon, the Sun raises the water where the Moon depresses it; and depresses where the Moon raises the water; so that the tides are made only by the difference of their actions.

It must be observed, that the spring-tides happen not directly on the new and full Moons, but rather a day or two after, when the attractions of the Sun and Moon have conspired together for a considerable time. In like manner the neap-tides happen a day or two after the quarters, when the Moon's attraction has been lessened by the Sun for several days together.

98. *When the Moon is in her PERIGÆUM, or nearest approach to the Earth, the tides increase more than in the same circumstances at other times.*

FOR according to the laws of gravitation, the Moon must attract most when she is nearest to the Earth.

99. *The spring-tides are greatest about the time of the EQUINOXES, that is, about the latter ends of March and September, than at other times of the year; and the neap-tides then are less.*

BECAUSE the longer diameter of the spheroid, or the two opposite floods, will at that time be in the Earth's equator; and consequently will describe a great circle of the Earth; by whose diurnal rotation those floods will move swifter, describing a great circle in the same time they used to describe a lesser circle parallel to the equator, and consequently the waters being thrown more forcibly against the shores, they must rise higher.

100. The following observations have been made on the rise of the tides.

1st. The morning-tides generally differ in their rise from the evening-tides.

2d. The new and full Moon spring-tides rise to different heights.

3d. In winter, the morning-tides are highest.

4th. In

4th. In summer, the evening-tides are highest.

So that after a period of about six months, the order of the tides are inverted; that is, the rise of the morning and evening-tides will change places, the winter morning high-tides becoming the summer evening high-tides.

Some of these effects arise from the different distances of the Moon from the Earth, after a period of six months, when she is in the same situation with respect to the Sun; for, if she is in perigee at the time of new moon, in about six months after she will be in perigee about the time of full Moon.

These particulars being known, a pilot may chuse that most convenient, when he would conduct a ship in or out of a port, where there is not sufficient depth at low-water.

Small inland seas, such as the Mediterranean and Baltic, are little subject to tides; because the action of the Sun and Moon is always nearly equal at both extremities of such seas. In very high latitudes the tides are also very inconsiderable: For the Sun and Moon acting towards the equator, and always raising the water towards the middle of the torrid zone, the neighbourhood of the poles must consequently be deprived of those waters, and the sea must, within the frigid zones, be low, relative to other parts.

101. All the things hitherto explained would exactly obtain, was the whole surface of the Earth covered with Sea: But since it is not so, and there being a multitude of islands, beside continents, lying in the way of the tide, which interrupt its course; therefore in many places near the shores, there arise a great variety of other appearances beside the foregoing ones, which require particular solutions, wherein the situation of the shores, straits, shoals, winds, and other things, must necessarily be considered. For instance:

102. As the sea * has no visible passage between Europe and Africa, let them be supposed one continent, extending from 72 degrees north to 34 degrees south, the middle between these two would be in latitude 19 degrees north, near Cape Blanco on the west coast of Africa. But it is impossible that the flood-tide should set to the westward upon the west coast of Africa (for the general tide following the course of the Moon, must set from east to west), because the continent for above 50 degrees, both northward and southward, bounds that sea on the east; and therefore if any regular tide, as proceeding from the motion of the sea, from east to west, should reach this place, it must either come from the north of Europe southward, or from the south of Africa northward, to the said latitude upon the west coast of Africa.

103. This opinion is further corroborated, or rather fully confirmed, by common experience, that the flood-tide sets to the southward along the west coast of *Norway*, from the north cape to the *Naze*, or entrance of the *Baltic Sea*, and so proceeds to the southward along the east coast of *Great Britain*, and in its passage supplies all those ports with the tide one after another, the coast of Scotland having the tide first, because it

proceeds from the northward to the southward; and thus on the days of the full, or change, it is high-water at *Aberdeen* at 12 h. 45 m. but at *Tinmouth Bar*, the same day, not till 3 h. From thence rolling to the southward, it makes high-water at the *Spurn* a little after 5 h. but not till 6 h. at *Hull*, by reason of the time required for its passage up the river; from thence passing over the *Wellbank* into *Yarmouth Road*, it makes high-water there a little after 8 h. but in the *Pier* not till 9 h., and it requires near an hour more to make high-water at *Yarmouth* town; in the mean time setting away to the southward, it makes high-water at *Harwich* at 10 h. 30 m., at the *Nore* at 12, at *Gravesend* 1 h. 30 m., and at *London* at 3 h. all in the same day: And although this would seem to contradict that hypothesis, of the natural motion of the tide being from east to west, yet as no tide can flow west from the main continent of *Norway* or *Holland*, or out of the *Baltic*, which is surrounded by the main continent except at its entrance, it is evident that the tide we have been now tracing by its several stages from *Scotland* to *London*, is supplied by the tide whose original motion is from east to west; yet as water always inclines to the level, it will in its passage fall towards any other point of the compass, to fill up vacancies where it finds them, and yet not contradict, but rather confirm, the first hypothesis.

104. While the tide, or high water, is thus gliding to the southward along the east coast of *England*, it also sets to the southward along the west coasts of *Scotland* and *Ireland*, a branch of it falls into St George's channel, the flood running up north-east, as may be naturally inferred from its being high-water at *Waterford* above three hours before it is high-water at *Dublin*, or thereabout on that coast; and it is three quarters of an hour ebb at *Dublin* before it is high-water at the *Isle of Man*, &c.

But not to proceed farther in particulars than our own, or the *British* channel, we find the tides set to the southward from the coast of *Ireland*, and in its passage a branch of it falls into the *British* channel between the *Lizard* and *Ushant*; its progress to the southward may be easily proved, by its being high-water on the day of the full and changes at *Cape Clear* after 4 h., and at *Ushant* about 6 h., and at the *Lizard* after 7 h. The *Lizard* and *Ushant* may properly be called the chaps of the *British* channel, between which the flood sets to the eastward along the coasts of *England* and *France*, till it comes to the *Goodwin* or *Galliper*, where it meets the tide before mentioned, which sets to the southward, along the coast of *England* to the *Thames*, where these two tides meeting, contribute very much towards sending a powerful tide up the *Thames* to *London*: And by the bye, when the natural course of the tide is interrupted by a sudden shift of the wind; and between this northern and southern tide, driving one back, and the other in, has been known to occasion twice high-water in 3 or 4 hours, which, by those who did not consider this natural cause, was looked upon as a prodigy.

105. But now it may be objected, that this course of the flood-tide, east, or east-north-east, up the channel, is quite contrary to the hypothesis of the general motion of the tides being from east to west, and consequently of its being high-water where the moon is vertical, or any where else in the meridian.

In

In answer. This particular direction of any branch of the tide doth not at all contradict the general direction of the whole; a river, whose course is west, may supply canals which may wind north, south, or even east, and yet the river keep its natural course; and if the river ebb and flow, the canals supplied by it would do the same, but not keep exact time with the river, because it would be flood, and the river advanced to some height, before the flood reached the farther part of the canals; and the more remote, the longer time it would require; and it may be added, that if it was high-water in the river just when the moon was on the meridian, she would be far past it, before it could be high-water in the remotest part of those canals, or ditches, and the flood would set according to the course of those canals that received it, and could not set west up a canal of a different position; and as *St. George's* channel, the *British* channel, &c. are no more in proportion to the vast ocean, than these canals are to a large navigable river; it will evidently follow; that among those obstructions and confinements, the flood may set upon any other point of the compass as well as west, and may make high-water at any other time as well as when the Moon is upon the meridian, and yet no way contradict the general theory of the tide before asserted.

106. When the time of high-water at any place is, in general, mentioned, it is to be understood on the days of the syzygies, or days of new and full Moon; when the Sun and Moon pass the meridian of that place at the same time: Among pilots it is customary to reckon the time of flood, or high-water, by the point of the compass the Moon bears on, allowing $\frac{1}{4}$ of an hour for each point, at that time: Thus, on the full and change days, in places where it is flood at noon, the tide is said to flow north and south, or at 12 o'clock; in other places on the same days, where the Moon bears 1, 2, 3, 4, or more points to the east or west of the meridian, when it is high-water, the tide is said to flow on such point: Thus, if the Moon bears S. E. at flood, it is said to flow S. E. and N. W.; or 3 hours before the meridian, that is, at 9 o'clock; if it bears S. W., it flows S. W. and N. E., or at 3 hours after the meridian; and in like manner for other points of the Moon's bearing.

107. In some places it is high water on the shore, or by the ground, while the tide continues to flow in the stream, or offing; and according to the length of time it flows longer in the stream than on the shore, it is said to flow tide, and such part of tide; allowing 6 hours to a tide; Thus 3 hours longer in the offing than on the shore, makes tide and half-tide; an hour and half longer, makes tide and quarter-tide; three quarters of an hour longer, makes tide and half-quarter tide; &c.

108. Along an extent of coast next to the ocean, such as the western coast of parts of Africa, and the eastern and western coasts of parts of South America, it is generally high-water about the same hour: But ports on the coasts of narrow seas, or within land, have the times of their high-water sooner or later on the same day, according as those ports are farther removed from the tide's way, or have their entrances more or less contracted.

109. The times of high-water, in any place, fall about the same hours after a period of about 15 days, or between one spring-tide and another

another: But during that period, the times of high-water fall each day later by about 48 minutes.

110. From the observations of many persons, there have been collected the times when it is high-water on the days of the new and full Moon, on most of the sea coasts of Europe, and many other places: These times are usually put in a table against the names of the places digested in an alphabetical order; the like is followed in this work, only they are not given in a table by themselves, but make a column in the table of the latitudes and longitudes of places; which column is not filled up against many of the names in that table, for want of a sufficient collection of observations: This may be supplied by those who have opportunity and inclination.

The use of such a table is to find the time when it is high-water at any of the places mentioned therein: But as this depends upon knowing the time when the Moon comes to the meridian, and this on the Moon's age, and this on the knowledge how to find some of the common notes in the Calendar; therefore it was thought convenient to introduce in this place a compendium of Chronology, wherein will be contained the several articles above mentioned.

SECTION VIII.

Of Chronology.

III. CHRONOLOGY is the art of estimating, and comparing together, the times when remarkable events have happened, such as are related in history.

An *ÆRA*, or *EPOCH*A, is a time when some memorable transaction occurred; and from whence some nations date, and measure their computations of time.

Some have dated their events from the creation }
of the world.

Others from the deluge, or flood.

The Greeks from their Olympiads of 4 years }
each.

The Romans from the building of Rome.

The Astronomers from Nabonassar King of }
Babylon.

Some Historians from the death of Alexander }
the Great.

The Christians from the birth of Christ.

The Mahometans from the flight of Mahomet, }
and called the Hegira,

Years of the World.	Years before Christ.
0000	4004
1656	2348
3228	776
3251	753
3257	747
3676	328
4004	A. D.
4626	622

In order to assign the distance between these, and other events, the ancients found it necessary to have a large measure of time, the limits of which was naturally pointed out to them by the return of the seasons, and this interval they called a year.

112. The most natural division of the year appeared to them to be the returns of the new Moon; and as they observed 12 new Moons to happen within the time of the general return of the seasons, they therefore first divided the year into 12 equal parts, which they called months; and as they reckoned about 30 returns of morning and evening between the times of the new moon and new moon, therefore they reckoned their month to consist of 30 days, and their year, or 12 months, to contain 360 days; and this is what is generally understood by the lunar year of the ancients.

113. But in length of time it was found, that this year did not agree with the course of the Sun, the seasons gradually falling later in the year than they had been formerly observed; this put them upon correcting the method of estimating their year, which they did from time to time, by taking a day or two from the month as often as they found it too long for the course of the Moon; and by adding a month, called an intercalary month, as often as they found 12 lunar months to be too short for the return of the four seasons and fruits of the Earth: This kind of year, so corrected from time to time by the priests, whose business it was, is what is to be understood by the *luni-solar* year; which was anciently used in most nations, and is still, among the *Arabs* and *Turks*.

As a great variety of methods were used in different countries to correct the length of the year, some by intercalating days in every year, and others by inserting months and days in certain returns, or periods of years; and these different methods being observed by some eminent men to create a considerable difference in the accounts of time kept by neighbouring nations, thereby introducing a confusion in the chronological order of times, they therefore invented certain periods of years, called *cycles*, with which they compared the most memorable occurrences.

114. At length *Julius Cæsar* observing the confusion which this variety of accounts occasioned; and knowing that his order, as Emperor of the Romans, would be followed by a very considerable part of the world: He therefore, about 40 years before the birth of Christ, decreed that every fourth year should consist of 366 days, and the other three of 365 days each. This he did in consequence of the information given him by *Sylogenes*, an eminent mathematician of Alexandria in Egypt; for at that time the philosophers of the Alexandrian school knew, from a length of experience, that the year consisted of about 365 days and a quarter; and this was the reason of ordering every fourth year to consist of 366 days, thereby compensating for the quarter day omitted in each of the preceding three years. This method, called the *Julian Account*, or *Old Style*, continued to be used in most Christian states until the year 1582.

The Astronomers, since the time of *Julius Cæsar*, have found that the true length of the solar year, or common year, is 365 days, 5 hours, 48 minutes, 55 seconds, nearly, being less than the Julian of 365 days, 6 hours, by about 11 minutes, 5 seconds, which is about the 130th

part of 86400, the seconds contained in a day; so that in 130 Julian years, there would be one day gained above 130 solar years; and in 400 Julian years, there would be gained 3 days, 1 hour, 53 minutes, 20 seconds; consequently one day omitted in every 130 common years, would bring the current account of time to agree very nearly with the motion of the Sun.

115. In the year of our Lord 325, when the *Council of Nice* settled the day for the celebration of Easter, the *Vernal Equinox* (that is, the day in the spring when the Sun rose at six and set at six) happened on the 21st of March: But about the year 1580 the *Vernal Equinox* fell on the 11th of March, making a difference of about 10 days. Now *Gregory* the XIIIth, who was Pope at that time, observing that this difference of time in the falling out of the *Equinox* would affect the intention of the *Nicene Council* concerning the time of the year appointed by them for the celebration of Easter: He therefore, in the year 1581, published a *Bull*, ordering that in the year 1582 the 5th of October should be called the 15th, and so on; whereby the 10 days taken off would cause the time of the *Vernal Equinox* to fall on the 21st of March, as at the time of the *Nicene Council*: and because a little more than three days were gained in every 400 years, by the *Julian* account; therefore, to prevent any future difference, every century, or number of 100 years, not divisible by 4, such as 17 hundred, 18 hundred, 19 hundred, &c.; should contain only 365 days, which, by the *Julian* account, should have contained 366; and the centuries divisible by 4, such as 16 hundred, 20 hundred, 24 hundred, &c. should be leap-years of 366 days: whereby the three days would be omitted, which the anticipation of the equinoxes would gain in 400 years; the small excess of 1 hour, 53 minutes, 20 seconds, not amounting to a whole day in less than 5082 years, being rejected as inconsiderable; the intermediate years to be reckoned as they used to be in the *Julian* or *Old Stile*. This Pope's alteration, called the *Gregorian* or *New Stile*, was received in most of the Christian states: But some at that time, chose to continue the *Julian*; among whom were the English; and they in the year 1752 reformed their account, and introduced among themselves a new one, that nearly corresponds with the *Gregorian*.

A certain length of the year being once settled, and a regular account of time in consequence thereof being so fitted, as to conform invariably to the seasons; it was natural for the States who received such account, to fit thereto a register of the days in each month, and therein to note the days when any remarkable occurrence was to be commemorated; and such register has obtained the name of *Calendar*.

116. The Calendar now in use among most of the Christian states consists of 12 months, called *January, February, March, April, May, June, July, August, September, October, November, December*; these months are called *civil*; the number of days in each may be readily remembered by the following rule.

117. Thirty days has *November, April, June, and September*;

February has twenty-eight alone, All the rest have thirty-one, When the year consists of 365 days: But in every fourth, which consists of 366 days, *February* has 29. This additional day was intercalated

lated after the 24th of February, which in the Old Roman Calendars was called the *sixth of the calends of March*, and being this year reckoned twice over, the year was called *Bissextile*, or LEAP-YEAR.

Beside the months, time is also divided into weeks, days, hours, minutes, &c. a year containing 52 weeks, a week 7 days, a day 24 hours, an hour 60 minutes, &c.

In the Calendars it has been usual to mark the seven days of the week with the seven first letters of the alphabet; always calling the first of January A, the 2d B, the 3d C, the 4th D, the 5th E, the 6th F, and the 7th G, and so on, throughout the year; and that letter answering to all the Sundays for a year, is called the DOMINICAL LETTER.

According to this disposition, the letters answering to the first day of every month in the year will be known by the following rule.

118. At Dover Dwell's George Brown, Esquire,
Good Christopher Finch, And David Frier.

Where the first letter of each word answers to the letter belonging to the first day of the months in the order from January to December.

119. A year of 365 days contains 52 weeks and 1 day; and a leap-year has 52 weeks and 2 days; therefore the first and last days of a common year fall on the same week day, suppose it *Monday*; then the next year begins on a *Tuesday*, the next year on *Wednesday*, and so on to the eighth year, which would be on *Monday* again, did every year contain 365 days; also the Dominical letter would run backwards through all the seven letters: But this round of seven years is interrupted by the leap-years; for herein *February* having a 29th day annexed, the first Dominical letter in *March* must fall a day sooner than in the common year; so that leap-year has two Dominical letters, the one (supposing G) serving for *January* and *February*, and the other, which is the preceding letter (F) serves for the rest of the Sundays in that year.

120. The SOLAR CYCLE, or cycle of the Sun, is a period of 28 years, in which all the varieties of the Dominical letters will have happened, and the same order of the letters will return as were 28 years before. At the birth of Christ 9 years had past in this cycle.

For the changes, were all the years common ones, would be 7,

But the interruptions by leap-year being every fourth year;

Therefore the changes will be 4 times 7, or 28 years.

This return of the Dominical letters is constant in the *Julian* account: But in the *Gregorian*, where among the complete centuries, or hundredth years, only every fourth is leap-year, the other three hundredth years, which according to the *Julian*, would be leap-years, are by the *Gregorians* only common years of 365 days: in these all the letters must be removed one place forwards in a direct order; and either year, instead of having two Dominical letters (as suppose D, C) will have only one (as D), the Dominical letters moving retrograde.

121. The LUNAR CYCLE, or cycle of the Moon (and sometimes called the *Metonic Cycle*, from *Meton*, an Athenian, who invented it about 432 years before the time of *Christ*), is a period of nineteen years, containing all the variations of the days on which the new and full Moons happen; after which, they fall on the same days they did 19 years before.

The PRIME, or GOLDEN NUMBER, is the number of years elapsed in this cycle.

At the birth of *Christ*, the golden number was 2.

For many years after the *Nicene Council*, it was thought that 19 solar years, or 228 solar months, were exactly equal to 235 synodical, or lunar, months; and that the same yearly *golden numbers* set in their calendars against the days when the new Moons happened throughout one lunar cycle, would invariably serve for the new Moons of corresponding years throughout every successive lunar cycle. But later observations shew, that this cycle is less than 19 years by a little more than one hour, twenty-eight minutes, whereby the new Moons will, in a little less than 311 years, happen a day earlier than by the Metonic account; and consequently all those festivals depending on the new Moons, will in time be removed into different seasons of the year than at their first institution: thus the new Moons in the year 1750 happened above $4\frac{1}{2}$ days earlier than the times shewn by the calendar. But were the golden numbers, when once prefixed to the proper new Moon days in a Metonic period, to be set a day earlier at the end of every 310,7 years, a pretty regular correspondence might be preserved between the solar and lunar years.

122. The EPACT of any year, is the Moon's age at the beginning of that year; that is, the days past since the last new Moon.

The time between new Moon and new Moon is in the nearest round numbers $29\frac{1}{2}$ days; therefore the lunar year consisting of 12 lunations, must be equal to 354 days, which is 11 days less than the solar year of 365 days: Now supposing the solar and lunar years to begin together, the epact is 0, the beginning of the next solar year, the epact is 11; the 3d year the epact is 22, the 4th 33, &c. But when the epact exceeds 30, then an intercalary month of 30 days is added to the lunar year, making it consist of 13 months; so that the epact at the beginning of the 4th year is only 3, the 5th 14, the 6th 25, the 7th 36, or only 6, on account of the intercalary month; and so on to the end of the cycle of 19 years; at the expiration whereof, the same epacts would run over again, was the cycle perfect; and the epact would always be 11 times the prime.

123. By the Nicene Council it was enacted.

1st. That Easter-day should be celebrated after the vernal equinox, which at that time happened on the 21st of March.

2d. That it should be kept after the full, or 14th day, of that Moon which happened nearest to the 21st of March in common years, and nearest the 20th of March in leap-years.

3d. That the Sunday next following the 14th, or day of full Moon, should be Easter-Sunday: which must always fall between the 20th or 21st of March, and the 25th of April.

124. The MOON'S SOUTHRING at any place, is the time when she comes to the meridian of that place, which is every day later by about $\frac{1}{4}$ of an hour, or 48 minutes; occasioned by the hours in a day being divided by the 30 times she passes the meridian from new Moon to new Moon.

The

The Sun and Moon come to the meridian at the same time on the day of new Moon, or change; also the Moon comes to the opposite part of the same meridian, when in opposition, or at full Moon: Hence between new and full, she comes to the meridian in the afternoon; at full she souths at midnight, and after full at past midnight, or in the morning; likewise when at change, she rises nearly with the Sun, at full she rises about the time the Sun sets.

125. The ROMAN INDICTION is a cycle of 15 years, used by the ancient Romans for the times of taxing the provinces. Three years of this cycle were elapsed at the birth of Christ.

The DIONYSIAN PERIOD is a cycle of 532 years, arising by multiplying together 28 and 19, the solar and lunar cycles, and was contrived by *Dionysius Exiguus*, a Roman abbot, about the year of Christ 527, as a period for comparing of chronological events.

The JULIAN PERIOD contains 7980 years; it arises by multiplying together 28, 19, 15, the cycles of the Sun, Moon, and Indiction. This was also contrived as a period for chronological matters; and its beginning is put 710 years before the usual date of the creation.

From the principles laid down in the preceding articles, depend the solution of the following problems.

126. PROBLEM I. To find whether any given year be leap-year.

RULE. Divide the given year by 4, if 0 remains, it is leap-year; if 1, 2, or 3 remains, it is so many years after.

Observing that the years 1800, 1900, 2100, &c. are common years.

EXAM. I. Is 1772 leap-year?

$$\begin{array}{r} 4 \overline{) 1772} (443 \end{array}$$

Remains 0, so it is leap-year.

EXAM. II. Is 1775 leap-year?

$$\begin{array}{r} 4 \overline{) 1775} (443 \end{array}$$

Remains 3 years past leap-year.

127. PROBLEM II. To find the years of the solar, lunar, and indiction cycles.

RULE. To the given years add 9 for the solar, 1 for the lunar, 3 for the indiction: Divide the sums in order by 28, 19, 15; the remainder in each shews the year of its respective cycle.

EXAM. Required the years of the solar, lunar, and indiction cycles for the year 1773?

$$\begin{array}{r} 1773 \\ 9 \overline{) 1782} (63 \\ 28 \overline{) 1782} (63 \end{array} \qquad \begin{array}{r} 1773 \\ 1 \overline{) 1774} (93 \\ 19 \overline{) 1774} (93 \end{array} \qquad \begin{array}{r} 1773 \\ 3 \overline{) 1776} (118 \\ 15 \overline{) 1776} (118 \end{array}$$

Remains 18=solar cycle. 7=lunar cyc. or golden N°. 6=indict. cycle. Whereby it appears { 18th year of the 64th solar cycle } since the birth of Christ
that the year 1773 { 7th year of the 94th lunar cycle }
is the 6th year of the 119th indiction cycle

128. PROBLEM III. *To find the Dominical letter till the year 1800.*

RULE. To the given year add its fourth part, divide the sum by 7; the remainder taken from 7 leaves the index of the letter in common years, reckoning A 1, B 2, C 3, &c.

But in leap-year, this letter and its preceding one (in the retrograde order which these letters take), are the Dominical letters.

EXAM. I. *For the year 1773.*

$$\begin{array}{r} 4)1773 \\ 443 \\ \hline 7)2216(316 \end{array}$$

4

Remains 4. Then $7-4=3=e$.
So c is the Dominical letter.

EXAM. II. *For the year 1776.*

$$\begin{array}{r} 4)1776 \\ 444 \\ \hline 7)2220(317 \end{array}$$

1

Remains 1. Then $7-1=6=f$.
So g f are the Dominical letters.

And in this manner were the following numbers computed.

For the Dominical letters during the 18th century.

Solar cycles	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Dom. letters	DC	B	A	G	F	E	D	C	B	A	G	F	E	D
Solar cycles	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Dom. letters	G	F	E	D	C	B	A	G	F	E	D	C	B	A

The year 1800 being a common year, stops the above order, and the following are the Dominical letters for the 19th century.

Solar cycles	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Dom. letters	ED	C	B	A	G	F	E	D	C	B	A	G	F	E
Solar cycles	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Dom. letters	A	G	F	E	D	C	B	A	G	F	E	D	C	B

129. PROBLEM IV *To find the epact till the year 1900.*

RULE. Multiply the golden number for the given year by 11, and divide the product by 30; from the remainder take 11, leaves the epact.

If the remainder is less than 11, add 19 to it, and it gives the epact.

Ex. I. *Find the epact for 1773.*

The Golden number is 7. (127)
Multiply by 11

$$\begin{array}{r} 30)77(2 \\ 66 \\ \hline 11 \end{array}$$

Remains 17, more than 11.
Then $17-11=6$ the epact.

Ex. II. *To find the epact for 1776.*

The golden number is 10 (127)
Multiply by 11

$$\begin{array}{r} 30)110(3 \\ 90 \\ \hline 20 \end{array}$$

Remains 20, more than 11.
Then $20-11=9$ the epact.

And thus might the following numbers be found.

Gold N^o 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19.
Epacts 29 11 22 3 14 25 6 17 28 9 20 1 12 23 4 15 26 7 18.
The epacts here proceed by the difference 11, rejecting 30's.

130. PRO-

130.

PROBLEM V. *To find the Moon's age.*

RULE. To the epact add the number and day of the month; their sum, if under 30, is the Moon's age.

But if above 30, the excess in months of 31 days, or the excess above 29, in a month of 30 days, shews the age, or days since last conjunction.

When the solar and lunar years begin together, the Moon's age on the 1st of each month, or the monthly epacts, are called the numbers of the months.

And are $\left\{ \begin{array}{cccccccc} 0 & 2 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ \text{Jan. Feb. Mar. Apr. May, June, July, Aug. Sept. Oct. Nov. Dec.} \end{array} \right.$

EXAM. I. *What is the Moon's age on the 14th of October, 1773?*
 The epact is 6 (129)
 The No of month 8
 The day of month 14
 The sum is 30 = Moon's age,
 as the month has 31 days.

EXAM. II. *What is the Moon's age on the 20th of March, 1778?*
 The epact is 9. (129)
 Then $9 + 1 + 20 = 30$ is the sum of the epact, number and day of month.
 And $30 - 30 = 0$ is the Moon's age, as the month has 31 days.

131. The day of next new Moon is readily found by taking her age from 30.

The day of new Moon in any month is equal to the difference between the sum of the year's and month's epacts, and 29 or 30; according as the month has 30 or 31 days. Thus:

On Oct. 14 the Moon is 30 days old. On March 20, Moon is 0 days old.
 So new Moon on October 16. So new Moon on the 20th.
 Or $6 + 8 = 14$ is the sum of epacts. Or $9 + 1 = 10$, the sum of epacts.
 Then $30 - 14 = 16$ the day of new Moon. Then $30 - 10 = 20$, the day of new Moon in October, 1773.

132. PROBLEM VI. *The day of the month, in any year, being given, to know on what week-day it will fall.*

RULE. Find the Dominical letter (128): also the week-day the first of the proposed month falls on (118); and hence the name of the proposed day of the month will be known; observing that the 1st, 8th, 15th, 22d, and 29th days of any month fall on the same week-days.

Ex. I. <i>On what day of the week does the 14th of Oct. fall, in 1773?</i>	Ex. II. <i>In 1776, on what week-day does the 20th of March fall?</i>
The Dominical letter is C. (128)	The Dominical letter is F. (128)
The 1st of October is A. (118)	The 1st of March is D. (118)
Therefore October 3d is Sunday.	Then March 3d is Sunday.
Consequently 14th is Thursday.	And so March 20th is Wednesday.

133. PROBLEM VII. *To find when Easter-day will fall in any year between 1700 and 1899*

RULE. Find on what day of March the new Moon falls nearest to the 21st in common years, or nearest the 20th in leap-years; then the Sunday next after the full, or 15th day of that new Moon, will be Easter-day.

If the 15th day fall on a Sunday, the next Sunday is Easter-day.

Ex. I. <i>When does Easter-day fall in the year 1773?</i>	Ex. II. <i>Required the time of Easter-day in the year 1776?</i>
The Dominical letter is C. (128)	The Dominical letter is F. (128)
March 21, Moon's age is 28 (130)	March 20, Moon's age 0. (130)
New Moon on March 23.	New Moon on March 20
The 15th day is April 7.	Full Moon on April 4.
April the 1st is G, or Thursday.	April 1st is G, or Monday.
Then Easter-Sunday is April 11th.	Then Easter-Sunday is April 7th.

134. Easter-day is always found by the full Paschal Moons, and these are readily seen to the following curious table, which was in the year 1750 communicated in the Royal Society by the Earl of Macclesfield, and published in the Philosophical Transactions for the same year, and its use shewn in the following precepts.

“ To find the day, on which the Paschal limits, or full Moon, falls in any given year; look, in the column of golden numbers belonging to that period of time wherein the given year is contained, for the golden number of that year; over-against which, in the same line continued to the column intitled Paschal full Moons, you will find the day of the month, on which the Paschal limit, or full Moon, happens in that year. And the Sunday next after that day is Easter-day in that year, according to the Gregorian account.”

“ A TABLE, shewing, by means of the golden numbers, the several days on which the Paschal limits, or full Moons, according to the Gregorian account, have already happened, or will hereafter happen; from the Reformation of the Calendar in the year 1582, to the year 4199, inclusive.”

Golden numbers from the year 1583 to 1699, and so on to 4199, all inclusive.														Pachal full Moon.
1583	1700	1900	2100	2300	2500	2700	2900	3100	3300	3500	3700	3800	4100	Days of the month and Sun. letters.
to 10	to 10	to 10	to 10	to 10	to 10	to 10	to 10	to 10	to 10	to 10	to 10	to 10	to 10	March 21 C
1699	1899	2199	2299	2399	2499	2599	2899	3099	3399	3499	3599	3699	3799	4099
3	14	—	6	17	6	17	—	9	—	1	12	1	12	4
—	3	14	—	6	—	14	—	17	9	—	1	—	12	—
11	—	3	14	—	14	—	6	17	—	9	—	9	—	23 E
19	—	11	3	14	3	14	—	—	17	17	—	—	—	24 F
8	19	—	11	—	11	—	3	14	—	6	17	6	—	25 G
—	8	19	—	11	—	11	—	3	14	—	6	—	17	26 A
16	—	8	19	—	19	—	11	—	14	—	14	—	—	27 B
5	—	16	—	8	—	16	—	19	—	19	—	—	—	28 C
—	5	—	16	—	8	—	11	—	11	—	14	3	14	29 D
13	—	5	—	16	—	5	—	13	—	11	—	3	—	30 E
2	13	—	5	—	16	—	8	19	—	11	—	3	14	31 F
10	—	2	13	—	13	—	5	16	—	11	—	11	—	1 G
—	10	—	13	—	13	—	16	—	19	8	19	—	—	2 A
18	—	10	—	2	13	—	—	5	16	—	8	19	—	3 B
7	18	—	2	13	—	2	13	—	5	16	—	8	19	4 C
—	7	18	—	10	—	10	—	5	16	—	16	8	19	5 D
15	—	7	18	—	10	—	13	—	5	16	—	8	19	6 E
4	15	—	7	18	—	18	—	10	13	5	16	—	—	7 F
—	4	15	—	7	18	—	10	2	13	—	5	16	—	8 G
12	—	4	—	15	—	18	—	2	13	2	13	—	5	9 A
1	12	—	15	—	15	—	18	—	10	2	13	—	—	10 B
—	1	12	—	4	—	4	—	10	18	—	2	13	—	11 C
9	—	12	—	4	—	4	—	7	18	10	10	2	—	12 D
—	9	—	12	—	12	—	15	—	7	18	—	10	—	13 E
17	—	9	—	1	—	1	—	15	—	7	18	—	10	14 F
6	17	—	17	—	12	—	4	—	15	7	18	—	—	15 G
14	6	—	6	—	17	—	9	—	12	—	7	18	—	16 A
—	14	—	9	—	17	—	1	12	—	15	4	15	7	17 B
—	—	6	—	9	—	9	—	1	12	4	15	4	15	18 C

His Lordship also gave with the above table, an account of the principles upon which he constructed it; which the more inquisitive readers may consult, if they please.

135. PROBLEM VIII. *To find the time of the Moon's southing on a given day.*

RULE. The Moon's age in days multiplied by $\frac{48}{60}$, or by $\frac{4}{5}$, or by

0, 8, gives, in hours and tenth parts, the time of her southing, nearly.

That time, if less than 12 hours, is the time after midday. But if greater, the excess is the time after last midnight.

Ex. I. *At what time does the Moon come to the meridian of London, on the 14th of October, 1773?*

The Moon's age is 30 days. (130)
which multiplied by 0,8

$\frac{24,0}{100}$ hours,

Moon souths at noon.

Moon souths at 0 h. 0 m. afternoon.

Each tenth part of an hour being 6 minutes, any number of such tenth parts multiplied by 6, produces minutes.

136. PRO-

136. PROBLEM IX. *To find the time of high-water at any place.*

RULE. To the time of the Moon's southing, add the time the Moon has passed the meridian on the full and change days to make high-water at that place; the sum shews the time of high-water on the given day.

The time of high water, on the full and change days, is found in the right-hand column of the geographical table, art. 137, against the name of the place.

Ex. I. <i>On the 14th of October, 1773, at what time will it be high-water at London?</i>	Ex. II. <i>Required the time when it will be high-water at Ushant on March 20th, 1776?</i>
Moon souths at 0 h. 0 m. noon. (135)	Moon souths at 0 h. 0 m. (135)
H.W. at Lond. 3 0 on syzygies.	High-water at Ushant 3 45 P. M.
Sum 3 0	Sum 3 45
H. W. at 3 h. 0 m. afternoon	High-water at 3 45 P. M. on
on the day proposed.	the day proposed.

The v, viii, ix, problems preceding, have solutions, such as are common in books of pilotage, and which in some cases will produce conclusions considerably wide of the truth; it has therefore been judged necessary to consider these articles in a more accurate manner, which will be given in Book IX. of Days works.

SECTION IX.

137. *A Geographical Table.*

Containing the latitudes and longitudes of the chief towns, islands, bays, capes, and other parts of the sea-coasts in the known world; collected from the most authentic observations and charts extant; with the times of high-water on the days of the new and full Moon.

The longitudes are reckoned from the meridian of London. By the latitude and longitude of an island, or harbour, is meant the middle of that place.

Note, B. stands for bay; C. for cape; R. for river; Po. for port; Pt. for point; I. for Isle; St. for saint; G. for gulf; M. for mount, Eu. for Europe; Am. for America; Atl. for Atlantic; Ind. for Indian; Med. Sea for Mediterranean Sea; Wh. Sea for White Sea; Archip. for Archipelago; Nov. Sco. for Nova Scotia; Phil. I. for Philippine Isles; Adriat. for Adriatic; Eng. for England: Beside other contractions which will be easily understood.

Nam

Names of places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H Water.
A						
I. Abacco, { N. point	Am.	Bahama I.	Atl. Ocean.	27 12 N.	77 05 W.	
or Lucayos { S. point				26 15 N.	77 01 W.	
Abbregrak	Eu.	France	Eng. Channel	48 32 N.	4 15 W.	4h. 3om.
St. Abbfhead	Eu.	Scotland	Germ. Ocean	55 55 N.	1 56 W.	
I. Abdeleur.	Africa	Anian	Indian Sea	11 55 N.	51 45 E.	
Aberdeen	Eu.	Scotland	Germ. Ocean	57 06 N.	01 44 W.	0 45
Abo	Eu.	Finland	Baltic Sea	60 27 N.	22 17 E.	
Abrolhos bank	Am.	Brazil	Atl. Ocean	18 22 S.	38 45 W.	
Abrolho bank N. part	Am.	Bahama	Atl. Ocean	21 33 N.	69 50 W.	
Achen	Asia.	I. Sumatra	Indian Ocean	5 15 N.	95 55 E.	
Aden	Asia.	Arabia	Indian Sea	12 55 N.	45 35 E.	
I. Admiralties	Eu.	Nova Zem.	North Ocean	75 05 N.	52 50 E.	
I. Agalega, or Gallega	Africa	Madagascar	Indian Ocean	10 15 S.	54 46 E.	
C. St. Agnes	Am.	Patagon	Indian Ocean	53 55 S.	62 47 W.	
Agra	Asia	India	Mogul's	26 43 N.	76 49 E.	
I. St. Agusta	Eu.	Dalmatia	Adriatic Sea	42 40 N.	18 57 E.	
C. Ajuga	Am.	Peru	South Ocean	6 38 S.	80 50 W.	
B. Alagoa.	Africa	C. offrs	South Ocean	25 30 S.	33 33 E.	
I. Aland	Eu.	Sweden	Baltic Sea	60 20 N.	21 30 E.	
R. Albany	Am.	New Wales	Hud. Bay	52 35 N.	85 18 W.	
I. Alboran	Africa	Algier	Medit. Sea	36 00 N.	2 27 W.	
Aliborough	Eu.	England	Germ. Ocean	52 20 N.	1 25 E.	9 45
I. Alderney	Eu.	England	Eng. Channel	49 48 N.	2 11 W.	12 00
Aleppo	Asia	Syria	Medit. Sea	35 45 N.	37 25 E.	
Alexandretta	Asia	Syria	Medit. Sea	36 35 N.	36 25 E.	
Alexandria	Africa	Egypt	Medit. Sea	31 11 N.	30 21 E.	
I. Algeranca	Africa	Canaries	Atl. Ocean	29 45 N.	12 13 W.	
Algier	Africa	Algier	Medit. Sea	36 49 N.	2 18 E.	
Allicant	Eu.	Spain	Medit. Sea	38 34 N.	0 07 W.	
I. Alicur, Lipari II.	Eu.	Italy	Medit. Sea	38 31 N.	14 37 E.	
Alkofir	Africa	Egypt	Red Sea	26 20 N.	34 41 E.	
B. All Saints, or	Am.	Brasil	Atl. Ocean	13 05 S.	38 45 W.	
Todos Sanctos	Eu.	Spain	Medit. Sea	36 51 N.	2 15 W.	
Almeria	Africa	Zanguebar	Indian Ocean	5 45 S.	52 30 E.	
Almirante,	Asia	Arabia	Red Sea	4 30 S.	55 40 E.	
limits				28 20 N.	34 19 E.	6 00
Altur	Am.	Terra Firma	Atl. Ocean	0 30 S.	47 35 W.	
S. Amazons,	Asia	Molucca I.	Indian Ocean	4 25 N.	127 25 E.	
mouths	Am.	Chili	South Ocean	25 30 S.	83 20 W.	
I. Amboyne	Eu.	Dutchland	Germ. Ocean	53 30 N.	6 20 E.	7 30
II. Ambrofa	Asia	China	South Ocean	24 30 N.	118 45 E.	
I. Ameyland	Eu.	Dutchland	Germ. Ocean	52 23 N.	05 04 E.	3 00
I. Amoy	Asia	S. Continen	Indian Ocean	37 55 S.	75 15 E.	
Amsterdam	Africa	S. Continen	South Ocean	21 25 S.	179 41 W.	
I. Amsterdam	Eu.	Eth. Coast	Alt. Ocean	2 36 S.	5 35 E.	
I. Amsterdam	Eu.	Italy	Mediterran.	43 38 N.	13 36 E.	
I. Ancona	Asia	India	B. Bengal	14 00 N.	93 05 F.	
II. Andaman,	Asia	India	Indian Ocean	10 08 N.	93 35 E.	
limits				10 00 N.	73 40 E.	
I. Andaro	Am.	Mexico	Atl. Ocean	12 30 N.	81 35 W.	
I. St. Andero	Africa	Madagascar	Indian Ocean	15 46 S.	45 22 E.	
Sotoverto	Eu.	Scotland	Germ Ocean	56 18 N.	2 37 W.	2 15
C. St. Andrea	Am.	Bahama I.	Atl. Ocean	25 00 N.	77 58 W.	
St. Andrew's	Africa	Madagascar	Indian Ocean	23 30 N.	77 00 W.	
Is. Andrews { N. point	Eu.	Turky	Archipelago	37 00 S.	58 40 E.	
Is. Andrews { S. point	Eu.	Italy	Medit. Sea	36 27 N.	33 38 E.	
Is. Anga'ay	Africa	Ethiopia	N. Al. Ocean	41 42 N.	16 16 E.	
C. St. Ang-lo	Africa	Caffers	Indian Ocean	34 45 S.	9 35 E.	
Mount St. Angelo	Am.	Antil Is.	Atl. Ocean	18 15 N.	62 57 W.	
R. d'Angra						
C. d'Anquilhas						
I. Anguilla.						

I. Anhot

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water
I. Anholt	Eu.	Denmark	Sound	56 40 N.	12 00 E.	oh. com.
C. Anne.	Am.	New Eng.	West Ocean	42 50 N.	70 27 W.	
C. Queen Anne	Am.	Greenland	North Ocean	64 15 N.	50 30 W.	
Q. Anne's Foreland	Am.	N. Main	Hudson's Str.	64 08 N.	74 36 W.	
Annapolis Royal	Am.	Nova Scotia	B. Fundy	45 00 N.	64 00 W.	
I. Antego	Am.	Caribbe Isles	Atl. Ocean	16 57 N.	61 56 W.	
Antibes	Eu.	France	Medit. Sea	43 35 N.	07 13 E.	
I. Ante- coft	Am.	Canada	{ B. St. Lau- rence	50 00 N.	62 08 W.	
Antiochetta	Asia	Syria	Medit. Sea	49 33 N.	36 17 E.	
C. d'Antifer	Eu.	France	Eng. Channel	49 47 N.	0 34 E.	
C. Antonio	Am.	Isle Cuba	Atl. Ocean	21 45 N.	84 05 W.	
I. St. Antonio	Africa	Cape Verd	Atl. Ocean	17 45 N.	24 23 W.	
C. St. Antony	Am.	Magellan	St. Atl. Ocean	36 45 S.	53 42 W.	
Antwerp	Eu.	Flanders	R. Scheld	51 13 N.	4 29 E.	
B. Apalxy	Am.	Florida	G. Mexico	30 00 N.	83 53 W.	6 00
I. Apalioria	Asia	India	Indian Ocean	9 08 S.	101 40 W.	
Aquapulco	Am.	Mexico	South Sea	17 10 N.	96 03 W.	
Aquatulco	Am.	Mexico	South Sea	15 27 N.	39 00 E.	6 00
Archangel	Eu.	Russia	White Sea	64 34 N.	42 35 W.	
I. d'Arcas	Am.	Mexico	G. Mexico	20 45 N.	92 35 W.	
Arica	Am.	Peru	South Sea	10 27 S.	71 05 W.	
I. Arran	Eu.	Ireland	St. Geo. Ch.	54 48 N.	8 59 W.	11 0
I. Ascending	Am.	Brasil	St. Atl. Ocean	20 42 S.	28 30 W.	
I. Ascension	Am.	Brasil	St. Atl. Ocean	7 57 S.	13 54 W.	
I. Affinaria, Sardinia	Eu.	Italy	Medit. Sea	41 06 N.	8 36 E.	
R. Ashley	Am.	Carolina	Atl. Ocean	33 22 N.	79 50 W.	0 45
R. Aslene	Africa	Guinea	St. Atl. Ocean	5 30 N.	20 20 W.	
I. Aftores	Africa	Madagascar	Indian Ocean	10 22 S.	53 25 E.	
Athens	Eu.	Turkey	Archipelago	37 40 N.	24 10 E.	
Atkin's Key	Am.	Bahama Isles	Atl. Ocean	22 07 N.	73 46 W.	
Atwood's Keys	Am.	Bahama Isles	Atl. Ocean	21 22 N.	72 40 W.	
C. Ava	Asia	Japan	South Ocean	34 45 N.	141 00 E.	
If. Aves, Sotovent	Am.	Terra Firma	Atl. Ocean	11 30 N.	66 43 W.	
C. St. Augustine	Am.	Brasil	Atl. Ocean	8 48 S.	35 00 W.	
C. St. Augustine	Asia	Mindanao	South Ocea	6 40 N.	126 25 E.	
St. Augustine	Am.	Florida	Atl. Ocean	30 10 N.	10 29 W.	4 30
Aydhah	Africa	Egypt	Red Sea	21 53 N.	36 26 E.	
Aylah.	Asia	Arabia	Red Sea	29 03 N.	35 41 E.	
Babelmondel Straits	Africa	Abyssinia	Red Sea	12 50 N.	43 50 E.	
C. Baba	Asia	Natalia	Archipelago	39 33 N.	26 22 E.	
I. Bachian	Asia	Molucca Isles	South Sea	00 40 N.	123 00 E.	
I. Bahama	Am.	Bahama Isles	Atl. Ocean	26 45 N.	78 35 W.	
Bahama Bank, N. pt.	Am.	Bahama Isles	Atl. Ocean	27 50 N.	78 43 W.	
Baker's dozen	Am.	Labrador	Hudson's Bay	57 0		
Balafor	Asia	India	B. Bengal	20 00 N.	79 54 E.	
Baldivia	Am.	Chili	South Ocean	39 38 S.	72 45 W.	
I. Bali	Asia	Sunda Isles	Indian Ocean	8 05 S.	114 30 E.	
Baltimore	Eu.	Ireland	West Ocean	51 16 N.	9 26 W.	4 30
I. Banca { S. end	Asia	Sunda Isles	Indian Ocean	3 15 S.	107 10 E.	
I. Banda { N. W. end	Asia	Sunda Isles	Indian Ocean	1 50 S.	105 30 E.	
I. Banda	Asia	Molucca Isles	Indian Ocean	04 30 N.	127 25 E.	
Banjar	Asia	I. Borneo	Indian Ocean	2 27 S.	113 50 E.	
Bantam	Asia	I. Java	Indian Ocean	6 15 S.	106 25 E.	
B. Bantry	Eu.	Ireland	Atl. Ocean	51 45 N.	10 46 W.	
I. Barbadoes	Am.	Caribbe Isles	Atl. Ocean	13 00 N.	59 50 W.	
Bridge town	Africa	Samaga	N. Atl. Ocean	21 50 N.	16 26 W.	
C. Barbas	Am.	Caribbe Isles	Atl. Ocean	18 06 N.	61 35 W.	
I. Barbuda	Eu.	Greenland	North Ocean	78 18 N.	20 06 E.	
C. Barcam	Eu.	Spain	Medit. Sea	41 26 N.	2 18 E.	
Barcelona	Eu.	France	Eng. Channel	49 38 N.	1 16 W.	7 30

Names of places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Pargatz Point	Eu.	Iceland	North Ocean	66° 30' N.	0°	
I. Barfey	Eu.	Wales	St. Geo. Ch	52° 44' N.	17° 12' W.	
C. Barlo	Eu.	Russia	White Sea	66° 30' N.	5° 00' W.	
I. Bartholomew	Am.	Caribbe Isles	Atl. Ocean	17° 52' N.	38° 00' E.	
I. de Bas	Eu.	France	Eng. Channel	48° 50' N.	62° 35' W.	
Baffra	Afia.	Arabia	Indian Sea	29° 45' N.	4° 00' W.	3h. 45m
C. Baflos, or Baxos	Africa	Arabian	Indian Sea	4° 12' N.	47° 40' E.	
Baflos de Banhos	Africa	Zanguebar	Indian Ocean	5° 00' S.	47° 07' E.	
Baflos de Chagos	Afia.	India	Indian Ocean	6° 42' S.	48° 08' E.	
I. Baflos des Indes	Africa	Zanguebar	Indian Ocean	21° 19' S.	68° 20' E.	
Batavia	Afia	I. Java	Indian Ocean	6° 15' S.	106° 15' E.	
Bayonne	Eu.	France	B. Biscay	43° 30' N.	1° 25' W.	3 30
Bayona Isles	Eu.	Spain	Atl. Ocean	41° 45' N.	9° 01' W.	
Brachy Head	Eu.	England	Eng. Channel	50° 43' N.	0° 25' E.	0 00
Bear-bay	Eu.	Greenland	North Ocean	79° 10' N.	24° 15' E.	
N. Bear } S. Bear }	Am.	Labradere	Hudson's Bay	54° 40' N.	83° 0'	12 00
I. Beerenberg	Eu.	Labradore	North Ocean	54° 25' N.	4° 30' E.	
Belcher's Isles	Am.	Labradore	Hudson's Bay	56° 0' N.	83° 4'	
Belfast	Eu.	Ireland	Irish Sea	54° 43' N.	5° 52' W.	10 0
Bellise	Eu.	France	B. Biscay	47° 21' N.	3° 13' W.	3 30
Straits of Belleisle	Eu.	France	Atl. Ocean	51° 48' N.	51° 08' W.	
Bell Sound	Am.	Newfoundl.	North Ocean	77° 15' N.	12° 40' E.	
Bencola	Afia	I. Sumatra	Indian Ocean	4° 00' S.	102° 55' E.	
Bengal	Afia	India	B. Bengal	22° 00' N.	92° 45' E.	
Bergen	Eu.	Norway	Western Oc.	60° 10' N.	6° 14' E.	
Berlin	Eu.	Germany	R. Elbe	52° 33' N.	13° 32' E.	
I. Bermudas	Am.	Bahama Isles	Atl. Ocean	32° 25' N.	60° 38' W.	7 00
I. Bermaja	Am.	Mexico	G. Mexico	21° 40' N.	92° 53' W.	
Berwick	Eu.	England	Germ. Ocean	55° 45' N.	1° 50' W.	2 30
Berry Point	Eu.	England	Eng. Channel	50° 37' N.	3° 49' W.	
Birds Island	Am.	Acadia	G. St. Lawr.	47° 51' N.	60° 54' W.	
Bilboa	Eu.	Spain	B. Biscay	43° 26' N.	3° 18' W.	
I. du Ric	Am.	Acadia	R. St. Lawr.	48° 30' N.	68° 36' W.	2 0
Blackney	Eu.	England	Germ. Ocean	53° 20' N.	0° 55' E.	6 00
Black Point	Eu.	Greenland	North Ocean	78° 00' N.	10° 50' E.	
Black Isle	Eu.	Nova Zem.	North Ocean	72° 52' N.	52° 35' E.	
C. Blanco	Africa	Negroland	Atl. Ocean	20° 45' N.	17° 23' W.	9 45
C. Blanco	Am.	Patagonia	Atl. Ocean	47° 07' S.	64° 05' W.	
C. Blanco	Eu.	Greenland	North Ocean	77° 58' N.	20° 04' E.	
C. Blanco	Am.	Mexico	South Sea	9° 42' N.	85° 55' W.	
I. Blanco, Sotavento	Am.	Terra Firma	Atl. Ocean	11° 42' N.	64° 20' W.	
Blanchart Race	Eu.	France	Eng. Channel	49° 42' N.	2° 08' W.	0 00
Blâques	Eu.	Ireland	Atl. Ocean	52° 00' N.	11° 56' W.	
Blavet, or Port Louis	Eu.	France	B. Biscay	47° 45' N.	3° 13' W.	3 0
Bocachica	Am.	Terra Firma	Caribb. Sea	10° 20' N.	75° 30' W.	
C. Bojador	Africa	Negroland	Atl. Ocean	26° 12' N.	13° 27' W.	0 00
R. Bofhaya	Afia	Siberia	South Sea	52° 48' N.	157° 00' E.	
I. Bombay	Afia	India	Indian Ocean	19° 42' N.	73° 03' E.	
Bona	Africa	Tunis	Mediterran.	37° 08' N.	7° 10' E.	
C. Bona	Africa	Tunis	Mediterran.	37° 10' N.	10° 00' E.	
C. Bona vista	Am.	Newfoundl.	Atl. Ocean	49° 36' N.	50° 10' W.	
I. Bona vista	Africa	C. Verd Isles	Atl. Ocean	16° 05' N.	21° 50' W.	
C. Bona fortuna	Eu.	Russia	White Sea	65° 35' N.	38° 25' E.	
I. Bonayre, Sorvento	Am.	Terra Firma	Atl. Ocean	11° 52' N.	67° 20' W.	
B. Bonaventura	Am.	Terra Firma	South Sea	3° 18' N.	76° 50' W.	
C. Bon Esperance	Africa	Caffers	Indian Ocean	33° 55' S.	18° 35' E.	3 00
Bordeaux	Eu.	France	B. Biscay	44° 50' N.	0° 30' W.	3 00
East Point } West Point } North Point } South Point }	Afia		Indian Ocean	1° 12' N.	117° 10' E.	
				3° 15' N.	108° 57' E.	
				7° 05' N.	113° 40' E.	
				9° 32' S.	112° 05' E.	

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Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
I. Bor-neo	Asia		Indian Ocean	5° 00' N.	112° 15' E.	
neo { Succadano				5° 50' S.	108° 35' E.	
I. Bornholm	Eu.	Sweden	Baltic Sea	55° 12' N.	15° 50' E.	
Boston	Eu.	England	Germ. Ocean	53° 10' N.	0° 25' E.	
Boston	Am.	New Eng.	Atl. Ocean	42° 34' N.	70° 25' W.	
Boulogne	Eu.	France	Eng. Channel	50° 44' N.	1° 42' E.	roh. 30m.
I. Bourbon, St. Den.	Africa	Madagascar	Indian Ocean	20° 52' S.	55° 35' E.	
I. St. Brandon	Africa	Madagascar	Indian Ocean	16° 45' S.	64° 48' E.	
B. Brandwyp	Eu.	Greenland	North Ocean	79° 50' N.	26° 20' E.	
I. Bravas	Africa	C. Verd	Atl. Ocean.	14° 30' N.	23° 44' W.	
Bremen	Eu.	Germany	R. Weser	53° 30' N.	9° 00' E.	6 00
Breefoud, a sand	Eu.	Dutchland	Germ. Ocean	53° 12' N.	5° 15' E.	4 30
Breflau	Eu.	Silesia	R. Oder	51° 03' N.	17° 13' E.	
Bret	Eu.	France	B. Biscy	48° 23' N.	4° 26' W.	3 45
B. Breff	Am.	New Britain	West Ocean	52° 10' N.	52° 30' W.	
Bridge Town	Am.	I. Barbadoes	Atl. Ocean	13° 00' N.	59° 50' W.	
Bridlington Bay	Eu.	England	Germ. Ocean	54° 07' N.	00° 04' E.	3 45
Brill	Eu.	Dutchland	Germ. Ocean	51° 56' N.	4° 10' E.	1 30
Brion Isle	Am.	Acadia	G. St. Lawr.	47° 50' N.		
Bristol	Eu.	England	St. Geo. Ch.	51° 28' N.	2° 30' W.	6 45
B. { Louisbourg				45° 54' N.	59° 50' W.	
I. Scateri				46° 08' N.	59° 27' W.	
North Cape				47° 06' N.	60° 7' W.	
I. Mathias				2° 00' S.	147° 50' E.	
North point				2° 30' S.	148° 40' E.	
S. W. point				6° 00' S.	146° 37' E.	
Strait Dampier				6° 15' S.	146° 15' E.	
C. St. George				5° 30' S.	150° 55' E.	
I. St. John				4° 20' S.	152° 40' E.	
Buchanels	Eu.	Scotland	Germ. Ocean	57° 29' N.	1° 23' W.	3 00
Buenos Ayres	Am.	Brasil	Atl. Ocean	34° 35' S.	57° 36' W.	
Burgaford point	Eu.	Iceland	North Ocean	66° 03' N.	16° 34' W.	
Burlings, rocks	Eu.	Portugal	Atl. Ocean	39° 30' N.	9° 13' W.	
Burlington	Eu.	England	Germ. Ocean	54° 00' N.	0° 08' E.	
Button's Isles	Am.	New Britain	Hudf. Straits	60° 35' N.	65° 20' W.	6 50
C						
I. Cabrera	Eu.	Italy	Mediterran.	43° 10' N.	9° 11' E.	
Cadiz	Eu.	Spain	Atl. Ocean	36° 33' N.	5° 56' W.	4 30
Caen	Eu.	France	Eng. Channel	49° 11' N.	0° 16' W.	9 00
Cagliari, I. Sardinia	Eu.	Italy	Medit. Sea	39° 25' N.	9° 38' E.	
Calabar { Old	Africa	Guinea	Eth. Ocean	4° 30' N.	8° 10' E.	
Calabar { New				5° 00' N.	7° 00' E.	
C. Calaberno	Asia	Natolia	Archipelago	38° 42' N.	26° 44' E.	
Calais	Eu.	France	Eng. Channel	50° 58' N.	01° 56' E.	11 30
C. Calamodon	Asia	India	B. Bengal	10° 22' N.	80° 40' E.	
Caldera	Asia	I. Mindano	South Ocean	7° 00' N.	121° 25' E.	
I. Caldyy	Eu.	England	St. Geo. Ch.	51° 33' N.	5° 14' W.	5 15
Calecut	Asia	India	Indian Ocean	11° 15' N.	75° 39' E.	
Isles Caicos, or }						
Cankroff, Nor. }	Am.	Bahama Isles	Atl. Ocean	21° 48' N.	71° 50' W.	
part						
Cairo	Africa	Egypt	R. Nile	30° 02' N.	31° 31' E.	
I. Great Camanis	Am.	West Indies	Atl. Ocean	19° 18' N.	80° 29' W.	
I. Little Camanis	Am.	West Indies	Atl. Ocean	19° 42' N.	79° 20' W.	
Cambodia	Asia	India	Indian Ocean	10° 35' N.	104° 45' E.	
C. Cambron, or }	Africa	Algiers	Medit. Sea	37° 18' N.	4° 58' E.	
Carbon	Am.	New Spain	Atl. Ocean	15° 35' N.	83° 29' W.	
C. Cameron	Africa	Guinea	Atl. Ocean	3° 30' N.	9° 10' E.	
R. Camerones	Am.	Magellan	Atl. Ocean	44° 50' S.	64° 10' W.	
B. Camerones	Am.	Dutchland	Germ. Ocean	53° 33' N.	5° 30' E.	1 30
Camfer, a sand	Eu.					

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Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Camin	Eu.	Germany	Baltic Sea	54 04 N.	15 40 E.	
Campeachy	Am.	Yucatan	Atl. Ocean	19 36 N.	90 53 W.	
I. Canaria	Africa	Canaries	Atl. Ocean	28 07 N.	14 26 W.	3h. om.
C. Candelo	Eu.	Russia	N—Ocean	69 25 N.	45 30 E.	
{ C. St. John, W. end Candia C. Solomon, E. end	Eu.	Turky	Medit. Sea	{ 35 12 N. 35 19 N. 34 57 N.	{ 23 54 E. 25 23 E. 27 06 E.	
Candia	Asia	I. Ceylon	Indian Ocean	7 54 N.	81 53 E.	
I. Candu	Asia	India	Indian Ocean	7 30 S.	77 55 E.	
C. Canfo	Am.	Novia Scotia	Atl. Ocean	45 18 N.	60 48 W.	
C. Canfo Passage	Am.	Novia Scotia	Atl. Ocean	45 50 N.	60 00 W.	
C. Cantin	Africa	Barbary	Atl. Ocean	32 49 N.	9 09 W.	0 00
Cantire, Mul	Eu.	S—o—land	West Ocean	55 22 N.	5 45 W.	
Canton	Asia	China	South Ocean	23 08 N.	113 08 E.	
I. Capri	Eu.	Italy	Medit. Sea	40 34 N.	14 11 E.	
I. Capraya	Eu.	Italy	Medit. Sea	43 03 N.	10 15 E.	
C. Carapar	Asia	India	B. Bengal	19 22 N.	86 05 E.	
Caraccas	Am.	Terra Firma	Atl. Ocean	10 06 N.	66 45 W.	
Caries, or Hecluits Isles	Am.	Greenland	Baffin's Bay	77 15 N.	62 00 W.	
Crinda Point	Am.	California	South Ocean	38 24 N.	124 25 W.	
Crillile	Eu.	England	Irish Sea	54 47 N.	2 35 W.	
C. Carmel	Asia	Syria	Levant	33 08 N.	35 35 E.	
Caroline Isles, limits	Asia	—	South Sea	{ 7 10 N. 12 00 N.	{ 137 25 E. 127 25 E.	
C. Carthage	Africa	Barbary	Medit. Sea	36 52 N.	10 31 E.	
Carthage	Am.	Terra Firma	Caribbean Sea	10 27 N.	75 21 W.	
Carthagene	Eu.	Spain.	Medit. Sea	37 37 N.	1 03 W.	
Cary's Swan's Nest	Am.	—	Hudson's Bay	62 20 N.	83 30 W.	8 15
Cassets	Eu.	I. Guernsey	Eng. Channel	49 50 N.	2 26 W.	
C. Cassander	Eu.	Turky	Archipelago	40 02 N.	23 41 E.	
I. St. Cartharines	Am.	Brazil	Atl. Ocean	27 45 N.	47 58 W.	
Cat Ile { N. point S. point	Am.	Bahama	Atl. Ocean	{ 24 50 N. 23 48 N.	{ 75 38 W. 75 35 W.	
Cathnes Point, or Dinnat Head	Eu.	Scotland	West Ocean	58 46 N.	3 17 W.	9 00
Canea	Eu.	I. Sicily	Medit. Sea	42 40 N.	20 30 E.	
C. Catocha	Am.	New Spain	Caribbean Sea	20 48 N.	86 35 W.	
I. Cayenne	Am.	Terra Firma	Atl. Ocean	4 56 N.	52 10 W.	
N. E. point				1 48 N.	124 37 E.	
N. point				1 40 N.	120 50 E.	
W. point				3 00 S.	117 55 E.	
S. W. point Ma- cassar	Asia	Spice Isles	Indian Ocean	4 40 S.	119 05 E.	
S. point				5 40 S.	119 55 E.	
S. E. point				5 00 S.	121 40 E.	
I. Cephalonia	Eu.	Turky	Medit. Sea	38 20 N.	20 11 E.	
Ceuta	Africa	Barbary	Medit. Sea	35 45 N.	4 42 W.	
Lafanapatam, N. part				9 47 N.	80 55 E.	
Trinquemaile, S. E. end	Asia	India	Indian Ocean	8 40 N.	81 40 E.	
C. Gallo, S. West end				6 27 N.	82 10 E.	
Chandennagar	Asia	Bengal	River Ganges	6 15 N.	80 20 E.	
Charles Town	Am.	Carolina	Asheley River	22 51 N.	88 34 E.	
C. Charles	Am.	Virginia	Atl. Ocean	33 22 N.	79 50 W.	3 0
I. of { East End. Charles } West End.	Am.	Labrador	Hudson's Str.	{ 62 46 N. 62 48 N.	{ 74 15 W. 75 30 W.	10 15
C. Charles	Am.	New Brit in	West Ocean	51 50 N.	51 10 W.	

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Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
I. Charlton	Am.	New Wales	Hudson's Bay	52° 30' N.	82° 00' W.	
B. Chebucto	Am.	Nova Scotia	Atl. Ocean	44° 45' N.	63° 18' W.	
Chaignecto	Am.	Nova Scotia	B. Fundy	46° 15' N.	63° 11' W.	oh. 45 m.
Cherbourg	Eu.	France	Eng. Channel	49° 38' N.	01° 33' W.	7 30
Cherry Isle	Eu.	Greenland	North Ocean	74° 35' N.	18° 05' E.	
Chester	Eu.	England	Irish Sea	53° 10' N.	2° 25' W.	
Chiddock	Eu.	England	Eng. Channel	50° 47' N.	3° 00' W.	
C. Chidley	Am.	New Britain	Hud. Straits	60° 22' N.	64° 00' W.	
I. Chiloe { N. point	Am.	Patagonia	South Sea	41° 45' S.	73° 05' W.	
{ S. point	Am.	Patagonia	South Sea	43° 50' S.	73° 05' W.	
C. Chiokotkago	Asia	Siberia	North Ocean	64° 00' N.	174° 45' W.	
Christiana	Eu.	Norway	Sound	59° 25' N.	10° 30' E.	
Christiansople	Eu.	Sweden	Baltic Sea	55° 55' N.	15° 10' E.	
Christiansadt	Eu.	Sweden	G. Bothnia	62° 47' N.	22° 50' E.	
I. St. Christopher's	Am.	Carib. Isles	Atl. Ocean	17° 15' N.	62° 50' W.	
R. St. Christopher's	Africa	Caffres	Indian Ocean	32° 47' S.	30° 00' E.	
C. Chukchenie	Asia	Siberia	North Ocean	66° 30' N.	171° 10' W.	
C. Churchill }	Am.	New Wales	Hudson's Bay	53° 00' N.	93° 17' W.	
R. Churchill }	Am.	New Wales	Hudson's Bay	58° 47' N.	94° 15' W.	7 20
I. Chufan	Asia	China	Chinese Sea	30° 00' N.	121° 50' E.	
Gvita Vecchia	Eu.	Italy	Medit. Sea	42° 50' N.	11° 51' E.	
C. Cleare	Eu.	Ireland	West Ocean	51° 18' N.	9° 50' W.	4 30
I. Closte	Asia	India	Indian Ocean	22° 00' S.	95° 40' E.	
Cochin	Asia	India	Indian Ocean	9° 50' N.	76° 05' E.	
I. Cocos	Asia	India	Indian Ocean	12° 20' S.	98° 10' E.	
I. Cocos	Am.	Mexico	South Ocean	5° 00' N.	88° 45' W.	
C. Cod.	Am.	New Engl.	Atl. Ocean	42° 15' N.	69° 27' W.	
Colcheffer	Eu.	England	Germ. Ocean	52° 00' N.	0° 58' E.	
C. Cold	Eu.	Greenland	North Ocean	79° 00' N.	10° 00' E.	
R. Colorado	Am.	New Spain	G. California	31° 40' N.	115° 25' W.	
I. Colgau	Eu.	Russia	North Ocean	69° 20' N.	45° 00' E.	
Collioure	Eu.	Spain	Medit. Sea	42° 31' N.	3° 10' E.	
C. Colone	Asia	Turky	Archipelago	37° 43' N.	24° 41' E.	
C. Colonne	Eu.	Natolia	Archipelago	39° 10' N.	27° 04' E.	
C. Colonna	Eu.	Italy	Medit. Sea	38° 56' N.	18° 05' E.	
Comana	Am.	Terra Firma	Atl. Ocean	10° 00' N.	65° 07' W.	
C. Comarin	Asia	India	Indian Ocean	7° 45' N.	78° 17' E.	
C. Comfort	Am.	New Wales	Hudson's Bay	64° 45' N.	82° 30' W.	3 00
Concarneau	Eu.	France	B. Biscay	47° 54' N.	3° 50' W.	
C. Conception	Am.	California	South Sea	35° 40' N.	126° 01' W.	
B. Conception	Am.	Newfoundl.	Atl. Ocean	48° 25' N.	50° 07' W.	
Conception	Am.	Chili	South Sea	36° 43' S.	73° 35' W.	
R. Congo	Am.	Congo	Eth. Ocean	5° 45' S.	11° 53' E.	
I. Coningen	Asia	New Zealand.	South Ocean	34° 30' S.	164° 25' E.	
Coningsburg	Eu.	Poland	Baltic Sea	54° 44' N.	21° 53' E.	2 15
Conquet	Eu.	France	Eng. Channel	48° 30' N.	4° 35' W.	
C. Conquibaco	Am.	Terra Firma	Atl. Ocean	12° 15' N.	69° 57' E.	
Constantinople	Eu.	Turky	Archipelago	41° 00' N.	28° 58' E.	
Copenhagen	Eu.	Denmark	Baltic Sea	55° 41' N.	12° 50' E.	
Copernic	Eu.	Norway	Sound	59° 20' N.	10° 10' E.	
I. Copland	Eu.	Ireland	Irish Sea	54° 40' N.	6° 40' W.	
I. Coquet	Eu.	England	Irish Sea	55° 20' N.	1° 25' W.	3 00
R. Coquimbo	Am.	Chili	Germ. Ocean	29° 54' S.	71° 10' W.	
C. Corbau	Asia	Natolia	South Ocean	38° 03' N.	26° 58' E.	
Cordoue	Eu.	France	Archipelago	45° 30' N.	01° 10' W.	
Corea, South limit	Asia	China	B. Biscay	34° 59' N.	124° 25' E.	
I. Corfu	Eu.	Turky	South Ocean	39° 50' N.	19° 48' E.	
C. Corientes.	Africa	C. Caffres	Mediterran.	24° 08' S.	36° 49' E.	
C. Corientes	Am.	Mexico	Indian Ocean	20° 18' N.	108° 00' W.	
Corinth	Eu.	Turky	South Ocean	37° 30' N.	23° 00' E.	
Carle	Eu.	Ireland	Archipelago.	51° 45' N.	7° 30' W.	6 30

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Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
C. Corfe	Africa	Guiney	Eth. Sea	° 5 12 N.	° 0 23 W.	3h. 30m
{ C. Corfo, North point Bonficio, South point	Eu.	Italy	Mediterran.	{ 42 53 N. 41 22 N.	{ 9 40 E. 9 26 E.	
I. Corvo	Eu.	Azores	Atl. Ocean	39 48 N.	31 23 W.	
I. Colmoledo	Africa	Madagascar	Indian Ocean	10 28 S.	51 40 E.	
I. Coudre	Am.	Canada	R. St. Lawr.	47 31 N.	69 2 W.	
Cow and Calif	Eu.	Ireland	West Ocean	51 21 N.	9 36 W.	
I. Cozumel	Am.	Jucatan	Atl. Ocean	19 36 N.	86 35 W.	
R. Crocei	Asia	China	E. Nankin	34 06 N.	120 10 E.	
Cromer	Eu.	England	Germ. Ocean	53 05 N.	0 56 E.	7 00
Crooked I. N. point	Am.	Bahama	Atl. Ocean	22 47 N.	73 50 W.	
Crois Ile	Eu.	Russia	White Sea	66 31 N.	36 33 E.	
Crois Point	Eu.	Nova Zem.	North Ocean	72 00 N.	53 12 E.	
St. Cruz	Africa	Barbary	Atl. Ocean	30 36 N.	9 35 W.	
I. St. Cruz	Am.	Antilles I.	Atl. Ocean	17 55 N.	65 02 W.	
{ C. Antonio, W. point P. de Mais, E. point				{ 21 45 N. 20 03 N.	{ 84 05 W. 74 52 W.	
Hollow Cape	Am.	Antilles I.	Atl. Ocean	19 42 N.	77 25 W.	
St. Jago				20 03 N.	75 51 W.	
St. Mary				21 26 N.	78 10 W.	
Le St. E'sprit				21 56 N.	79 50 W.	
Havannah				23 12 N.	81 45 W.	
B. Hondy				22 54 N.	82 40 W.	
Cebs Isles	Am.	New Wales	Hudson's Bay	54 15 N.	82 34 W.	
Cubello	Asia	Ind. Malab.	Indian Ocean	7 50 N.	71 55 E.	
B. Cumberland	Am.	North Main	Davis's Str.	66 40 N.	65 20 W.	
Curafo	Am.	Terra Firma	Atl. Ocean	11 56 N.	68 20 W.	
I. Cuzzola	Eu.	Turky	Medit. Sea	42 50 N.	16 55 E.	
Cusco	Am.	Peru	Inland	12 25 S.	73 35 W.	
{ C. Baffa, W. end C. St. Andr. E. end				{ 35 04 N. 35 40 N.	{ 33 04 E. 35 08 E.	
C. de Gasse, S. point	Asia	Syria	Medit. Sea	34 35 N.	33 41 E.	
C. Grego, S. E. point				34 57 N.	34 36 E.	
D						
Dahul	Asia	India	Arabian Sea	18 24 N.	73 33 E.	
Dahlak	Asia	Arabia	Red Sea	15 50 N.	41 44 E.	
I. Dageroort, Light-house	Eu.	Livonia	Baltic Sea	58 55 N.	22 32 E.	
Dantzic	Eu.	Po'and	Baltic Sea	54 22 N.	18 36 E.	
Str. Dardanelis	Eu.	Turky	Archipelago	40 10 N.	26 26 E.	
Gulph Darien	Am.	Terra Firma	Caribbean Sea	8 45 N.	76 35 W.	
Dartmouth	Eu.	England	Eng. Channel	50 27 N.	3 36 W.	6 30
I. Dauphin	Am.	Louisiana	G. Mexico	29 40 N.	87 51 W.	6 00
St. David's Head	Eu.	Wales	St. Geo. Ch.	51 55 N.	5 22 W.	
Fort St. David's	Asia	India	Crom. Coast	12 05 N.	80 55 E.	
I. Defcada	Am.	Carib. Isles	Atl. Ocean	16 36 N.	61 0 W.	
C. Defire	Eu.	Nova Zem.	North Ocean	77 45 N.	79 20 E.	
C. De-foalition	Am.	Greenland	North Ocean	61 45 N.	47 00 W.	
Devil's Isles	Eu.	Greenland	North Ocean	80 00 N.	11 43 E.	
Dewpoint	Asia	India	B. Bengal	16 07 N.	81 47 E.	
I. Diego Royes	Asia	Ind. Malab.	Indian Ocean	{ 0 45 S 0 30 N	{ 70 25 E 68 10 E	
I. Diego Garcia	Africa	India	Indian Ocean	8 45 S.	68 10 E	
Str. Diemen	Asia	Japan Isles	South Ocean	31 12 N.	130 55 E.	

I. Dieu

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
I. Dieu	Eu.	France	Bay of Biscay	46 26 N.	2 20 W.	
Diepe	Eu.	France	Eng. Channel	49 55 N.	1 09 E.	10h. 30m.
Diggs's, or Dudley's Cape	Am.	Greenland	Baffin's Bay	75 48 N.	59 07 W.	
C. Diggs	Am.	Labrador	Hudson's Bay	62 41 N.	78 50 W.	
Po. Diu	Asia	India	Indian Ocean	21 37 N.	70 28 E.	
C. Dobbs	Am.	North Wales	Hudson's Bay	65 00 N.	81 25 W.	
Dofare	Asia	Arabia	Indian Ocean	16 24 N.	53 40 E.	
St. Dominge,	Am.	Antilles	Atl. Ocean	18 25 N.	69 30 W.	
Hispaniola	Am.	Caribbe	Atl. Ocean	15 15 N.	61 08 W.	
I. Dominico	Eu.	Dutchland	R. Maes	52 00 N.	4 26 E.	
Dordrecht	Africa	Ajan	Indian Ocean	10 15 N.	50 44 E.	
C. Dorfi	Eu.	Turky	Archipelago	38 02 N.	25 12 E.	
C. Doro	Eu.	Dutchland	Germ. Ocean	51 47 N.	4 40 E.	3 00
Dort	Eu.	Livonia	Baltic Sea	58 20 N.	23 00 E.	
I. Dofel	Africa	Zanguebar	Indian Ocean	5 15 N.	49 24 E.	
Is. Dos Banhos	Eu.	England	Eng. Channel	51 07 N.	1 13 E.	11 30
Dover	Eu.	England	Germ. Ocean	51 25 N.	1 21 E.	1 15
Downs	Africa	Egypt	Red Sea	19 56 N.	37 40 E.	
Po. Drade	Am.	California	South Sea	38 45 N.	128 35 W.	
Po. Drake, Sir Francis	Am.	Norway	North Ocean	63 16 N.	10 55 E.	
Dronthem	Eu.	Ireland	Irish Sea	53 12 N.	6 55 W.	9 15
Dublin	Eu.	Scotland	Germ. Ocean	55 58 N.	2 22 W.	2 30
Dundalk	Eu.	Ireland	Irish Sea	53 57 N.	6 28 W.	
Dundee	Eu.	Scotland	Germ. Ocean	56 26 N.	2 48 W.	2 15
Dungarvan	Eu.	Ireland	Atl. Ocean	51 57 N.	7 55 W.	4 30
Dungensis	Eu.	England	Eng. Channel	51 00 N.	0 51 E.	9 45
Dungbay Head	Eu.	Scotland	Germ. Ocean	58 45 N.	2 57 W.	
Dunkirk	Eu.	France	Germ. Ocean	51 02 N.	2 27 E.	0 00
Dunnofe	Eu.	I. Wight	Eng. Channel	50 38 N.	1 23 W.	9 45
Durazzo	Eu.	Turky	Medit. Sea	41 58 N.	25 00 E.	
Edinburgh	Eu.	Scotland	Germ. Ocean	55 58 N.	3 00 W.	4 30
Edystone	Eu.	England	Eng. Channel	50 12 N.	4 20 W.	5 30
I. Elba	Eu.	Italy	Mediterran.	43 52 N.	10 38 E.	
R. Elbe mouth	Eu.	Germany	Germ. Ocean	54 18 N.	7 10 E.	0 00
Elbing	Eu.	Poland	Baltic Sea	54 12 N.	20 35 E.	
Elfsburg	Eu.	Sweden	Baltic Sea	56 00 N.	13 35 E.	
Elfsnore	Eu.	Denmark	Baltic Sea	56 00 N.	13 23 E.	
I. Elutheria	N. pt. Am.	Bahama	Atl. Ocean	25 45 N.	76 42 W.	
Embsden	Eu.	Germany	Germ. Ocean	24 57 N.	75 53 W.	
R. Emes mouth	Eu.	Germany	Germ. Ocean	53 05 N.	7 26 E.	0 00
Enchuyfen	Eu.	Dutchland	Zuyder Sea	53 10 N.	7 20 E.	7 30
I. Engano, or Trompeuse	Asia	Sumatra	Indian Ocean	52 43 N.	5 06 E.	0 00
B. Enhorn	Eu.	Greenland	North Sea	6 00 S.	102 35 E.	
Ephesus	Asia	Natolia	Archipelago	78 45 N.	26 05 E.	
Eftaples	Eu.	France	Eng. Channel	38 00 N.	27 53 E.	
Eufatia	Am.	Caribbe	Atl. Ocean	50 34 N.	1 42 E.	11 00
I. Exuma	Am.	Bahama	Atl. Ocean	17 33 N.	62 58 W.	
Fairhead	Eu.	Ireland	West. Ocean	23 25 N.	75 35 W.	
C. Falcon	Eu.	Barbary	Medit. Sea	55 19 N.	6 20 W.	
I. Falkland	E. end Inf. A. Am.	Patagon	Atl. Ocean	36 03 N.	0 14 W.	
Falmouth	Eu.	England	Eng. Channel	51 30 S.	55 40 W.	
C. Falo	Eu.	Turky	Archipelago	52 15 S.	56 47 W.	
				50 12 N.	5 12 W.	5 30
				40 12 N.	24 27 E.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
C. Faló	Africa	Zanguebar	Indian Ocean	8 52 S.	59 55 E.	
Falsterbom	Eu.	Sweden	Baltic Sea	55 20 N.	13 36 E.	
I. Fana	Eu.	Turky	Medit. Sea	40 14 N.	19 32 E.	
R. Farate	Africa	Egypt	Red Sea	21 40 N.	36 29 E.	
Faro Head, or C. } Wiath }	Eu.	Scotland	West. Ocean	58 44 N.	4 50 W.	
C. Farwell	Am.	Greenland	North Ocean	59 37 N.	44 30 W.	
C. Fartack	Asia	Arabia	Indian Ocean	15 41 N.	51 31 E.	
C. Fear	Am.	Carolina	Atl. Ocean	34 04 N.	78 09 W.	
I. Fernando Noronha	Am.	Brazil	Atl. Ocean	3 50 S.	30 35 W.	
I. Felicour, Lipari Isles	Eu.	Italy	Medit. Sea	38 33 N.	14 51 E.	
I. Fermina	Africa	Turky	Archipelago	37 24 N.	25 05 E.	
I. Fernandezpo	Africa	Guiney	Atl. Ocean	3 02 N.	3 33 E.	
I. Ferro	Africa	Canaries	Atl. Ocean	27 48 N.	17 26 W.	
C. Finisterre	Eu.	Spain	Atl. Ocean	43 15 N.	9 20 W.	
I. Florida	Asia	Corea	South Sea	33 30 N.	127 25 E.	
Flamborough Head	Eu.	England	Germ. Ocean	54 08 N.	0 11 E.	4 h. oom.
I. Flores	Eu.	Azores	Atl. Ocean	39 23 N.	30 22 W.	
C. Florida	Am.	Florida	G. Mexico	25 50 N.	80 20 W.	7 30
Flushing	Eu.	Dutchland	Germ. Ocean	51 33 N.	3 20 E.	0 45
I. Fly	Eu.	Dutchland	Germ. Ocean	53 16 N.	5 35 E.	7 30
Forbisher's Straits	Am.	Greenland	Atl. Ocean	62 05 N.	47 18 W.	
North Foreland	Eu.	England	Germ. Ocean	51 28 N.	1 10 E.	9 45
South Foreland	Eu.	England	Eng. Channel	51 12 N.	1 18 E.	9 45
Foreland Fair	Eu.	Ireland	North Ocean	55 05 N.	6 30 W.	
Foreland Merchants	Eu.	Greenland	North Ocean	79 18 N.	10 50 E.	
I. Formentaria	Am.	Spain	North Ocean	63 20 N.	17 05 W.	
I. Formigo	Eu.	Azores	Medit. Sea	38 33 N.	1 15 E.	
C. Formosa	Africa	Guiney	Atl. Ocean	37 55 N.	21 50 W.	
R. Formosa	Africa	Guiney	Eth. Sea	6 10 N.	5 43 E.	
I. Formosa { N. point	Asia	China	Eth. Sea	4 22 N.	4 49 E.	
I. Forteventura { S. point	Asia	China	Eth. Sea	21 25 N.	121 25 E.	
I. W. end	Africa	Canaries	Indian Ocean	22 00 N.	120 40 E.	
Foulnefs	Africa	Canaries	Atl. Ocean	28 23 N.	13 25 W.	
Foulfound	Eu.	England	Germ. Ocean	52 57 N.	0 58 E.	6 45
Foye	Eu.	Greenland	North Ocean	77 30 N.	12 50 E.	
I. France, P. Louis	Eu.	England	Eng. Channel	50 25 N.	4 30 W.	5 15
C. St. Francis	Africa	Madagascar	Indian Ocean	20 10 S.	57 33 E.	
I. St. Francisco	Am.	Peru	South Sea	0 30 N.	80 35 W.	
R. St. Francisco	Africa	Zanguebar	Indian Ocean	6 23 S.	53 22 E.	
Fredericksstadt	Am.	Brazil	Atl. Ocean	10 55 S.	36 30 W.	
Fretum Borough	Eu.	Norway	Sound	59 00 N.	11 10 E.	
C. Frio	Am.	Rahama	Atl. Ocean	22 30 N.	73 40 W.	
R. Fugor	Eu.	Russia	North Ocean	70 00 N.	61 20 E.	
I. Fuego	Am.	Brazil	Atl. Ocean	23 00 S.	40 11 W.	
B. Fuhian	Africa	Zanguebar	Indian Ocean	00 10 N.	42 05 E.	
I. Fyal	Asia	De Vera	Atl. Ocean	14 55 N.	23 28 W.	
	Eu.	China	South Sea	23 00 N.	112 35 E.	
		Azores	Atl. Ocean	38 38 N.	27 05 W.	
G						
I. Galla	Am.	Terra Firma	South Sea	2 40 N.	79 35 W.	
R. Gallega	Am.	Patagon	Atl. Ocean	51 37 S.	65 35 W.	
I. Gallego	Am.	Terra Firma	South Sea	1 40 N.	104 35 W.	
Gallipoli	Eu.	Italy	Medit. Sea	40 19 N.	18 08 E.	
Gallipoly	Eu.	Turky	Archipelago	40 36 N.	27 02 E.	
I. Gallita	Africa	Barbary	Medit. Sea	37 42 N.	9 03 E.	
C. Gallo	Asia	I. Ceylon	Indian Ocean	6 15 N.	80 20 E.	
Is. Gallopega	Am.	Peru	South Sea	2 00 N.	89 00 W.	
Gally Head	Eu.	Ireland	West Ocean	52 40 N.	9 30 W.	

Galway

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Galway	Eu.	Ireland	West. Ocean	53 10 N.	10 03 W.	
R. Gambia	Africa	Negroland	Atl. Ocean	13 00 N.	14 58 W.	
I. Gambo	Asia	India	Indian Ocean	3 05 S.	77 25 E.	
C. Gardafuy	Africa	Anian	Indian Ocean	11 48 N.	50 25 E.	
R. Garonne	Eu.	France	B. Biscay	45 30 N.	1 05 W.	3 h. com.
Galpey Bay	Am.	Acadia	G. St. Lawr.	49 51 N.	63 44 W.	1 30
C. de Gatt	Eu.	Spain	Medit. Sea	36 32 N.	2 05 W.	
C. Gear	Africa	Barbary	Atl. Ocean	30 35 N.	10 01 W.	
Genoa	Eu.	Italy	Medit. Sea	44 25 N.	8 41 E.	
B. St. George	Am.	Newfoundl.	Atl. Ocean	48 19 N.	56 00 W.	
I. St. George	Asia	Natolia	Archipelago	38 47 N.	25 07 E.	
I. St. George	Eu.	Azores	Atl. Ocean	38 45 N.	25 55 W.	
St George's Fort	Asia	India	B. Bengal	13 08 N.	80 39 E.	
Gibraltar	Eu.	Spain	Medit. Sea	36 13 N.	4 53 W.	0 00
I. Gilolo { N. point	Asia	Spice Islands	Indian Ocean	2 30 N.	128 00 E.	
I. Gilolo { S. point	Asia	Spice Islands	Indian Ocean	1 30 S.	129 25 E.	
Glasgow	Eu.	Scotland	R. Clyd	55 52 N.	4 05 W.	
Goa	Asia	India	Malabar	15 51 N.	73 50 E.	
Goes	Eu.	Dutchland	Germ. Ocean	51 39 N.	4 05 E.	
Golf's trike	Am.	Terra Firma	Caribbe Sea	10 20 N.	67 40 W.	
Gombroon	Asia	Persia	Persian Gulf	27 40 N.	55 20 E.	
I. Gomero	Africa	Canaries	Atl. Ocean	28 08 N.	16 35 W.	
C. Gondewar	Asia	India	B. Bengal	16 55 N.	82 55 E.	
C. Good Hope	Africa	Capfers	Indian Ocean	33 55 S.	18 35 E.	3 00
I. Gorea	Africa	Negroland	Atl. Ocean	14 40 N.	17 00 W.	
I. Gorgona	Eu.	Italy	Medit. Sea	43 21 N.	9 11 E.	
I. Goto	Eu.	Sweden	Baltic Sea	58 00 N.	20 15 E.	
I. Goto	Asia	Corea	South Sea	56 58 N.	19 37 E.	
Gottenberg	Eu.	Sweden	Sound	57 40 N.	19 50 E.	
R. Grand	Am.	Paraguay	Atl. Ocean	34 25 N.	11 44 E.	
Granville	Eu.	France	Eng. Channel	51 58 S.	50 35 W.	
I. Gratiola	Africa	Canaries	Atl. Ocean	48 50 N.	1 32 W.	7 00
I. Gratiola	Eu.	Azores	Atl. Ocean	29 37 N.	26 08 W.	
C. Gratos a Dios	Am.	New Spain	Caribbe Sea	39 06 N.	12 10 W.	
Graveline	Eu.	France	Eng. Channel	48 48 N.	22 15 W.	
Gravefend	Eu.	England	R. Thames	50 59 N.	2 13 E.	0 00
I. Grenada	Am.	Caribbe	Atl. Ocean	51 35 N.	0 20 E.	1 30
C. Gremia	Eu.	England	R. Thames	51 52 N.	61 39 W.	
Gripwald	Eu.	Turky	Archipelago	51 28 N.	0 05 E.	
Grimethy	Eu.	Germany	Baltic Sea	40 33 N.	26 20 E.	
Groin, or C. Cornma	Eu.	England	B. Biscay	54 14 N.	15 34 E.	
I. Groy	Eu.	Spain	Germ. Ocean	53 30 N.	0 56 E.	3 00
I. Gurdaleupe	Am.	Newfoundl.	Atl. Ocean	43 28 N.	9 20 W.	
Guayaquil	Am.	Caribbe	Atl. Ocean	50 39 N.	51 35 W.	
I. Guernsey	Eu.	Peru	Atl. Ocean	16 20 N.	61 25 W.	
Gulf	Eu.	England	South Sea	2 10 S.	80 10 W.	
H	Eu.	England	Eng. Channel	49 30 N.	2 47 W.	1 30
Hacluit's Headland	Eu.	Greenland	St. Geo. Ch.	50 06 N.	6 00 W.	
I. Hai- { N. E. point	Asia	China	North Ocean	79 55 N.	12 00 E.	
I. Hai- { S. W. point	Asia	China	Indian Ocean	19 45 N.	110 13 E.	
Halifax	Am.	Nova Scotia	West Ocean	18 22 N.	108 13 E.	
I. Hall	Am.	Greenland	Atl. Ocean	44 36 N.	63 26 W.	7 30
Halliford	Eu.	Iceland	North Ocean	63 56 N.	44 26 W.	
Hamburgh	Eu.	Germany	R. Elbe	64 30 N.	27 15 W.	
Hare Isle	Am.	Canada	R. St. Lawr.	53 41 N.	10 38 E.	6 00
Harlem	Eu.	Dutchland	Germ. Ocean	48 00 N.	63 26 W.	3 30
Hartland Point	Eu.	England	Bristol Chan.	52 24 N.	4 10 E.	9 00
Hartlepool	Eu.	England	Germ. Ocean	51 06 N.	4 35 W.	3 00
Harwich	Eu.	England	Germ. Ocean	54 40 N.	0 56 W.	3 00
	Eu.	England	Germ. Ocean	52 11 N.	1 18 E.	11 15

C. Hatter.

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
C. Hatteras	Am.	Carolina	Atl. Ocean	35° 24' N.	76° 20' W.	
Havannah	Am.	I. Cuba	Atl. Ocean	23° 12' N.	81° 11' W.	
Havre de Grace	Eu.	France	Eng. Channel	49° 30' N.	0° 17' E.	gh.oom.
I. St. Helena	Africa	Caffers	Atl. Ocean	16° 00' S.	5° 53' W.	
Helig's Sound	Eu.	Greenland	North Ocean	79° 15' N.	12° 50' E.	
Is. Heligh Land	Eu.	Norway	North Ocean	65° 15' N.	9° 30' E.	
C. Henrietta Maria	Am.	New Wales	Hudson's Bay	55° 10' N.	84° 10' W.	12 00
C. Henry	Am.	Virginia	Atl. Ocean	36° 57' N.	76° 23' W.	11 15
I. Heys	Eu.	France	Bay of Biscay	46° 24' N.	2° 14' W.	
High Mount	Eu.	Greenland	North Ocean	83° 23' N.	26° 40' E.	
C. Hinpölen	Am.	Maryland	Atl. Ocean	39° 10' N.	74° 50' W.	
{ C. Tiberon, W. pt.				18° 17' N.	7° 04' W.	
				18° 19' N.	73° 1' W.	
St. Louis				19° 50' N.	73° 08' W.	
C. St. Nichol.				18° 28' N.	72° 42' W.	
N. W. pt.				18° 25' N.	69° 30' W.	
Po. Grave				19° 05' N.	68° 30' W.	
St. Domingo				21° 41' N.	73° 15' W.	
C. Raphael N.				25° 33' S.	108° 10' E.	
{ E. pt.				20° 30' S.	112° 15' E.	
				34° 45' S.	112° 00' E.	
R. Hogfies	Am.	Bahama	Atl. Ocean	72° 32' N.	179° 45' E.	1 30
{ B. Seadogs				53° 23' N.	4° 40' W.	
				16° 18' N.	85° 23' W.	
{ N. W. limit.				22° 54' N.	82° 40' E.	
{ S. W. limit.				49° 24' N.	0° 20' E.	9 00
Holy Cape	Asia	Siberia	North Ocean	76° 22' N.	23° 40' E.	
Holy Head	Eu.	Wales	Irish Sea	55° 42' S.	66° 00' W.	
C. Honduras	Am.	New Spain	Caribbean Sea	76° 41' N.	13° 36' E.	
B. Hondy, I. Cuba	Arp.	Antilles	Atl. Ocean	49° 45' N.	1° 55' E.	
Honfleur	Eu.	France	Atl. Ocean	21° 45' N.	89° 15' W.	
Hope Isle	Eu.	Greenland	R. Seine	53° 50' N.	0° 28' W.	6 00
C. Horn	Am.	Ter. Fuego	North Ocean	53° 55' N.	0° 24' E.	5 13
Hornfound	Eu.	Greenland	South Ocean	21° 07' N.	73° 01' W.	
La Hougue	Eu.	France	North Ocean	36° 00' N.	139° 40' E.	
R. Hughly	Asia	India	Eng. Channel	9° 47' N.	80° 55' W.	
Hull	Eu.	England	B. Bengal	15° 07' N.	22° 35' W.	
R. Humber, Ent.	Eu.	England	B. Humber	18° 45' N.	78° 00' W.	
I. Hynago	Am.	Bahama	Germ. Ocean	17° 40' N.	76° 37' W.	
I				17° 58' N.	76° 05' W.	
				37° 30' N.	76° 00' W.	
Jado	Asia	Japan	Atl. Ocean	22° 54' S.	42° 40' W.	
C. Jafanapatan	Asia	I. Ceylon	South Sea	40° 40' N.	141° 25' E.	
I. Jago	Africa	C. Verd	Indian Ocean	31° 45' N.	126° 10' E.	
{ West end				6° 50' S.	105° 15' E.	
				7° 00' S.	115° 55' E.	
Port Royal	Am.	West Indies	Atl. Ocean	8° 30' S.	115° 55' E.	
East end				62° 20' N.	69° 00' W.	10 00
James Town	Am.	Virginia	B. Chesapeake	77° 40' N.	69° 10' E.	
R. Janeiro	Am.	Brazil	Atl. Ocean	78° 13' N.	12° 00' E.	
Japan Isles	Asia		South Sea	49° 07' N.	2° 26' W.	
{ C. Delo, W. pt.				31° 50' N.	35° 25' E.	
				55° 39' N.	6° 20' W.	
{ East limit				25° 50' N.	66° 33' E.	
				57° 33' N.	4° 02' W.	
Ice Cove	Am.	N. Main	Indian Ocean	12° 05' S.	45° 45' E.	
Ice Point	Eu.	Nova Zem.	Hudf. Straits	22° 00' N.	39° 27' E.	
Ice Sound	Eu.	Greenland	North Ocean	77° 40' N.	69° 10' E.	
I. Jerrey	Eu.	England	North Ocean	78° 13' N.	12° 00' E.	
Jerusalem	Asia	Palestine	Eng. Channel	49° 07' N.	2° 26' W.	
I. Ully, S. pt.	Eu.	Scotland	Inland	31° 50' N.	35° 25' E.	
R. Indus	Asia	India	West Ocean	55° 39' N.	6° 20' W.	
Inverness	Eu.	Scotland	Indian Ocean	25° 50' N.	66° 33' E.	
I. Joanna	Africa	Zanguebar	Germ. Ocean	57° 33' N.	4° 02' W.	
Joddah	Asia	Arabia	Indian Ocean	12° 05' S.	45° 45' E.	
			Red Sea	22° 00' N.	39° 27' E.	

E e

C. St. John

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
C. St. John	Am.	Newfoundl.	West. Ocean	50 25 N.	52 45 W.	6h. cm.
C. St. John	Africa	Maumbo	Eth. Ocean	1 17 N.	9 34 E.	
Fort St. John's	Am.	Newfoundl.	Atl. Ocean	47 39 N.	50 05 W.	
I. St. John { E. pt.	Am.	Canada	B. St. Lau-	46 36 N.	60 00 W.	
I. St. John { W. pt.			rence	47 35 N.	62 05 W.	
I. St. John de Nova	Africa	Madagascar	Indian Ocean	17 00 S.	44 02 E.	
St. John de Luz	Eu.	France	B. Biscay	43 10 N.	1 38 W.	3 30
Cape Jones	Am.	New Britain	Hudson's Bay	58 50 N.	79 00 W.	
Joppa	Asia	Syria	Levant	32 45 N.	36 00 E.	
Jones's Sound	Am.	Greenland	Baffin's Bay	71 07 N.	91 30 W.	
Ipswich	Eu.	England	Germ. Ocean	52 14 N.	1 00 E.	
Ispahan	Asia	Perfia	R. Zenderu	32 25 N.	52 55 E.	
I. Juan Fernandes	Am.	Chili	South Sea	34 10 S.	79 35 W.	4 45
Port. St. Julian	Am.	Patagonia	S. Atl. Ocean	48 51 S.	65 10 W.	
I. Ivica	Eu.	Spain	Medit. Sea	38 54 N.	1 15 E.	
K						
Kalmer	Eu.	Sweden	Baltic Sea	56 40 N.	17 25 E.	
Kambaya	Asia	India	Indian Ocean	23 36 N.	72 50 E.	
Kamchatka { Lower	Asia	Siberia	South Sea	56 11 N.	159 25 E.	
I. Karaginiskoy { Upper	Asia			54 48 N.	157 25 E.	
I. St. Katherine's	Am.	Brazil	South Sea	58 00 N.	162 10 E.	
Keco	Asia	Tonquin	Atl. Ocean	27 45 S.	47 58 W.	
Kegor	Eu.	Mulcovy	Indian Ocean	21 55 N.	106 10 E.	
R. Kennebeck	Am.	New Eng.	North Ocean	70 18 N.	74 00 E.	
Kentish Knock, a {	Eu.	England	Atl. Ocean	44 00 N.	69 45 W.	
land			Germ. Ocean	51 42 N.	1 45 E.	0 00
I. St. Kilda	Eu.	Scotland	West. Ocean	57 44 N.	8 18 W.	
I. Kilduin	Eu.	Lapland	North Ocean	69 30 N.	31 20 E.	7 30
Kinsale	Eu.	Ireland	Atl. Ocean	51 32 N.	1 20 W.	5 15
Klip	Eu.	Greenland	North Ocean	80 10 N.	12 22 E.	
R. Kola	Eu.	Lapland	North Ocean	69 20 N.	35 30 E.	
C. Kol	Eu.	Sweden	Sound	56 50 N.	13 13 E.	
Port Komol	Africa	Abyssinia	Red Sea	22 39 N.	36 17 E.	
Komero Isles	Africa	Zanguebar	Indian Ocean	10 48 S.	44 46 E.	
R. Kowimia	Asia	Siberia	North Ocean	13 10 S.	46 22 E.	
				70 40 N.	159 00 E.	
L						
Ladrone, or Marian {	Asia		South Ocean	21 00 N.	144 00 E.	
Isles				13 15 N.	42 55 E.	
C. Lagulias	Africa	Casraria	Indian Ocean	34 45 S.	21 40 E.	
Lancafter	Eu.	England	St. Geo. Ch.	54 42 N.	4 36 W.	
I. Lancerota	Africa	Canaries	Atl. Ocean	29 30 N.	12 20 W.	
Land's End	Eu.	England	St. Geo. Ch.	50 06 N.	6 00 W.	7 30
Langeneis	Eu.	Nova Zem.	North Ocean	74 40 N.	53 36 E.	
I. Lambay	Eu.	Ireland	Irish Sea	53 24 N.	7 30 W.	8 15
I. Lampadofa	Africa	Tunis	Medit. Sea	35 32 N.	12 45 E.	
B. St. Lazarus	Am.	Patagon	South Sea	48 42 S.	73 35 W.	
C. Ledo	Africa	Angola	Atl. Ocean	9 24 S.	12 55 E.	
Leith	Eu.	Scotland	Germ. Ocean	55 58 N.	2 59 W.	4 30
Leghorn	Eu.	Italy	Medit. Sea	43 33 N.	10 25 E.	
I. Lemnos	Asia	Natolia	Archipelago	40 02 N.	25 36 E.	
C. Lengus	Eu.	Turkey	Medit. Sea	40 44 N.	19 36 E.	
Leofiaff	Eu.	England	Germ. Ocean	52 38 N.	1 54 E.	9 45
Lepanto	Eu.	Turkey	Medit. Sea	38 20 N.	22 03 E.	
I. Lefow	Eu.	Denmark	Sound	57 05 N.	11 06 E.	
Leverpool	Eu.	England	Irish Sea	53 22 N.	2 30 W.	11 15
I. Lewes, N. pt.	Eu.	Scotland	West. Ocean	58 35 N.	6 37 W.	6 30
Liampo, or Ningpo	Asia	China	South Sea	30 00 N.	120 30 E.	
Lima	Am.	Peru	S. urh Sea	12 01 S.	76 45 W.	

Line

Names of Places.	Cont	Countries.	Coast.	Latitude.	Longitude.	H Water.
Lime	Eu.	England	Eng. Channel	50 45 N	0 15 W.	7h. om.
Limerick	Eu.	Ireland	R. Shannon	52 22 N.	9 48 W.	
L. Limosa	Eu.	Italy	Medit. Sea	36 08 N.	13 01 E.	
L. Lipari	Eu.	Italy	Medit. Sea	38 35 N.	15 31 E.	
L. Liqueo	Asia.	Japan	Medit. Sea	28 00 N.	127 30 E.	
L. Lisbon	Eu.	Portugal	R. Tagus	38 42 N.	8 52 W.	2 15
L. Libon Rock	Eu.	Portugal	Western Oc.	38 42 N.	9 25 W.	
L. Laiffa	Eu.	Dalmatia	Adriat. Sea	42 56 N.	18 32 E.	
Lizard	Eu.	England	Eng. Channel	49 57 N.	5 14 W.	7 30
Isles { S. W. end	Eu.	Norway	North Ocean	68 15 N.	10 20 E.	
Loft ut { N. E. end	Eu.	Norway	North Ocean	69 00 N.	12 00 E.	
R. Loire, Ent.	Eu.	France	B. Biscay	47 07 N.	2 05 W.	3 00
London	Eu.	England	R. Thames	51 32 N.	0 00	3 00
New London	Am.	New Engl.	West. Ocean	41 50 N.	72 14 W.	1 30
Londonderry	Eu.	Ireland	West. Ocean	55 01 N.	7 31 W.	
Long Ile	Am.	New Engl.	West. Ocean	41 00 N.	71 59 W.	3 00
L. Longo	Eu.	Dalmatia	Adriat. Sea	43 45 N.	17 58 E.	
Longland Head	Eu.	England	Germ. Ocean	51 47 N.	1 41 E.	10 30
Lookout Point	Eu.	Greenland	North Ocean	76 40 N.	16 25 E.	
C. Lopus	Africa	Loango	Atl. Ocean	0 47 S.	8 30 E.	
B. St. Louis	Am.	Louisiana	G. Mexico	28 50 N.	97 08 W.	
Louibourg	Am.	C. Breton	B. St. Lawr.	45 54 N.	59 50 W.	
Lubec	Eu.	Germany	Baltic Sea	54 00 N.	11 40 E.	
C. St. Lucar	Am.	California	South Sea	23 15 N.	109 42 W.	
R. Lucia	Africa	C. Caffers	Indian Ocean	27 52 S.	33 28 E.	
I. St. Lucia	Africa	C. de Verd	Atl. Ocean	17 25 N.	24 05 W.	
I. St. Lucia	Am.	Caribbe	Atl. Ocean	13 45 N.	61 00 W.	
N. E. point.	Asia.	Phil. Isles	South Sea	19 25 N.	121 45 E.	
C. Bajador	Eu.	Spain	Baltic Sea	55 42 N.	13 25 E.	5 15
Manila	Eu.	England	St. Geo. Ch	51 20 N.	4 04 W.	
S. W. point	Eu.	Ireland	W. Off. Ocean	52 24 N.	10 15 W.	6 00
E. point	Eu.	England	Germ. Ocean	52 55 N.	0 32 E.	
Lunden	Am.	Caribbe	Atl. Ocean	13 45 N.	61 00 W.	
Lundy	Eu.	England	South Sea	19 25 N.	121 45 E.	
Luppi's Head	Eu.	Ireland	W. Off. Ocean	52 24 N.	10 15 W.	
Lyun	Eu.	England	Germ. Ocean	52 55 N.	0 32 E.	
M	Am.	Caribbe	Atl. Ocean	13 45 N.	61 00 W.	
C. Mabou	Am.	Caribbe	Atl. Ocean	13 45 N.	61 00 W.	
Macao, or Makau	Asia.	China	South Sea	22 13 N.	113 51 E.	
Macassar	Asia.	I. Celebes	South Sea	4 40 S.	119 05 E.	
C. Michian	Eu.	Spain	B. Biscay	43 44 N.	3 05 W.	
C. St. Mary, S	Eu.	Spain	B. Biscay	25 24 S.	45 53 E.	
pt.	Eu.	Spain	B. Biscay	25 24 S.	45 53 E.	
B. St. Augustine	Am.	Florida	Atlantic Ocean	23 15 S.	44 25 E.	
Terra de Gada	Am.	Florida	Atlantic Ocean	19 36 S.	44 46 E.	
C. St. Andrew	Am.	Florida	Atlantic Ocean	15 46 S.	45 22 E.	
C. St. Sebastian	Am.	Florida	Atlantic Ocean	12 30 S.	49 13 E.	
C. de Ambrey	Am.	Florida	Atlantic Ocean	12 15 S.	50 15 E.	
N. pt.	Am.	Florida	Atlantic Ocean	12 15 S.	50 15 E.	
B d'Antongil	Am.	Madagascar	Indian Ocean	16 00 S.	49 40 E.	
Antavare	Am.	Madagascar	Indian Ocean	20 53 S.	47 48 E.	
Po. Dauphin	Am.	Madagascar	Indian Ocean	24 48 S.	48 06 E.	
deira { W. end	Am.	Madagascar	Indian Ocean	32 38 N.	17 06 W.	4
Madrid	Eu.	Spain	Atl. Ocean	32 25 N.	17 21 W.	12 0
Madura	Eu.	India	R. Manzanara	40 25 N.	03 39 W.	
R. Mues, mouth	Eu.	Dutchland	Germ. Ocean	52 06 N.	78 35 E.	
Sir. Le Maire	Am.	Patagonia	Atl. Ocean	55 00 S.	62 10 W.	1 30
Magadoca	Am.	Zangebar	Indian Ocean	2 53 N.	45 25 E.	
Sir. Ma. { E. ent.	Am.	Patagonia	Atl. Ocean	52 50 S.	72 35 W.	
ee llan { W. ent.	Am.	Patagonia	South Sea	52 36 S.	62 35 W.	

Names of Places.	Cont.	Countries.	Coast.	Latitude	Longitude.	H. Water.
Magislan	Asia	India	Malabar Coa.	12 10 N.	74 14 E.	
I. Maguana	Am.	Bahama I.	Atl. Ocean	22 36 N.	72 25 W.	
P. Mahon, Ile Minorca	Eu.	Spain	Medit. Sea	39 31 N.	3 53 E.	
Majorca, Ill. Majorca	Eu.	Spain	Medit. Sea	39 45 N.	2 25 E.	
C. Mala	Eu.	Turkey	Archipelago	37 20 N.	24 07 E.	
Malacca	Asia	India	Str. Malacca	2 12 N.	102 10 E.	
Malaga	Eu.	Spain	Medit. Sea	36 43 N.	4 02 W.	
Isles Mal- dive	Asia	India	Indian Ocean	7 20 N.	73 03 E.	
S. End.				0 20 S.	76 10 E.	
Maletroom Whirl- pool.	Eu.	Norway	West Ocean	68 08 N.	10 40 E.	
I. Malique	Asia	Maldiva I.	Indian Ocean	7 45 N.	72 40 E.	
St. Maloes	Eu.	France	Eng. Channel	48 39 N.	1 57 W.	6 h. com.
I. Malta	Eu.	Italy	Medit. Sea	35 54 N.	14 34 W.	
I. Man, W. end	Eu.	England	Irish Sea	53 45 N.	5 00 W.	9 00
Mangalor	Asia	India	Indian Ocean	13 02 N.	75 10 E.	
Manila	Asia	I. Luconia	South Sea	14 30 N.	120 25 W.	
I. Mansfield, N. pt.	Am.	New Britain	Hudson's Bay	62 38 N.	80 33 W.	
I. Manfia	Africa	Zanguebar	Indian Ocean	8 36 S.	40 40 E.	
I. Mardou	Eu.	Norway	Sound	58 14 N.	8 55 W.	
I. Margarita	Am.	Terra Firma	Atl. Ocean	11 15 N.	63 35 W.	
R. Maragnon	Am.	Brazil	Atl. Ocean	1 48 S.	44 17 W.	
Margate	Eu.	England	Eng. Channel	51 29 N.	1 10 W.	11 15
C. St. Maria	Eu.	Portugal	Atl. Ocean	36 45 N.	7 45 W.	
C. St. Maria, or Lucia	Eu.	Italy	Medit. Sea	40 04 N.	18 31 E.	
Marian, or N. lim.	Asia			21 00 N.	144 00 E.	
Ladrone Isles			South Ocean			
I. St. Marica	Eu.	Azores	Atl. Ocean	13 15 N.	142 55 E.	
St. Marics	Eu.	I. Scilly	Eng. Channel	37 00 N.	22 56 W.	
I. Marigallante	Am.	West Indies	Atl. Ocean	49 57 N.	6 45 W.	
I. Maritimo, Sicily	Eu.	Italy	Medit. Sea	16 00 N.	61 10 W.	
C. Marielo	Eu.	Turkey	Medit. Sea	38 04 N.	12 33 E.	
St. Martha	Am.	Terra Firma	Atl. Ocean	38 00 N.	26 00 E.	
I. St. Martin	Am.	West Indies	Atl. Ocean	11 26 N.	74 06 W.	
C. St. Martin	Africa	Caffers	Atl. Ocean	18 06 N.	62 30 W.	
C. St. Martin	Eu.	Spain	Atl. Ocean	32 08 S.	18 58 E.	
I. Martinique, Po.	Am.	West Indies	Medit. Sea	38 44 N.	0 25 E.	
Royal			Atl. Ocean	14 33 N.	60 54 W.	
Marfilles	Eu.	France	Medit. Sea	43 18 N.	5 27 E.	
C. St. Mary	Am.	Newfoundl.	Atl. Ocean	46 52 N.	52 15 W.	
C. St. Mary	Am.	Brazil	Atl. Ocean	34 52 S.	52 55 W.	
C. St. Mary	Asia	Natolia	Archipelago	37 46 N.	27 21 E.	
C. St. Mary	Eu.	Spain	N. Alt. Oce.	36 46 N.	7 49 W.	
C. Virgin Mary	Am.	Patagonia	S. Alt. Oce.	52 21 S.	67 29 W.	
I. Mascarenhas	Africa	Zanguebar	Indian Ocean	20 52 S.	55 35 E.	
I. Mascarin	Am.	Peru	South Ocean	1 20 S.	88 50 W.	
Mascat	Asia	Arabia	Indian Ocean	23 10 N.	57 40 E.	
Materland	Eu.	Sweden	Sound	57 58 N.	12 00 E.	
Mafulipatan	Asia	India	B. Bengal	16 28 N.	81 40 E.	
C. Matapan	Eu.	Turkey	Archipelago	36 25 N.	22 40 E.	
I. Matbare	Asia	Japan	South Ocean	26 30 N.	137 00 E.	
I. St. Matthew's	Africa	Guiney	Eth. Ocean	1 23 S.	6 11 W.	
I. Mauritius	Africa	Madagascar	Indian Ocean	20 10 S.	57 33 E.	
I. May	Africa	C. Verd	Atl. Ocean	15 19 N.	21 53 W.	
C. May	Am.	Penfilvania	Atl. Ocean	39 15 N.	74 43 W.	
I. Mayetta	Africa	Madagascar	Indian Ocean	12 53 S.	46 10 E.	
Mecca	Asia	Arabia	Red Sea	21 40 N.	41 00 E.	
Medina	Asia	Arabia	Red Sea	24 58 N.	39 53 E.	
I. Melada	Eu.	Dalmatia	Adriat. Sea	42 40 N.	19 34 E.	

Melinde

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Melinde	Africa	Zanguebar	Indian Ocean	3 07 S.	0 40 E.	
I. Melo	Eu.	Turkey	Archipelago	36 41 N.	25 05 E.	
Memel	Eu.	Courland	Baltic Sea	55 48 N.	22 23 E.	
Memiffan	Eu.	France	B. Biscay	44 20 N.	1 23 W.	3h. 30m.
I. Menado	Asia	I. Celebes	South Sea	1 36 N.	122 25 E.	
C. Mendozin	Am.	California	South Sea	41 20 N.	130 15 W.	
R. Metaparovus	Am.	Bahama	Atl. Ocean	21 58 N.	74 13 W.	
Metfina	Eu.	I. Sicily	Medit. Sea	38 21 N.	16 21 E.	
C. Mesurato	Africa	Tripoli	Medit. Sea	32 18 N.	16 36 E.	
I. Mety- lene	Asia	Natolia	Archipelago	39 21 N.	26 08 E.	
C. Sigre				39 11 N.	26 47 E.	
I. Meun	Eu.	Denmark	Baltic Sea	55 00 N.	13 15 E.	
Mexico	Am.	Mexico	Inland	20 00 N.	103 35 W.	
I. St. Michael	Eu.	Azores	Atl. Ocean	38 13 N.	22 50 W.	
Middlebourg	Eu.	Dutchland	Germ. Ocean	51 37 N.	3 58 E.	
Milford	Eu.	Wales	St. Geo. Ch.	51 45 N.	5 15 W.	
Milo, I. Milo	Asia	Turkey	Archipelago	36 41 N.	25 05 E.	
Milla's Isles	Am.	North Main	Hudson's Bay	64 36 N.	80 30 W.	
N. point				9 40 N.	124 25 E.	
S. E. pt. C. St.				6 40 N.	126 25 E.	
Augustine	Asia	Spice Islands	South Sea	7 00 N.	121 25 E.	
S. W. pt. Cal.				3 50 N.	124 43 E.	
S. E. pt. S. point				13 00 N.	119 37 E.	
I. Mindora	Asia	Philip. Isles	South Sea	39 58 N.	3 54 E.	
I. Mi- norca	Eu.	Spain	Mediterran.	40 24 N.	4 18 E.	
I. Miscon	Am.	Nova Scotia	G. St. Lawr.	47 45 N.	62 09 W.	
C. Miserata	Africa	Guinea	Atl. Ocean	6 25 N.	9 35 W.	
Missen Head	Eu.	Ireland	Atl. Ocean	51 16 N.	11 30 W.	
R. Mississippi, mouth	Am.	Louisiana	G. Mexico	29 00 N.	89 17 W.	
Mocha	Asia	Arabia	Red Sea	13 45 N.	44 04 E.	
Modon	Eu.	Turkey	Medit. Sea	36 55 N.	21 03 E.	
I. Mohilla	Africa	Zanguebar	Indian Ocean	11 55 S.	45 00 E.	
I. Monferat	Am.	West Indies	Atl. Ocean	16 37 N.	62 13 W.	
Mont Real	Am.	Canada	R. St. Lawr.	45 52 N.	73 11 W.	
I. Monte Christo	Eu.	Italy	Medit. Sea	42 17 N.	10 28 E.	
C. Monte Sancto	Eu.	Turkey	Archipelago	40 27 N.	24 39 E.	
Mount St. Michael	Eu.	France	Eng. Channel	48 39 N.	1 35 W.	
I. Margo	Asia	Natolia	Archipelago	36 55 N.	26 30 E.	
Morlaix	Eu.	France	Eng. Channel	48 30 N.	3 50 W.	
Mont Point	Eu.	England	St. Geo. Ch.	51 12 N.	4 40 W.	
Molambique	Africa	Zanguebar	Indian Ocean	15 00 S.	41 40 E.	
Moscow	Eu.	Russia	R. Moscow	55 45 N.	37 51 E.	
Molquitos Bank	Am.	Mexico	Atl. Ocean	14 45 N.	80 05 W.	
C. Mount	Africa	Guinea	Atl. Ocean	7 12 N.	10 44 W.	
Mount's Bay	Eu.	England	Eng. Channel	50 05 N.	5 45 W.	
Mouie River	Am.	New Wales	Hudson's Bay	51 25 N.	83 15 W.	
C. Muldon	Asia	Arabia	Perfian Gulf	26 04 N.	55 22 E.	
N						
C. Nabo	Asia	Japan	South Sea	40 35 N.	141 25 E.	
Nagafuck	Asia	Japan	South Sea	32 45 N.	128 50 E.	
Nankin	Asia	China	South Sea	32 07 N.	118 35 E.	
Nantes	Eu.	France	B. Biscay	47 13 N.	1 29 W.	
Nantuck Isle	Am.	New Eng.	West Ocean	41 34 N.	69 40 W.	
Naples	Eu.	Italy	Medit. Sea	40 51 N.	14 19 E.	
Narbonne	Eu.	France	Medit. Sea	43 11 N.	3 05 E.	
Narlinga	Asia	India	B. Bengal	18 05 N.	85 20 E.	
Narva	Eu.	Livonia	G. Finland	59 08 N.	29 18 E.	
I. Naffau	Asia	Sumatra	Indian Ocean	3 00 S.	100 25 E.	

C. Naffau

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
C. Naffau	Am.	Terra Firma	Atl. Ocean	7 53 N	58 07 W.	
Naffau Str.	Eu.	Russia	North Ocean	69 55 N.	57 30 E.	
B. Natal	Africa	Caffers	Indian Ocean	29 25 S.	33 10 E.	
I. Naxos	Eu.	Turkey	Archipelago	37 06 N.	25 58 E.	
Naze	Eu.	Norway	West Ocean	57 50 N.	7 32 E.	11h. 15m.
Nedules	Eu.	England	Eng. Channel	50 41 N.	1 48 W.	10 15
C. Negrailles	Afia	Pegu	B. Bengal	16 20 N.	94 15 E.	
C. Negro	Africa	Caffers	Atl. Ocean	16 30 S	11 30 E.	
C. Negro	Africa	Barbary	Medit. Sea	37 17 N.	9 09 E.	
Negropont	Eu.	Turkey	Archipelago	38 30 N.	24 05 E.	
Port Nelson	Am.	New Wales	Hudfon's Bay	57 07 N.	94 15 W.	
I. Nevis	Am.	Caribbee Isles	Atl. Ocean	57 35 N.	92 30 W.	8 20
Newcastle	Eu.	England	Germ. Ocean	16 55 N.	62 42 W.	
R. Nicaragua	Am.	New Spain	Atl. Ocean	55 03 N	1 28 W.	3 15
Nice	Eu.	Italy	Medit. Sea	11 40 N.	82 47 W.	
Is. Nicobar	Afia	Siam	B. Bengal	7 22 N.	94 40 E.	
J. St. Nicholas	Africa	C. Verd I.	Atl. Ocean	17 05 N.	23 28 W.	
Nicotera	Eu.	Italy	Medit. Sea	38 33 N.	16 30 E.	
Nieupont	Eu.	Flanders	Germ. Ocean	51 08 N	2 50 E.	12 00
Ninay	Afia	China	Sophr Sea	37 10 N	122 25 E.	
Ningpo, or Liampo	Afia	China	South Sea	30 00 N.	120 50 E.	
I. Nio	Eu.	Turkey	Archipelago	36 48 N.	26 02 E.	
I. Noel	Afia	India	Indian Ocean	10 30 S.	105 25 E.	
C. de Non	Africa	Barbary	Atl. Ocean	28 04 N.	10 32 W.	
Nombre de Dios	Am.	Terra Firma	Caribbee Sea	9 43 N.	78 35 W.	0 00
Nore	Eu.	England	R. Thames	51 28 N.	0 48 E.	
C. North	Am.	Terra Firma	Atl. Ocean	1 45 N.	49 00 W.	
N. Cape, I. Maggoroe	Eu.	Lapland	North Ocean	71 10 N.	25 55 E.	3 00
North Point	Eu.	Norway	North Ocean	62 15 N.	6 15 E.	
North-Bluff	Am.	North Main	Hudf. Straits	62 30 N.	70 59 W.	10 00
I. Nottingham, E. pt.	Am.	New Britain	Hudf. Straits	63 35 N.	77 48 W.	
O						
Oczakow	Eu.	Turkey	Black Sea	45 12 N.	34 40 E.	
I. Oeland	Eu.	Sweden	Baltic Sea	56 15 N.	18 35 E.	
Old head of Kingfale	Eu.	Ireland	Atl. Ocean	57 23 N.	17 36 E.	
I. Oleron	Eu.	France	B. Biscay	51 35 N.	8 58 W.	
Olinda	Am.	Brazil	S. Atl. Ocean	45 51 N.	1 25 W.	
Oliva	Eu.	Germany	Baltic Sea	8 13 S.	35 05 W.	
Olone	Eu.	France	B. Biscay	54 20 N.	18 30 E.	3 45
Oneglia	Eu.	Italy	Medit. Sea	46 32 N.	1 36 W.	
Oporto	Eu.	Portugal	Atl. Ocean	43 57 N.	7 52 E.	
Oran	Eu.	Barbary	Medit. Sea	40 53 N.	8 35 W.	
C. Orange	Am.	Terra Firma	Medit. Sea	35 45 N.	0 00 W.	
Orbitello	Eu.	Italy	Atl. Ocean	4 27 N.	50 50 W.	
I. Orchillo	Am.	Terra Firma	Medit. Sea	42 30 N.	12 00 E.	
Orfordness	Eu.	England	Caribb. Sea	11 32 N.	65 25 W.	9 45
Orkney Isles, limits	Eu.	Scotland	Germ. Ocean	52 17 N.	1 11 E.	
New Orleans	Eu.	Scotland	West. Ocean	59 24 N.	3 23 W.	3 00
I. Ormus	Am.	Louifiana	R. Mississippi	58 44 N.	2 11 W.	
C. del Oro, or Olerada	Afia	Perfia	G. Perfia	30 00 N.	90 00 W.	
R. Oronoque	Africa	Negroland	Atl. Ocean	27 30 N.	55 17 E.	
C. Oropefo	Am.	Terra Firma	Atl. Ocean	23 30 N.	14 31 W.	
C. Ortegai	Eu.	Spain	Atl. Ocean	8 08 N.	59 50 W.	
Ortona	Eu.	Spain	Medit. Sea	40 20 N.	0 49 E.	
I. Oruba	Eu.	Spain	B. Biscay	44 07 N.	8 00 W.	
Ofsend	Eu.	Italy	Medit. Sea	42 19 N.	14 37 E.	
C. Otranto	Am.	Terra Firma	Medit. Sea	12 03 N.	69 03 W.	
Ozaca	Eu.	Flanders	Caribbean Sea	51 14 N.	3 00 E.	12 00
	Eu.	Italy	Germ. Ocean	40 23 N.	17 41 E.	
	Afia	Japan	Medit. Sea	5 10 N.	124 05 E.	

C. Padron

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
C. Padon	Africa	Congo	Atl. Ocean	6° 00' S.	11° 40' E.	
Paia	Am.	Peru	South Sea	5° 20' S.	80° 35' W.	
C. Pailouri	Eu.	Turky	Archipelago	39° 59' N.	24° 03' E.	
Palermo, I. Sicily	Eu.	Italy	Medit. Sea	38° 10' N.	13° 43' E.	
Paliakate	Afia	India	B. Bengal	13° 40' N.	80° 50' E.	
I. Palma	Africa	Canaries	Atl. Ocean	28° 35' N.	17° 30' W.	
I. Palmaria	Eu.	Italy	Medit. Sea	41° 00' N.	13° 03' E.	
C. Palmaria	Afia	India	B. Bengal	20° 40' N.	87° 35' E.	
C. Palmas	Africa	Guiney	Atl. Ocean	4° 26' N.	5° 56' W.	
Panama	Am.	Mexico	South Ocean	8° 45' N.	79° 55' W.	
I. Panaria	Eu.	Italy	Medit. Sea	38° 40' N.	15° 41' E.	
Panorma	Eu.	Turky	Medit. Sea	40° 05' N.	21° 40' E.	
I. Pantalaria	Eu.	Italy	Medit. Sea	36° 55' N.	12° 31' E.	
R. Panuco	Am.	Mexico	G. Mexico	24° 02' N.	100° 13' W.	
R. Pariba	Am.	Br. fil.	Atl. Ocean	21° 26' S.	39° 50' W.	
Paris	Eu.	France	R. Seine	48° 50' N.	2° 25' E.	
C. Passero	Eu.	I. Sicily	Medit. Sea	36° 35' N.	15° 22' E.	
C. Patam	Afia	Malacca	Indian Ocean	7° 27' N.	101° 20' E.	
I. Patmos	Afia	Natolia	Archipelago	37° 22' N.	26° 48' E.	
R. Patahan	Afia	I. Sumatra	Sit. Malacca	0° 28' N.	102° 25' E.	
C. Paul	Eu.	Spain	Medit. Sea	37° 50' N.	0° 15' W.	
I. St. Paul	Am.	Newfoundl.	B. St. Lawr.	47° 15' N.	59° 57' W.	
I. St. Paul	Afia	France	Indian Ocean	38° 20' S.	75° 25' E.	
St. Paul de Leon	Eu.	Californi	Eng. Channel	48° 41' N.	3° 55' W.	
I. Paxeros	Am.	West Indies	Pac. Ocean	30° 18' N.	120° 45' W.	
I. Pearl, or Serana	Afia	India	B. Bengal	14° 55' N.	79° 00' W.	
Pekin	Afia	China	Inland	17° 00' N.	96° 58' E.	
I. Pelugofa	Eu.	Italy	Adriat. Sea	39° 54' N.	116° 28' E.	
I. Pemba	Africa	Zanguebar	Indian Ocean	42° 20' N.	18° 32' E.	
C. Pembroke	Am.	New Wales	Hudson's Bay	5° 38' S.	40° 09' W.	
I. Penguin	Am.	Newfoundl.	Atl. Ocean	63° 12' N.	82° 54' W.	
Penmark	Eu.	France	B. Biscay	50° 03' N.	50° 32' W.	
R. Penobscot	Am.	New Eng.	Atl. Ocean	47° 50' N.	4° 20' W.	
I. Pepsy, or Penguin	Am.	Paragonia	S. Atl. Ocean	44° 40' N.	68° 52' W.	
Pernambuco	Am.	Br. fil.	S. Atl. Ocean	48° 00' S.	64° 50' W.	
Petapoli	Afia	India	B. Bengal	8° 30' S.	35° 07' E.	
Petribourg	Eu.	Russia	Baltic Sea	16° 14' N.	81° 10' E.	
C. Petra	Afia	Natolia	Archipelago	59° 56' N.	30° 25' E.	
Peverel Point	Eu.	England	Eng. Channel	37° 02' N.	27° 38' W.	
Philadelphia	Am.	Pennsylvania	R. Delaware	50° 34' N.	1° 22' W.	
St. Philip	Africa	Benguela	Atl. Ocean	40° 50' N.	74° 00' E.	
I. Pianofa	Eu.	Italy	Medit. Sea	12° 22' S.	13° 20' E.	
I. Pico	Eu.	Azores	Atl. Ocean	42° 46' N.	10° 34' W.	
C. Pinas	Eu.	Spain	B. Biscay	38° 35' N.	27° 5' W.	
Mo. Pintados, or } St. Martin	Am.	Californi	South Ocean	43° 51' N.	6° 14' W.	
Piccadore Isles	Afia	China	South Sea	27° 30' N.	117° 15' W.	
Pirentia	Am.	Newfoundl.	Atl. Ocean	23° 30' N.	119° 25' E.	
R. Plata	Am.	I. La Plata	Atl. Ocean	47° 36' N.	51° 46' W.	9 00
R. Platewrack	Am.	Bahama Isles	Atl. Ocean	36° 00' S.	57° 40' W.	
Plymouth	Eu.	England	Eng. Channel	20° 04' N.	68° 37' W.	6 00
Policastro	Eu.	Italy	Medit. Sea	50° 26' N.	4° 27' W.	
I. Poliapate	Afia	Cambaya	Indian Ocean	40° 18' N.	15° 45' E.	
I. Poma	Eu.	Dalmatia	Adriat. Sea	9° 45' N.	109° 55' E.	
Pondicherry	Afia	India	B. Bengal	42° 57' N.	18° 14' E.	
Pontorion	Eu.	France	Eng. Channel	11° 56' N.	79° 53' E.	
I. Ponza	Eu.	Italy	Medit. Sea	48° 33' N.	1° 27' W.	
Pool	Eu.	England	Medit. Sea	40° 53' N.	13° 09' E.	
Porta Port	Eu.	Portugal	Atl. Ocean	51° 00' N.	1° 50' W.	
Port Mahon	Eu.	Spain	Medit. Sea	40° 53' N.	8° 35' E.	
Portland	Eu.	England	Eng. Channel	39° 51' N.	3° 53' E.	
				50° 30' N.	2° 48' W.	8 15

Port

Padon

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Port l'Orient	Eu.	France	B. Biscay	47° 45' N.	3° 18' W.	
Porto Bello	Am.	New Spain	Carrib. Sea	9° 33' N.	79° 45' W.	
Porto Rico	Am.	Antilles	Atl. Ocean	18° 30' N.	65° 35' W.	
Porto Rico	Am.	Antilles	Atl. Ocean	18° 25' N.	66° 05' W.	
Porto Rico	Am.	Antilles	Atl. Ocean	18° 30' N.	67° 15' W.	
Porto Sancho	Am.	Antilles	Atl. Ocean	18° 30' N.	16° 32' W.	
Portfall	Eu.	Canaries	Atl. Ocean	33° 07' N.	4° 43' W.	
Praken	Eu.	France	Eng. Channel	48° 36' N.	0° 46' W.	11h. 15m.
Prenau	Eu.	England	Eng. Channel	50° 49' N.	0° 46' W.	
C. Prior	Eu.	Coch. Chi.	Indian Ocean	17° 15' N.	106° 15' E.	
I. Providence	Eu.	Livonia	Baltic Sea	58° 26' N.	24° 58' E.	
I. Providence, or	Eu.	Guinea	Atl. Ocean	1° 47' N.	6° 39' E.	
St. Catherine	Am.	Spain	Atl. Ocean	43° 53' N.	8° 22' W.	
I. Pulí condor	Am.	Bahama	Atl. Ocean	24° 51' N.	77° 01' W.	
	Am.	Mexico	Atl. Ocean	13° 26' N.	80° 42' W.	
	Am.	Cambaya	Indian Ocean	8° 34' N.	107° 35' E.	
Quaqua, or Ivory Coast	Africa	Guinea	Eth. Sea	5° 00' N.	4° 00' W.	
Quebeck	Am.	Canada	R. St. Lawr.	46° 55' N.	69° 48' W.	7 30
Queda	Am.	Malaya	B. Bengal	6° 15' N.	100° 12' E.	
Quelpert	Am.	Korea	South Sea	33° 32' N.	128° 04' E.	
Quiloa	Africa	Zanguebar	Indian Ocean	9° 30' S.	39° 09' E.	
Quimper	Eu.	France	B. Biscay	48° 00' N.	4° 06' W.	
Quinam	Am.	Coch. Chi.	Indian Ocean	12° 52' N.	109° 10' E.	
Quiraba Isles	Africa	Zanguebar	Indian Ocean	11° 00' N.	41° 39' E.	
Quito	Am.	Peru	Inland	0° 13' S.	77° 50' W.	
R						
C. Race	Am.	Newfoundl.	Atl. Ocean	46° 50' N.	50° 50' W.	
Ragufa	Eu.	Dalmatia	Medit. Sea	42° 45' N.	20° 00' E.	
Rajapour	Am.	India	Indian Ocean	17° 19' N.	73° 50' E.	
Ramsgate	Eu.	England	Downs	51° 20' N.	1° 22' E.	
Ramhead	Eu.	England	Eng. Channel	50° 14' N.	4° 35' W.	
C. Ralsgate	Am.	Arabia	Indian Ocean	22° 46' N.	58° 48' E.	
Ravenna	Eu.	Italy	Medit. Sea	44° 26' N.	12° 21' E.	
C. Ray	Am.	Newfoundl.	Atl. Ocean	47° 45' N.	59° 4' W.	
I. Rec	Eu.	France	B. Biscay	46° 08' N.	1° 28' W.	3 00
Regio	Eu.	Italy	Mediterran.	38° 22' N.	16° 37' E.	
Cape Revolution	Am.	N—Main	Hudf. Straits	61° 29' N.	65° 10' W.	
Isles Revolution, E. } limit	Am.	N—Main	Hudf. Straits	61° 36' N.	64° 38' W.	
Revel	Eu.	Livonia	Baltic Sea	59° 22' N.	25° 33' E.	
Rhodes, N. } end	Am.	Natolia	Archipelago	36° 27' N.	28° 36' E.	
C. Tranquil, } S. end	Am.	Natolia	Archipelago	35° 55' N.	28° 23' E.	
Riga	Eu.	Livonia	Baltic Sea	56° 55' N.	24° 51' E.	
Ripraps, a sand	Eu.	England	Straits Dover	51° 53' N.	1° 25' E.	
Robin Hood's Bay	Eu.	England	Germ. Ocean	54° 25' N.	0° 08' W.	3 00
L. Roeca	Am.	Terra Firma	Alt. Ocean	11° 21' N.	66° 17' W.	4 15
Rochefort	Eu.	France	B. Biscay	46° 00' N.	1° 00' W.	3 45
Rochel	Eu.	France	B. Biscay	46° 10' N.	1° 11' W.	3 45
Rochester	Eu.	England	R. Medway	51° 26' N.	0° 30' E.	0 45
I. Rodrigue	Am.	Madagascar	Indian Ocean	19° 41' S.	63° 45' E.	
C. Romain	Am.	Terra Firma	Alt. Ocean	11° 40' N.	69° 05' W.	
Rome	Eu.	Italy	Medit. Sea	41° 54' N.	12° 45' E.	
I. Roncadore	Am.	Mexico	Alt. Ocean	13° 30' N.	78° 53' W.	
Rood Bay	Eu.	Greenland	North Ocean	79° 53' N.	14° 00' E.	
C. Roque	Am.	Brazil	Alt. Ocean	5° 00' S.	35° 43' W.	
I. Requepiz	Africa	Madagascar	Indian Ocean	9° 51' S.	64° 30' E.	

G. Rofet

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
G. Rofes	Eu.	Spain	Medit. Sea	42 10 N.	0 3 18 E.	
Rofock	Eu.	Germany	Baltic Sea	54 10 N.	12 50 E.	
I. Rotterdam	Asia	—	South Sea	20 05 S.	179 30 W.	
Rouen	Eu.	Dutchland	Germ. Ocean	51 56 N.	4 55 E.	3h.oom.
C. Rozant	Eu.	France	R. Seine	49 27 N.	1 10 E.	1 15
C. Roxo	Eu.	Portugal	Atl. Ocean	38 42 N.	9 35 W.	
Po. Royal	Africa	Negroland	Atl. Ocean	11 42 N.	14 33 W.	
C. Rôzier	Am.	I. Jamaica	Caribbean Sea	17 40 N.	76 37 W.	
I. Rugen	Am.	Nova Scotia	G. St. Lawr.	50 00 N.	63 45 W.	
I. Rum Key, or Samana	Eu.	Germany	Baltic Sea	54 32 N.	14 30 E.	
R. Rupert	Am.	Bahama	Atl. Ocean	23 00 N.	74 20 W.	
C. Rufato	Am.	New Britain	Hudfon's Bay	51 45 N.	78 40 W.	
Ruff Isles	Africa	Barca	Medit. Sea	32 53 N.	20 41 E.	
Rye	Eu.	Norway	North Sea	67 40 N.	10 25 E.	
S	Eu.	England	Eng. Channel	51 03 N.	0 45 E.	11 15
C. Sable	Am.	Nova Scotia	Atl. Ocean.	43 56 N.	66 25 W.	
I. Sable, W. end	Am.	Nova Scotia	Atl. Ocean	44 08 N.	59 00 W.	
I. Saddle back	Am.	North Main	Hudf. Straits	62 7 N.	68 13 W.	10 00
Saffia	Africa	Barbary	Atl. Ocean	32 30 N.	8 50 W.	
I. Safanjal bahre.	Africa	Egypt	Red Sea	27 05 N.	34 40 E.	
B. Saldanna	Africa	Caffers	Atl. Ocean	32 35 S.	19 30 E.	
I. Sal	Africa	C. Verd	Atl. Ocean	16 55 N.	21 41 W.	
Salerno	Eu.	Italy	Medit. Sea	40 39 N.	14 48 E.	
I. Salini, Lipari Is.	Eu.	Italy	Medit. Sea	38 39 N.	15 24 E.	
I. Salibuary	Am.	N.—Main	Hudfon's Bay	63 29 N.	76 47 W.	
Saltee	Africa	Barbary	Atl. Ocean	33 58 N.	6 20 W.	
Solomon Isles	Asia	—	South Sea	5 50 S.	171 05 W.	
Salonechi	Eu.	Turky	Archipelago	11 15 S.	178 35 W.	
I. Salvages	Africa	Canaries	N. Alt. Ocean	40 41 N.	23 13 E.	
I. Salvages { Upper } Lower }	Am.	North Main	Hudf. Straits	30 14 N.	14 59 W.	9 00
I. Samos	Asia	Natolia	Archipelago	62 32 N.	70 48 W.	11 10
C. Sambrough	Am.	Nova Scotia	West Ocean	37 46 N.	27 13 E.	
Sandwich	Eu.	England	Downs	44 31 N.	63 25 W.	
I. Sanguin	Asia	Philipp. Isles	South Sea	51 20 N.	120 E.	11 30
I. Sanien	Eu.	Norway	North Ocean	3 50 N.	122 30 E.	
Santa Cruz	Africa	Barbary	Atl. Ocean	69 30 N.	14 30 E.	
N. limit	Eu.	Italy	Medit. Sea	30 30 N.	9 35 W.	
S. pt. C. Tavoraro	Eu.	Italy	Medit. Sea	41 15 N.	9 31 E.	
Cagliari	Eu.	Italy	Medit. Sea	38 54 N.	9 15 E.	
Oristagni	Eu.	Italy	Medit. Sea	39 25 N.	9 38 E.	
Sarena	Am.	Chili	South Sea	39 53 N.	9 01 E.	
Scanderon	Asia	Syria	Levant	29 40 S.	71 15 W.	
Scarborough head	Eu.	England	Germ. Ocean	36 35 N.	36 25 E.	
I. Scarpanto	Asia	Natolia	Archipelago	54 18 N.	00 00 E.	3 45
I. Scararie, N. E. pt.	Am.	Acadia	West Ocean	35 45 N.	27 40 E.	
Scaw	Eu.	Denmark	Sound	46 08 N.	59 27 W.	
I. Schelling	Eu.	Dutchland	Germ. Ocean	57 34 N.	10 54 E.	
C. St. Nicholas	Asia	Natolia	Archipelago	53 27 N.	5 30 E.	
Scio	Eu.	England	St. Geo. Ch.	38 38 N.	26 12 E.	
C. Blanco	Eu.	Terra Firma	Caribbean Sea	38 24 N.	26 29 E.	
Seilly Isles	Am.	Turky	Archipelago	38 08 N.	26 20 E.	
Scots Settlement	Eu.	France	B. Biscay	50 00 N.	6 45 W.	3 45
I. Sea	Eu.	Turky	Archipelago	8 45 N.	76 35 W.	
Sermes	Eu.	France	B. Biscay	37 38 N.	24 53 E.	
I. Sebaldes	Am.	Patagonia	S. Alt. Ocean	48 00 N.	4 51 W.	
C. Sebastian	Am.	California	South Sea	50 53 S.	59 35 W.	
C. St. Sebastian	Africa	Madagascar	Indian Ocean	43 00 N.	126 00 W.	
				12 30 S.	49 13 E.	

F f

St. Sebastian

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
St. Sebastian	Eu.	Spain	B. Biscay	43 16 N.	2 05 W.	
Port Segura	Am.	Brazil	Atl. Ocean	16 57 S.	39 45 W.	
R. Senegal	Africa	Negroland	Atl. Ocean	15 40 N.	16 20 W.	roh. 3om.
I. Seranilha	Am.	West Indies	Atl. Ocean	16 20 N.	79 40 W.	
I. Sepigo	Eu.	Turky	Archipelago	36 09 N.	23 24 E.	
I. Sertes	Africa	Canaries	Atl. Ocean	32 35 N.	16 20 W.	
R. Sefios	Africa	Guinea	Atl. Ocean	5 48 N.	8 13 W.	
Seven Capes	Africa	Barbary	Medit. Sea	37 30 N.	6 15 W.	
Seven Stones, or Isles	Eu.	England	St. Geo. Ch.	50 10 N.	6 40 W.	4 30
R. Severn, Ent.	Eu.	England	St. Geo. Ch.	51 41 N.	3 05 W.	6 0
R. Severn	Am.	New Wales	Hudson's Bay	56 12 N.	88 57 W.	
R. Seyn, Ent.	Eu.	France	Eng. Channel	49 36 N.	0 30 E.	9 0
Seyn Head	Eu.	France	Eng. Channel	49 44 N.	0 34 E.	
Sheernefs	Eu.	England	R. Thames	51 25 N.	0 50 E.	0 00
Siam	Asia	India	Bay Siam	14 18 N.	100 55 E.	
R. Siam, Ent.	Asia	India	Bay Siam	13 15 N.	100 47 E.	
Siara	Am.	Brazil	Atl. Ocean	3 18 S.	39 50 W.	
E. end, Messina				38 10 N.	15 58 E.	
Catanea				37 22 N.	15 21 E.	
Syracufe				37 04 N.	15 31 E.	
S. end, C. Paf. faro	Eu.	Italy	Medit. Sea	36 35 N.	15 22 E.	
Alicata				37 11 N.	14 07 E.	
W. end, C. Boco				37 51 N.	21 43 E.	
Palermo				38 10 N.	13 43 W.	8 15
Sierra Leona	Africa	Guinea	Atl. Ocean	8 30 N.	12 07 E.	
Sillabar Road	Asia	I. Sumatra	Indian Ocean	4 00 S.	102 50 E.	
Str. Sincapore	Asia	Malacca	Indian Ocean	1 00 N.	104 30 E.	
R. Sinda, or Indus, } mouth	Asia	India	Indian Ocean	{ 24 30 N. 25 45 N. 18 58 N.	{ 63 10 E. 62 40 E. 38 24 E.	
Po. Shabak	Africa	Abyfinia	Red Sea	64 05 N.	82 12 W.	
Shark, or Seahorse } point	Am.	New Wales	Hudson's Bay	55 02 N.	1 20 W.	
Shields	Eu.	England	Germ. Ocean	23 15 N.	117 35 W.	
Shellock's Ile	Am.	California	South Sea	51 30 N.	1 55 W.	5 0
Shillocks	Eu.	Ireland	West. Ocean	{ 60 47 N. 59 54 N.	{ 0 10 W. 1 31 W.	3 00
I. Sherland, limits	Eu.	Scotland	West. Ocean	50 55 N.	6 30 W.	10 30
Shoreham	Eu.	England	Eng. Channel	57 50 N.	6 16 W.	5 30
I. Sky { N. pt. S. pt.	Eu.	Scotland	West. Ocean	{ 57 15 N. 60 00 N.	{ 81 30 W. 82 00 W.	
Sleepers Isles	Am.	New Britain	Hudson's Bay	58 35 N.	82 00 W.	
Great Sleeper				60 10 N.		
The Sleepers lay in a chain from the Great Sleeper down to Lat. 58° 50' N. & Long. 82° 20' W.						
Slime Head						
R. Slude	Eu.	Ireland	West. Ocean	53 20 N.	11 15 W.	
Sluyce	Am.	New Britain	Hudson's Bay	53 24 N.	78 50 W.	
C. Smith	Eu.	Dutchland	Germ. Ocean	51 19 N.	3 50 E.	
Smyrna	Am.	Labradore	Hudson's Bay	60 48 N.	80 55 W.	
I. Socatora	Asia	Natalia	Archipelago	38 28 N.	27 25 E.	
C. Spolomon	Africa	Anian	Indian Ocean	12 15 N.	52 55 E.	
R. Somme	Eu.	I. Candia	Medit. Sea	34 57 N.	27 06 E.	11 00
Soud Royal	Eu.	France	Eng. Channel	50 18 N.	1 40 E.	
Southampton	Eu.	Iceland	North Ocean	66 22 N.	15 15 W.	0 00
C. Southampton	Eu.	England	Eng. Channel	50 55 N.	1 00 W.	
South Cape	Am.	New Wales	Hudson's Bay	61 54 N.	86 14 W.	
C. Spartivento	Asia	Diemen's Ind	South Ocean	42 40 S.	130 05 E.	
	Eu.	Italy	Medit. Sea	37 50 N.	16 41 E.	

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
C. Sparte	Africa	Barbary	Atl. Ocean	35 42 N.	5 47 W.	
I. Spigie	Afia	New Zeld.	South Ocean	41 45 S.	162 50 E.	
I. Spirito Santo	Am.	Brasil	Atl. Ocean	20 24 S.	39 55 W.	5h. 15m.
Spurn	Eu.	England	Germ. Ocean	53 35 N.	0 30 E.	
I. Stampalia	Afia	Natolia	Archipelago	36 25 N.	26 55 E.	
I. Stancho	Afia	Natolia	Archipelago	36 50 N.	27 30 E.	
Start pt.	Eu.	England	Eng. Channel	50 09 N.	3 45 W.	6 45
C. St. John	Am.	Patagonia	Atl. Ocean	54 45 S.	60 35 W.	
C. St. Bartho-	Am.	Patagonia	Atl. Ocean	55 08 S.	60 45 W.	
lomew	Am.	Patagonia	Atl. Ocean	55 08 S.	60 45 W.	
Stavenger	Eu.	Norway	West. Ocean	58 47 N.	6 45 E.	
Setin	Eu.	Germany	Baltic Sea	53 36 N.	15 25 E.	
C. Stillo	Eu.	Italy	Medit. Sea	38 23 N.	17 07 E.	
Port Steven	Am.	Chili	South Sea	46 50 S.	82 36 W.	
Stockholm	Eu.	Sweden	Baltic Sea	59 20 N.	18 08 E.	
Stockton	Eu.	England	Germ. Ocean	54 33 N.	1 15 W.	5 15
Strallund	Eu.	Germany	Baltic Sea	54 23 N.	14 10 E.	
Strangford Bay	Eu.	Ireland	Irish Sea	54 23 N.	5 40 W.	10 30
I. Stromboli	Eu.	Italy	Medit. Sea	38 42 N.	15 48 E.	
Suez Town	Africa	Egypt	Red Sea	29 50 N.	33 27 E.	
Sukadana	Afia	I. Borneo	Indian Ocean	1 00 S.	110 40 E.	
I. Suma- { NW. end	Afia	India	Indian Ocean	5 15 N.	105 55 E.	
tra. { SE. end	Afia	India	Indian Ocean	5 07 S.	106 20 E.	3 30
Sunderland	Eu.	England	Germ. Ocean	54 55 N.	1 00 W.	
Str. Sundia	Afia	Siam	Indian Ocean	6 10 S.	105 35 E.	
Sutrinan	Am.	Terra Firma	Atl. Ocean	6 30 N.	55 30 W.	
Surat	Afia	India	Indian Ocean	21 10 N.	72 25 E.	
I. Surroy	Eu.	Lapland	North Ocean	71 00 N.	42 00 E.	
Swaken	Africa	Abysinia	Red Sea	19 30 N.	37 38 E.	
Swally Road	Afia.	India	Arabian Sea	21 55 N.	72 00 E.	
Swanicy	Eu.	Wales	St. Geo. Ch.	51 40 N.	4 25 W.	
Sweetnoise	Eu.	Lapland	North Ocean	68 08 N.	34 42 E.	
Swin, a sand	Eu.	England	Ent. Thames	51 37 N.	1 12 E.	12 00
Syracuse	Eu.	I. Sicily	Medit. Sea	37 04 N.	15 20 E.	
Syriam	Afia.	Pegu	B. Bengal	16 00 N.	96 40 E.	
Tadoufac Fort	Am.	Canada	R. St. Lawr.	48 00 N.	67 35 W.	
I. Tamarica	Am.	Brasil	Atl. Ocean	7 56 S.	35 05 W.	
Tamarin Town	Africa	I. Sokotra	Indian Ocean	12 30 N.	53 14 E.	9 00
B. Tanassarim	Afia	Malacca	B. Bengal	12 00 N.	98 40 E.	
I. Tandoxima	Afia	Japan	South Sea	30 30 N.	130 40 E.	
Tanger	Africa	Barbary	Atl. Ocean	35 55 N.	5 45 W.	
Tarento	Eu.	Italy	Medit. Sea	40 43 N.	17 31 E.	
C. Tatnam	Am.	New Wales	Hudson's Bay	57 35 N.	91 30 W.	3 00
R. Tees, mouth	Eu.	England	Germ. Ocean	54 36 N.	0 52 W.	
Tegoantepec	Am.	Mexico	South Sea	14 45 N.	96 23 E.	
Tellichery	Afia	India	Malabar Coast	11 42 N.	75 30 W.	
C. Telling	Eu.	Ireland	West. Ocean	54 40 N.	10 07 W.	3 00
I. Tenedos	Afia	Natolia	Archipelago	39 57 N.	26 14 E.	
I. Teneriff	Africa	Canaries	Atl. Ocean	28 13 N.	16 27 W.	
C. Tenes	Africa	Barbary	Medit. Sea	36 26 N.	1 53 E.	
I. Tercera	Eu.	Azores	Atl. Ocean	39 00 N.	24 52 W.	
Terra Nueva	Am.	N—Main	Hudf. Straits	62 4 N.	67 2 W.	9 50
Terrere	Eu.	Dutchland	Germ. Ocean	51 38 N.	3 35 W.	0 45
Tetuan	Africa	Barbary	Medit. Sea	35 27 N.	4 50 W.	
I. Texel	Eu.	Dutchland	Germ. Ocean	53 10 N.	4 59 E.	7 30
C. St. Thadzeus	Afia	Siberia	North Ocean	62 10 N.	175 05 E.	1 30
R. Thames, mouth	Eu.	England	Germ. Ocean	51 28 N.	1 10 E.	
C. St. Thomas	Africa	Caffers	Atl. Ocean	24 54 S.	15 25 E.	
I. St. Thomas	Africa	Guinea	Atl. Ocean	00 00	1 00 E.	
St. Thomas	Afia	India	B. Bengal	13 00 N.	80 00 E.	
C. Three Points	Am.	Terra Firma	Atl. Ocean	10 51 N.	62 41 W.	

F f 2

C. Three

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
C. Three Points	Africa	Guinea	Atl. Ocean	4° 48' N	1° 21' W.	
I. Tidore	Asia	Malacca Is.	Indian Ocean	0° 33' N.	126° 40' E.	
I. Timor	Asia	Molucca Is.	Indian Ocean	8° 20' S.	127° 40' E.	
Timmouth	Eu.	England	Germ. Ocean	10° 10' S.	123° 55' E.	3h.com.
I. Tino	Eu.	Turkey	Archipelago	55° 03' N	1° 17' W.	
I. Tobago	Am.	Caribbe	Atl. Ocean	37° 33' N.	25° 43' E.	
B. Todos Santos	Am.	Brazil	Atl. Ocean	11° 15' N.	60° 27' W.	
Tonquin	Asia	India	South Sea	13° 05' S.	38° 45' W.	
Tonberg	Eu.	Norway	Sound	20° 50' N.	105° 55' E.	
Topham	Eu.	England	Eng. Channel	58° 50' N	10° 05' E.	
Torbay	Eu.	England	Eng. Channel	50° 37' N.	3° 27' W.	6 00
Tornea	Eu.	Sweden	G. Bothnia	50° 34' N.	3° 38' W.	5 15
R. Tortosa	Eu.	Spain	Medit. Sea	65° 51' N	23° 18' E.	
I. Tortuga	Am.	Terra Firma	Atl. Ocean	40° 47' N	1° 03' E.	
I. Tory	Eu.	Ireland	West. Ocean	11° 00' N.	64° 23' W.	5 30
Toulon	Eu.	France	Medit. Sea	55° 09' N.	8° 30' E.	
C. Trefalgar	Eu.	Spain	Atl. Ocean	43° 07' N.	6° 02' E.	
I. Tremiti	Eu.	Italy	Medit. Sea	36° 06' N.	5° 38' W.	
C. de Tres forcas	Eu.	Barbary	Medit. Sea	42° 09' N.	15° 40' E.	
I. Traillies	Africa	India	Medit. Sea	35° 30' N.	2° 11' W.	
I. Trinity	Asia	Brazil	Indian Ocean	19° 30' S.	101° 25' E.	
I. Trinidad, E. pt.	Am.	Terra Firma	Atl. Ocean	20° 25' S.	23° 35' W.	
Trinity Bay, Ent.	Am.	Newfoundl.	Atl. Ocean	10° 38' N.	60° 27' W.	
Trist	Eu.	Corniola	Adriat. Sea	48° 55' N.	50° 20' W.	
Trinquemali	Asia	I. Ceylon	Indian Ocean	45° 51' N.	14° 03' E.	
Tripoli	Africa	Syria	Levant	8° 50' N.	83° 24' E.	
G. Triffe	Africa	Barbary	Medit. Sea	34° 53' N.	36° 07' E.	
I. Trifian d'Acunha	Am.	Terra Firma	Atl. Ocean	32° 54' N.	13° 10' E.	
Is. Tromfound	Eu.	Caffres	S. Atl. Ocean	10° 19' N.	67° 41' W.	
Truxilla	Eu.	Lapland	North Ocean	37° 12' S.	13° 23' W.	
Tunder	Am.	Peru	South Sea	70° 20' N.	19° 00' E.	
Tu-is	Africa	Denmark	West. Ocean	8° 00' S.	78° 35' E.	
Turin	Eu.	Barbary	Medit. Sea	55° 00' N.	9° 35' E.	
I. Turks	Am.	Italy	R. Po	36° 47' N.	10° 16' E.	
		Bahama	Atl. Ocean	45° 05' N.	7° 45' E.	
				21° 45' N.	70° 30' W.	
V						
Valencia	Eu.	Spain	Medit. Sea	39° 30' N	0° 40' W.	
St. Valery	Eu.	France	Eng. Channel	50° 10' N.	0° 56' E.	10 30
Valona	Eu.	Turkey	Medit. Sea	40° 55' N.	21° 15' E.	
Valpariso	Am.	Chili	South Ocean	33° 00' S.	72° 14' W.	
Vannes	Eu.	France	B. Biscay	47° 39' N.	2° 41' W.	3 45
C. Vela	Am.	Terra Firma	Atl. Ocean	47° 39' N.	2° 41' W.	
Venice	Eu.	Italy	Medit. Sea	12° 15' N.	71° 20' E.	
Vera Cruz	Am.	New Spain	G. Mexico	45° 25' N.	12° 10' E.	
C. Verd	Am.	Negroland	Atl. Ocean	19° 00' N.	97° 53' W.	
Uhma	Africa	Sweden	G. Bothnia	14° 43' N.	17° 05' W.	
Vicegapatam	Eu.	India	B. Bengal	63° 45' N.	21° 10' E.	
C. Victory	Am.	Patagonia	South Ocean	17° 30' N.	84° 02' E.	
Vienna	Eu.	Germany	R. Danube	52° 05' S.	72° 50' W.	
Vigo	Eu.	Spain	Atl. Ocean	48° 13' N.	16° 28' E.	
B. St. Vincent	Am.	Paraguay	Atl. Ocean	42° 00' N.	8° 35' W.	
C. St. Vincent	Eu.	Portugal	Atl. Ocean	23° 55' S.	45° 11' W.	
I. St. Vincent	Africa	C. Verd	Atl. Ocean	36° 41' N.	8° 42' W.	
I. St. Vincent	Am.	Caribbe	Atl. Ocean	17° 17' N.	23° 44' W.	
R. St. Vincent	Am.	Guinea	Eth. Ocean	13° 05' N.	61° 05' W.	
C. Virgin Mary	Am.	Patagonia	Atl. Ocean	4° 30' N.	7° 41' W.	
L. Virgins	Am.	Antil. Is.	Atl. Ocean	52° 15' S.	64° 20' W.	
C. Volo	Eu.	Turkey	Archipelago	18° 28' N	64° 41' E.	
R. Voltas	Africa	Guinea	Atl. Ocean	39° 07' N.	23° 23' E.	
C. Voltas	Africa	Caffres	Atl. Ocean	5° 52' N.	1° 10' E.	
				28° 04' S.	16° 18' E.	

Upd.

Names of Places.	Cont.	Countries.	Coast.	Latitude.	Longitude.	H. Water.
Uplal	Eu.	Sweden	R. Sala	59 52 N.	17 50 E.	
Uraniberg	Eu.	Denmark	Baltic Sea	55 54 N.	12 58 E.	
I. Uthant	Eu.	France	Eng. Channel	48 30 N.	4 55 W.	4h. 30m.
I. Uffica	Eu.	Italy	Medit. Sea	38 43 N.	13 33 E.	
I. Vulcano	Eu.	Italy	Medit. Sea	38 29 N.	15 33 E.	
W						
R. Wager	Am.	New Wales	Hudson's Bay	61 05 N.	89 37 W.	
C. Wallingham	Am.	New Britain	Hudf. Straits	62 39 N.	77 48 W.	12 00
Wardhus	Eu.	Lapland	North Ocean	70 35 N.	32 45 E.	
Warsaw	Eu.	Poland	R. Vistula	52 14 N.	21 10 E.	
Waterford	Eu.	Ireland	St. Geo. Ch.	52 07 N.	7 52 W.	6 30
Waiting Isle	Am.	Bahama	Atl. Ocean	23 42 N.	74 22 W.	
Wells	Eu.	England	Germ. Ocean	53 07 N.	1 00 E.	6 00
Western Isles	Afia	Diemen's land	South Ocean	42 50 S.	138 46 E.	
Western Isles { S. pt. N. pt.	Eu.	Scotland	West. Ocean	56 46 N.	7 40 W.	
Is. Westman	Eu.	Iceland	West. Ocean	58 35 N.	6 37 W.	
Is. Westrol	Eu.	Iceland	North Ocean	63 55 N.	17 30 W.	
Wexford	Eu.	Ireland	St. Geo. Ch.	52 13 N.	7 16 W.	
Weymouth	Eu.	England	Eng. Channel	52 40 N.	2 34 W.	7 00
Whale's Back	Eu.	Iceland	West Ocean	63 44 N.	17 05 W.	
Whale's Head	Eu.	Greenland	North Ocean	77 18 N.	21 30 E.	
Whale Rock	Eu.	Azores	Atl. Ocean	39 49 N.	22 35 W.	
Whitby	Eu.	England	Germ. Ocean	54 30 N.	0 50 W.	3 00
Whitehaven	Eu.	England	Irish Sea	54 25 N.	3 15 W.	
Wicklow	Eu.	Ireland	St. Geo. Ch.	52 50 N.	6 30 W.	
Window	Eu.	Courland	Baltic Sea	57 08 N.	22 20 E.	
W. end	Eu.	England	Eng. Channel	50 47 N.	1 25 W.	
S. end	Eu.	England	Eng. Channel	50 34 N.	1 25 W.	0 00
E. end	Eu.	England	Eng. Channel	50 41 N.	1 09 W.	
W. end	Eu.	England	Eng. Channel	50 41 N.	1 46 W.	
Is. Prince William	Afia	—	South Sea	16 45 S.	177 55 E.	0 45
Winchelsea	Eu.	England	Germ. Ocean	50 58 N.	0 50 E.	9 00
Wintertone's	Eu.	Sweden	Baltic Sea	53 02 N.	1 22 E.	
Wibuy in I. Gotland	Eu.	Sweden	Baltic Sea	57 40 N.	19 50 E.	
C. Wrath, or Faro head	Eu.	Scotland	West. Ocean	58 44 N.	4 50 W.	
Wybourg	Eu.	Finland	G. Finland	60 55 N.	30 20 E.	
Y						
Yamboa	Afia	Arabia	Red Sea	24 25 N.	38 54 E.	
Yarmouth	Eu.	England	Germ. Ocean	52 53 N.	1 40 E.	9 45
Yas de Amber	Africa	Zanguebar	Indian Ocean	0 00	47 15 E.	
Yellow River	Afia	China	South Sea	34 06 N.	120 10 E.	
Ylo	Am.	Peru	South Sea	17 16 S.	71 08 W.	
York Fort	Am.	New Wales	Hudson's Bay	57 14 N.	92 57 W.	9 10
York, New	Am.	New Engl.	Atl. Ocean	41 05 N.	74 51 W.	3 0
Youghall	Eu.	Ireland	St. Geo. Ch.	51 46 N.	7 46 W.	4 30
Z						
Zaratula	Am.	Mexico	South Sea	17 10 N.	105 00 W.	
I. Zant	Eu.	Italy	Adriat. Sea	37 50 N.	21 30 E.	
I. Zanzibar	Africa	Zanguebar	Indian Ocean	6 55 S.	40 10 E.	
Zaro	Eu.	Dalmatia	Medit. Sea	44 15 N.	16 55 E.	
NW. pt.	Afia	—	South Ocean	34 20 S.	165 40 E.	
C. Spigie	Afia	—	South Ocean	41 45 S.	162 50 E.	
Zenan city	Afia	Arabia	Inland	16 20 N.	47 44 E.	3 00
Zenic Sea	Eu.	Dutchland	Germ. Ocean			

Beside

Beside the times of high-water in the preceding table, the following times serve for coasts of considerable extent, and will nearly serve for the places on those coasts.

Finmark, or NNW. coast of Lapland, 1h. 30m. Jutland Isles 5h. om.
 Friesland coast 7 h. 30 m. Zeland coast 1 h. 30 m.
 Flanders coast 0 h. 00 m. Picardy and Normandy coasts 10 h. 30 m.
 Biscay, Gallician, and Portugal coasts 3 h. 00 m.
 Irish W. coast 3 h. 00 m. Irish S. coast 5h. 15m.
 Africa W. coast 3h. om. America W. coast 3 h. 0 m.
 America E. coast 4 h. 30 m.

LATITUDES and LONGITUDES of some places determined by the late
 TRANSIT of VENUS, June 3, 1764.

	Lat.	Long.
Batavia	6° 12' S.	106° 43' E.
Cadiz	36 31 N.	6 12 W.
Cajanebourg	64 13½ N.	27 50¼ E.
Cambridge	42 25 N.	71 10 W.
Fort Churchill	58 47½ N.	94 7 W.
Cape Francois	19 57 N.	72 10 W.
Gibraltar	36 5 N.	4 46 W.
Grypswald	54 4½ N.	13 21 E.
Gurjev	47 7 N.	52 7 E.
C. Hinlopen	38 47½ N.	75 3 W.
Jakutskoi	62 2 N.	129 52 E.
St. Joseph	23 3½ N.	109 35 W.
Kola	68 52½ N.	33 8 E.
Mexico	19 54 N.	100 5 W.
Noriton	40 10 N.	75 17½ W.
North Cape	71 10 N.	26 5 E.
Orenburg	51 46 N.	55 14 E.
Orsk	51 12½ N.	58 32¼ E.
Isle Otahite	17 29 S.	149 22 W.
Pekin	39 55 N.	116 31 E.
Petersburg	59 56 N.	30 28 E.
Ponoi	68 52½ N.	33 8 E.
Philadelphia	39 57 N.	75 13 W.
Quebec	48 47½ N.	71 12¼ W.
Fort Royal	14 36 N.	61 15 W.
Stockholm	59 20½ N.	18 8 E.
Umba	66 39½ N.	34 15 E.
Upsal	59 52 N.	17 43 E.
Isle Java		
Spain		
Finland		
New England.		
Hudson's Bay		
Isle Dominga		
Spain		
Pomerania		
Astracan		
Maryland		
Siberia		
California		
Lapland		
New Spain		
Pensylvania		
Lapland		
Astracan		
Astracan		
South Seas		
China		
Russia		
Lapland		
Pensylvania		
Canada		
Isle Martinique		
Sweden		
Russia		
Sweden		

END OF BOOK VI; AND OF VOL. I.

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